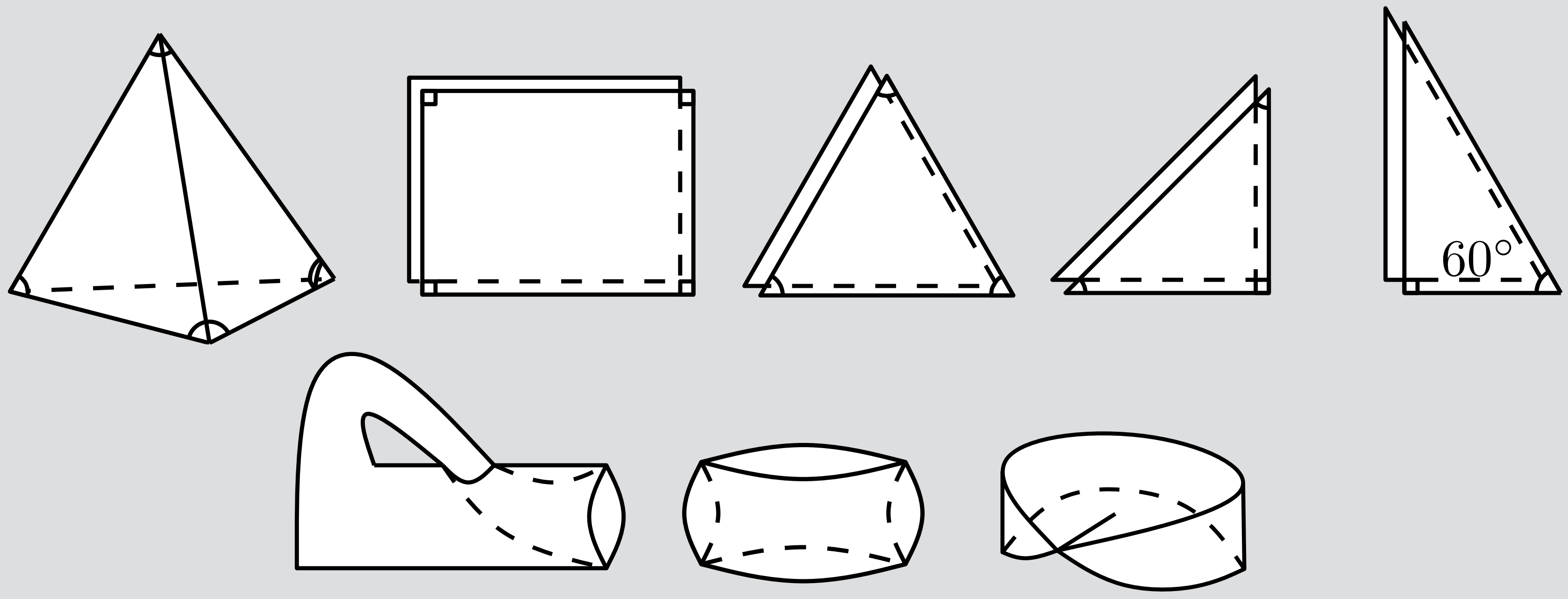
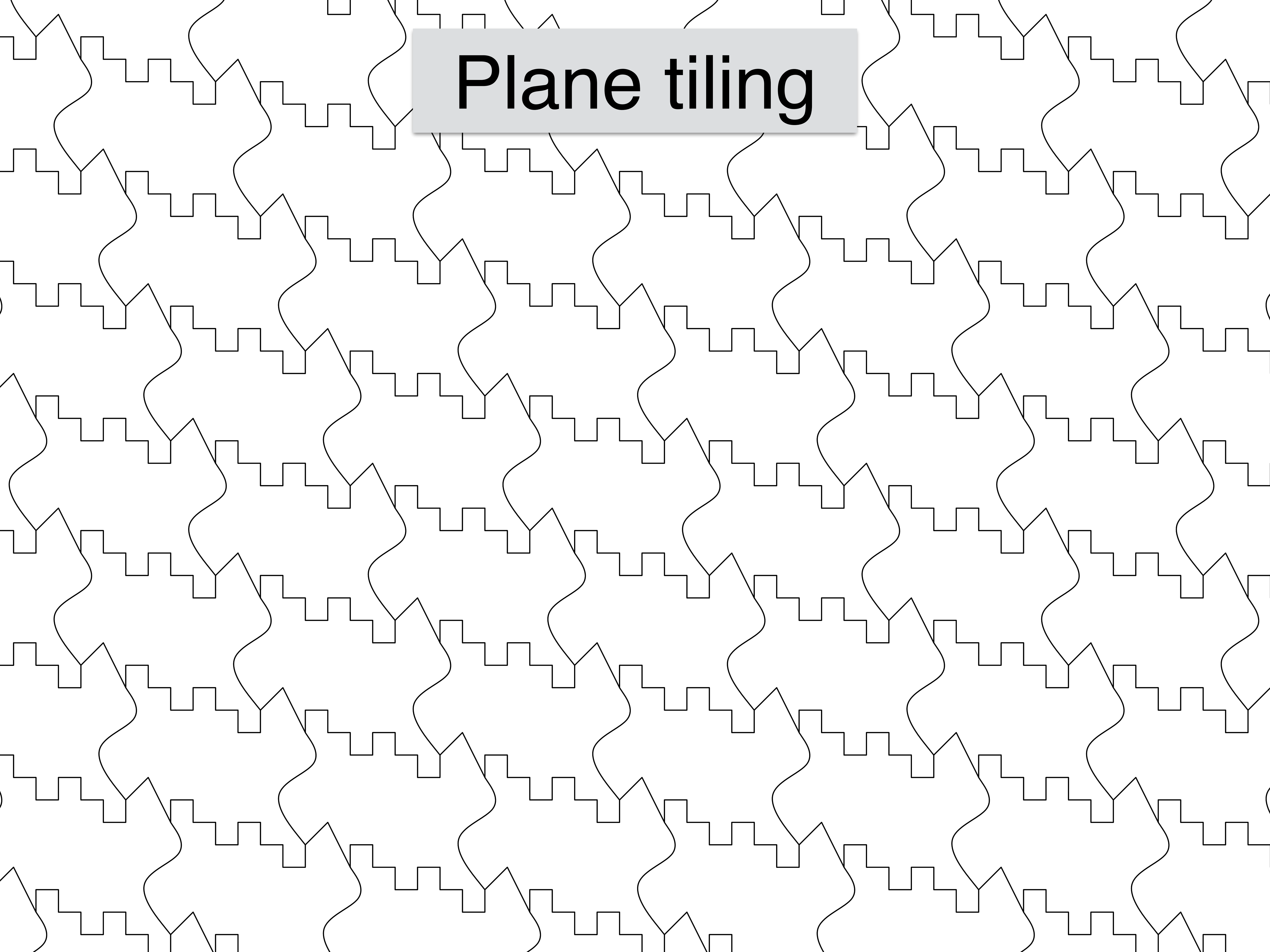
Some Results on Tile-makers



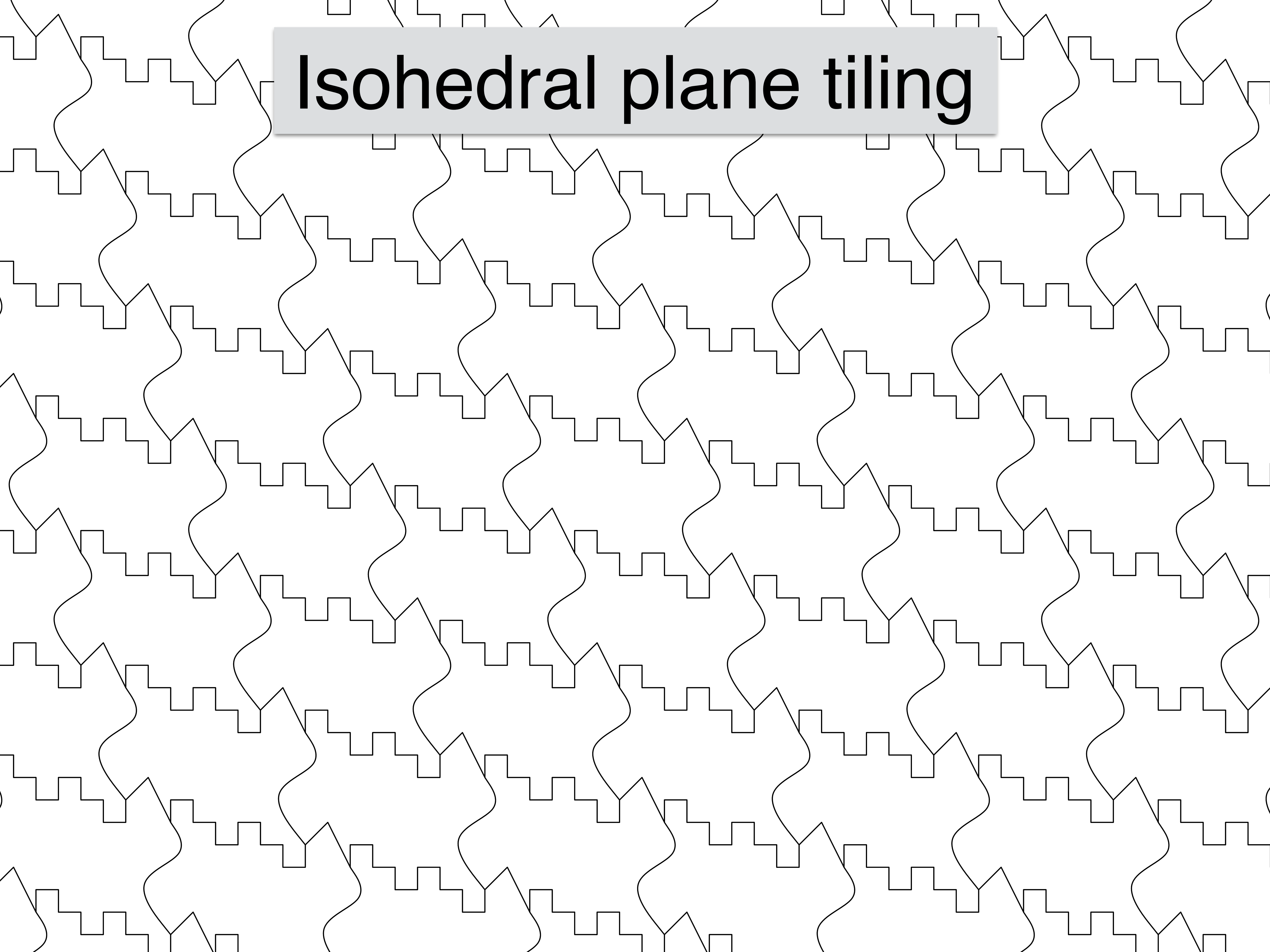
Stefan Langerman, Andrew Winslow

ULB UNIVERSITE LIBRE DE BRUXELLES

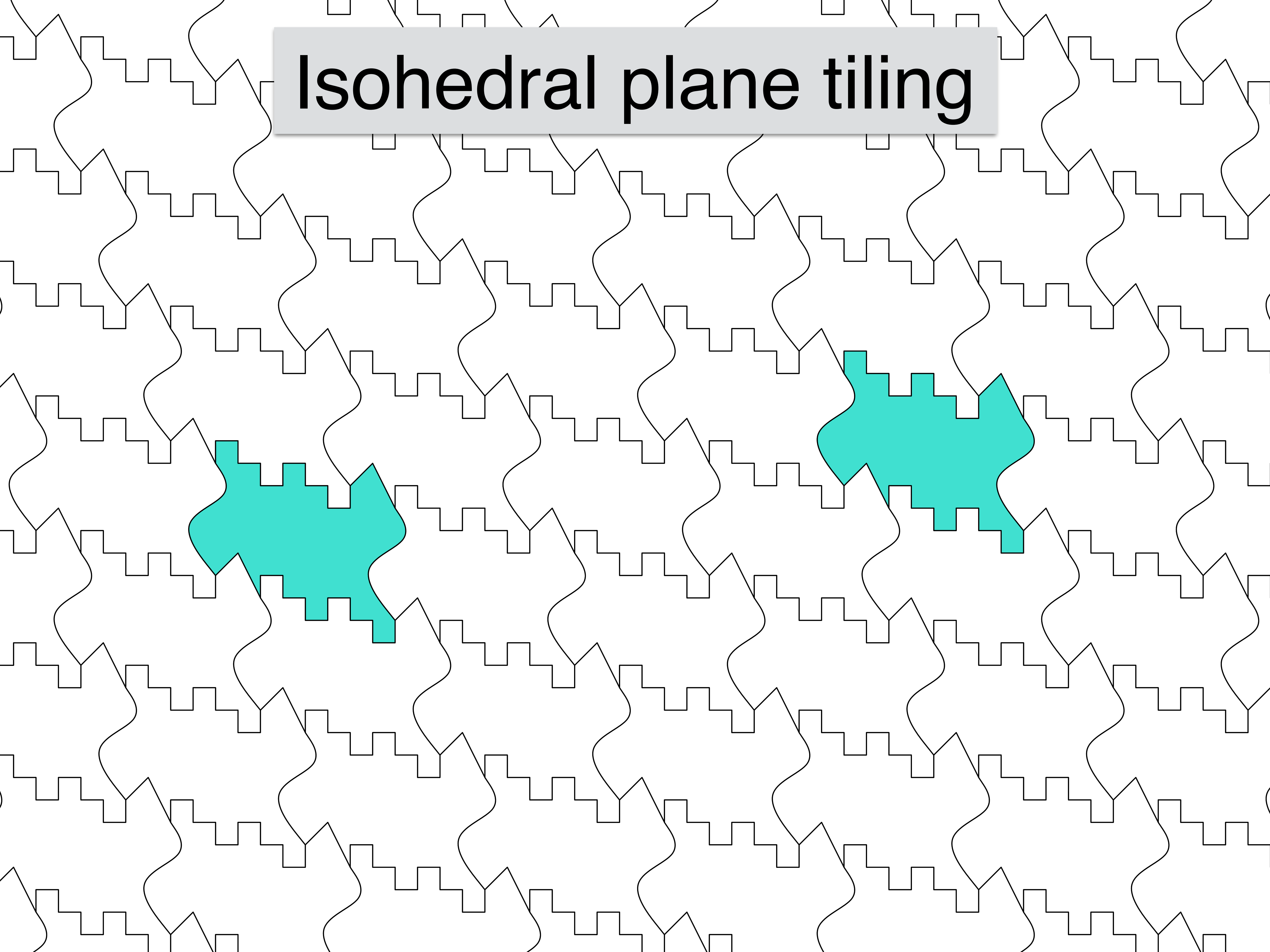
Plane tiling



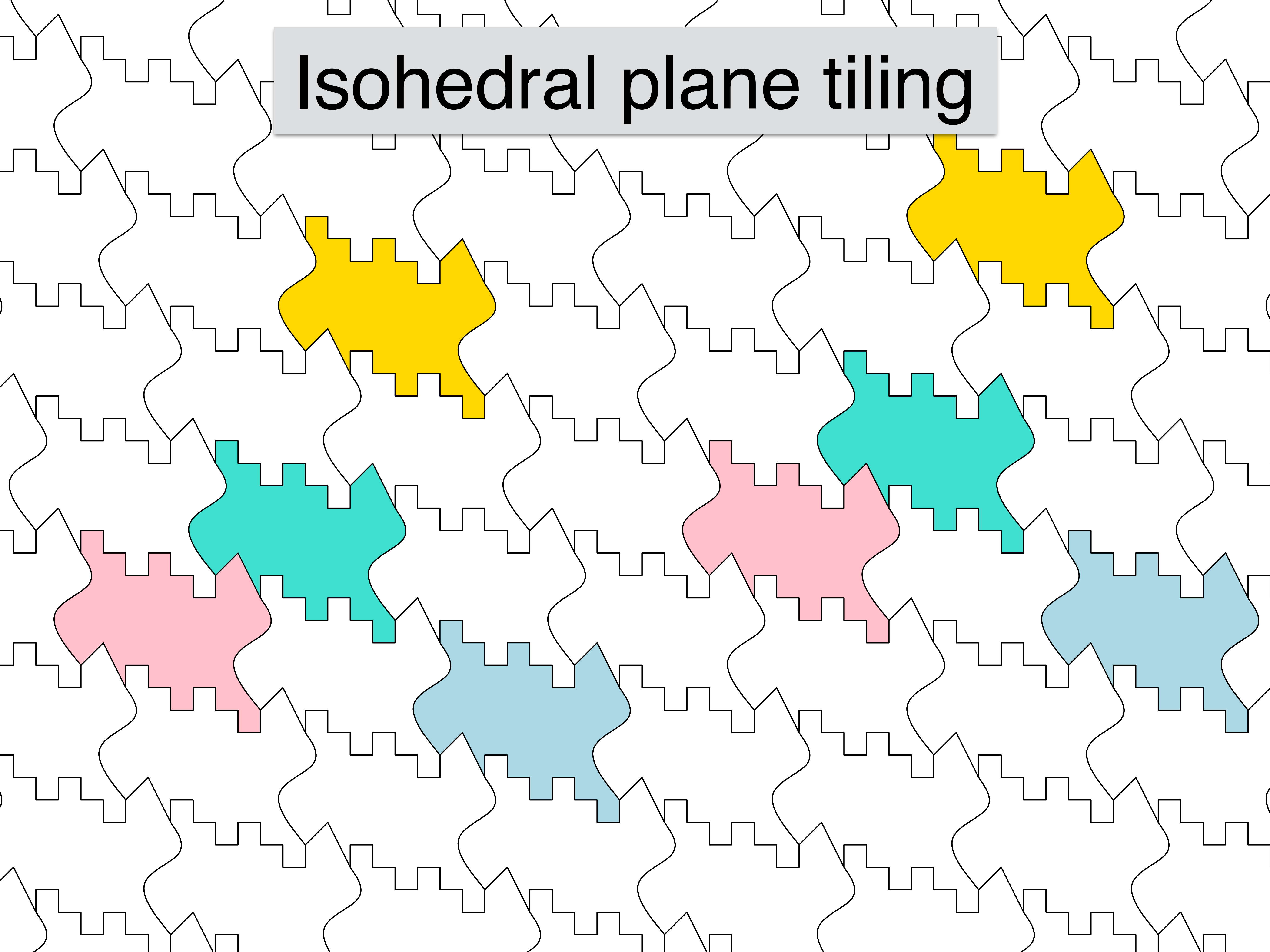
Isohedral plane tiling



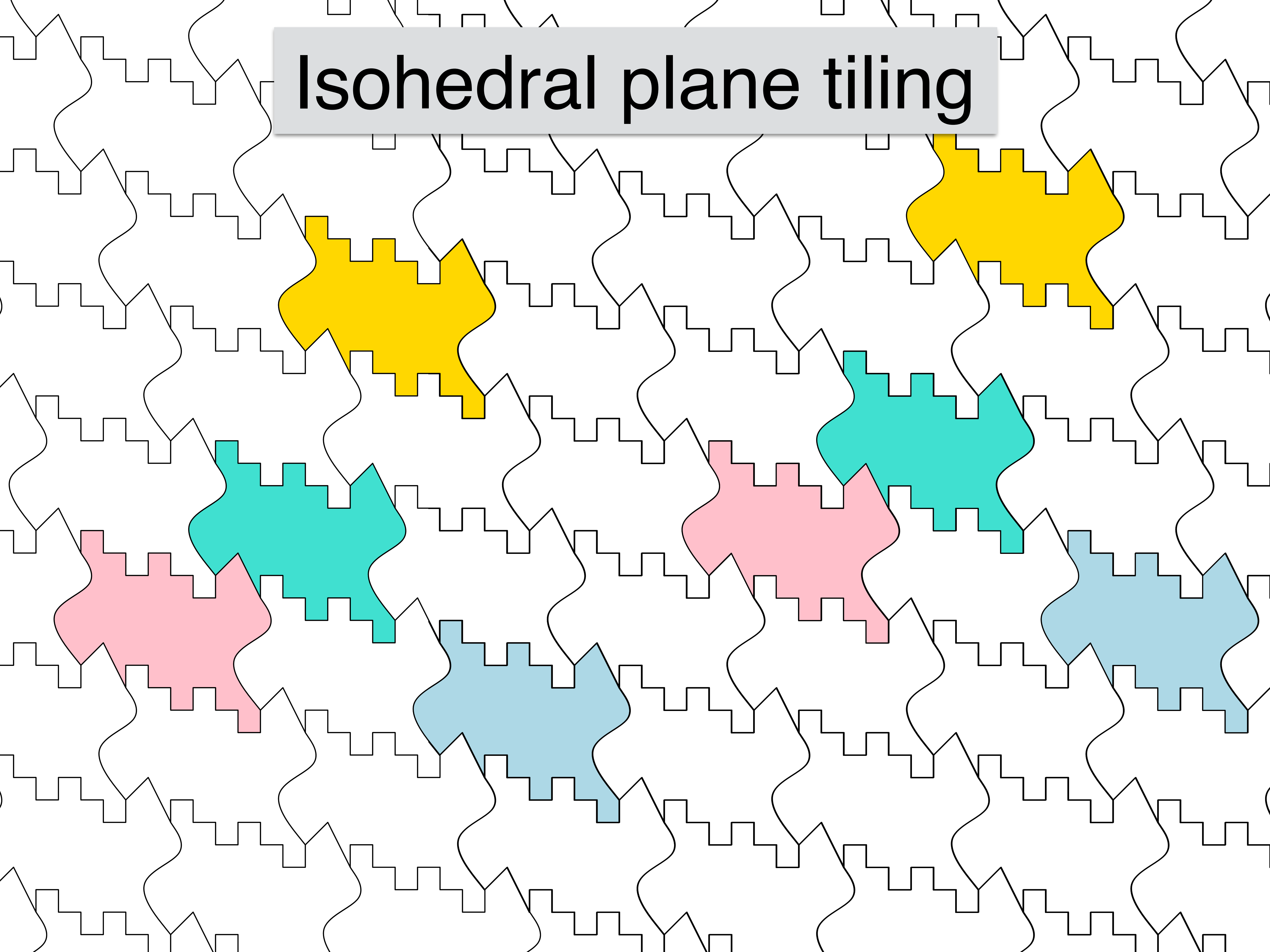
Isohedral plane tiling



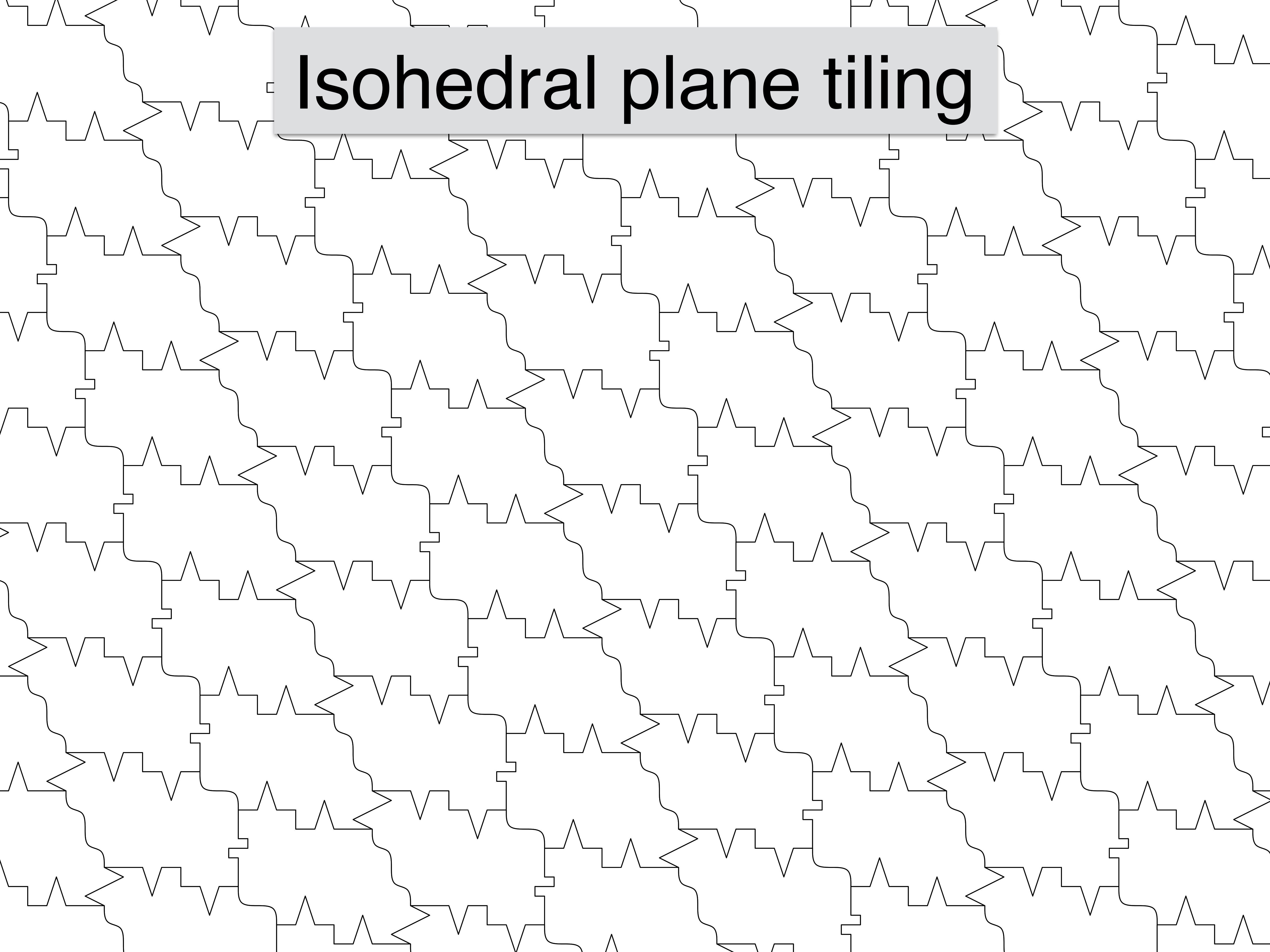
Isohedral plane tiling



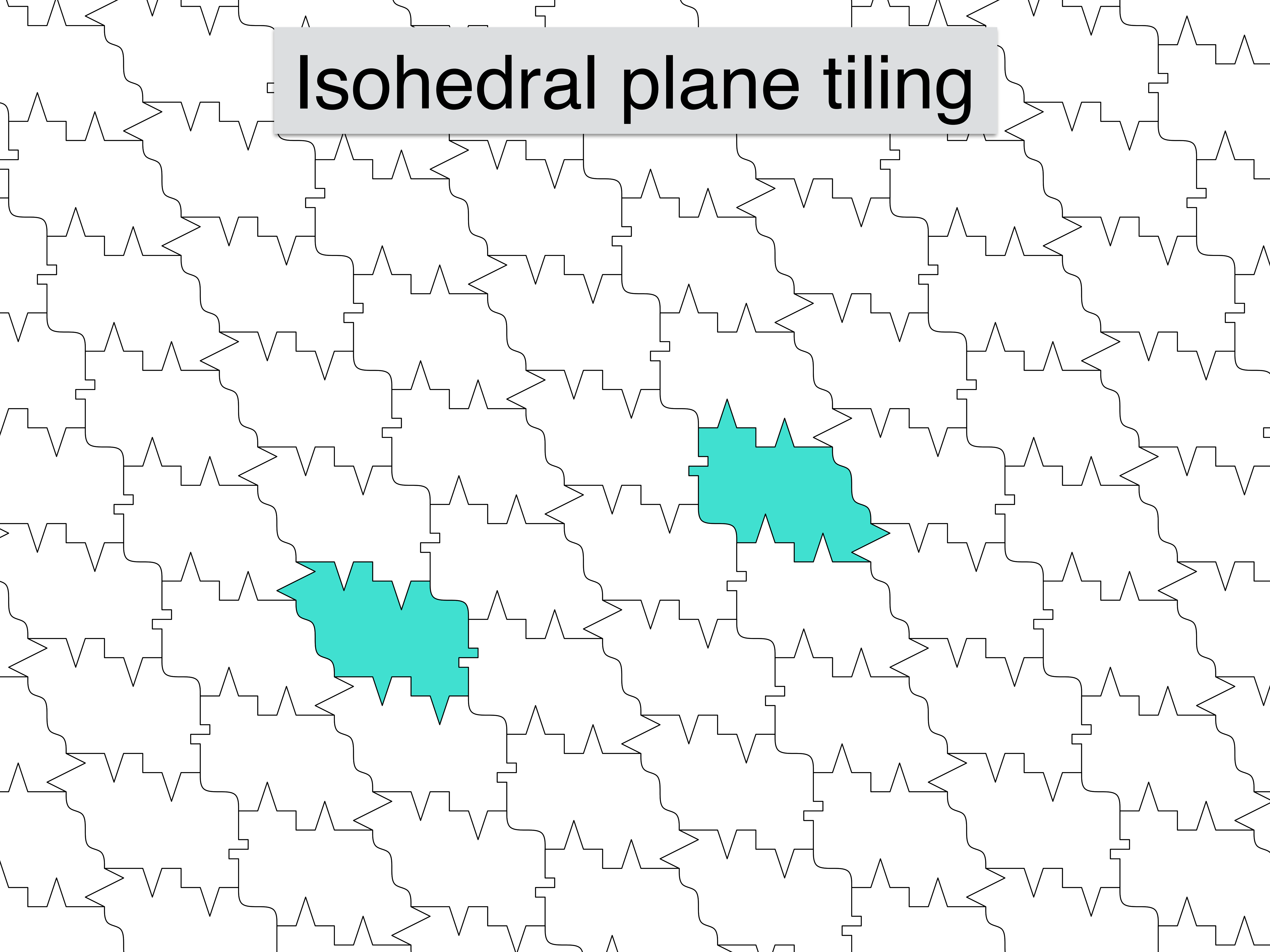
Isohedral plane tiling



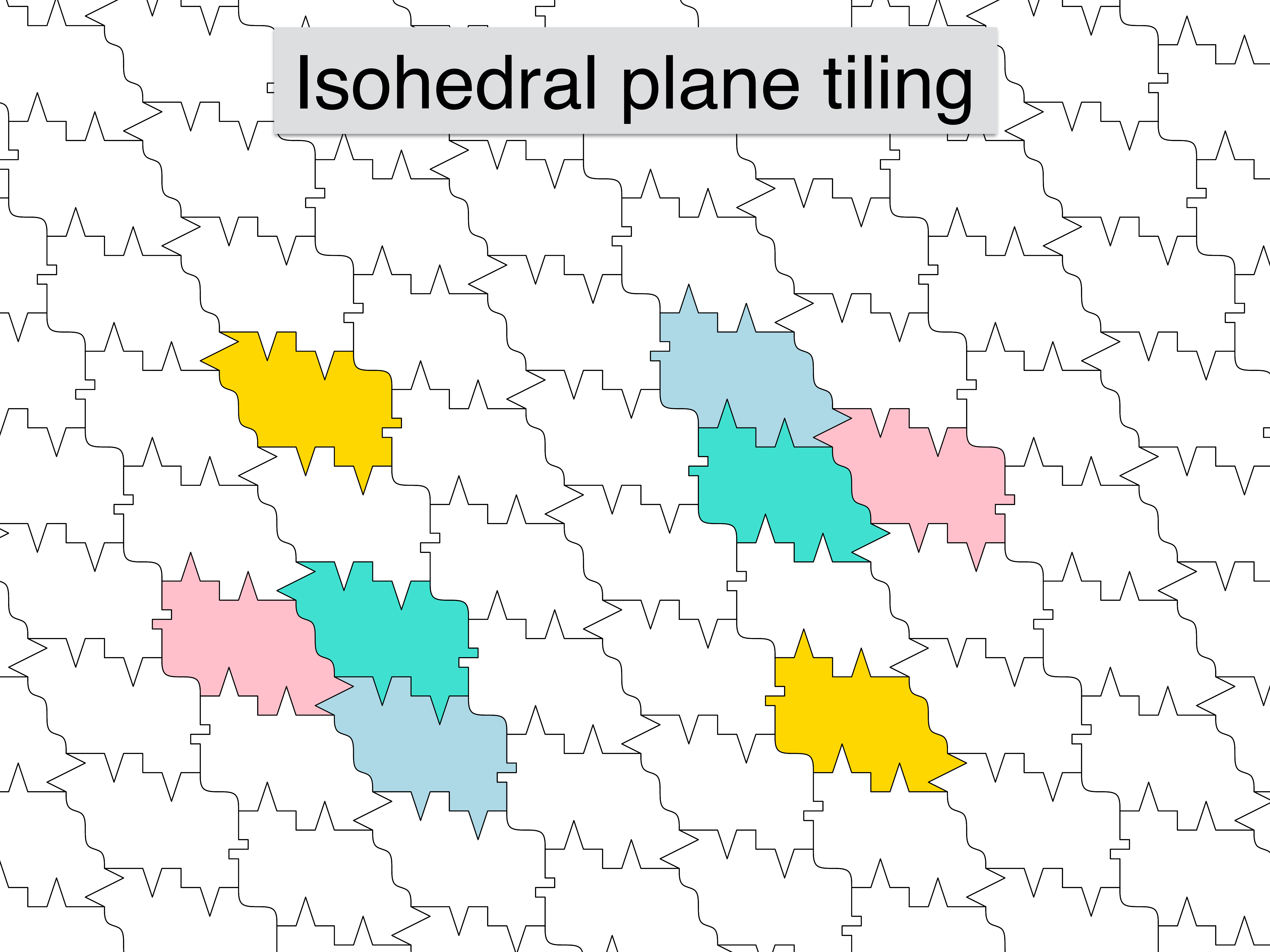
Isohedral plane tiling



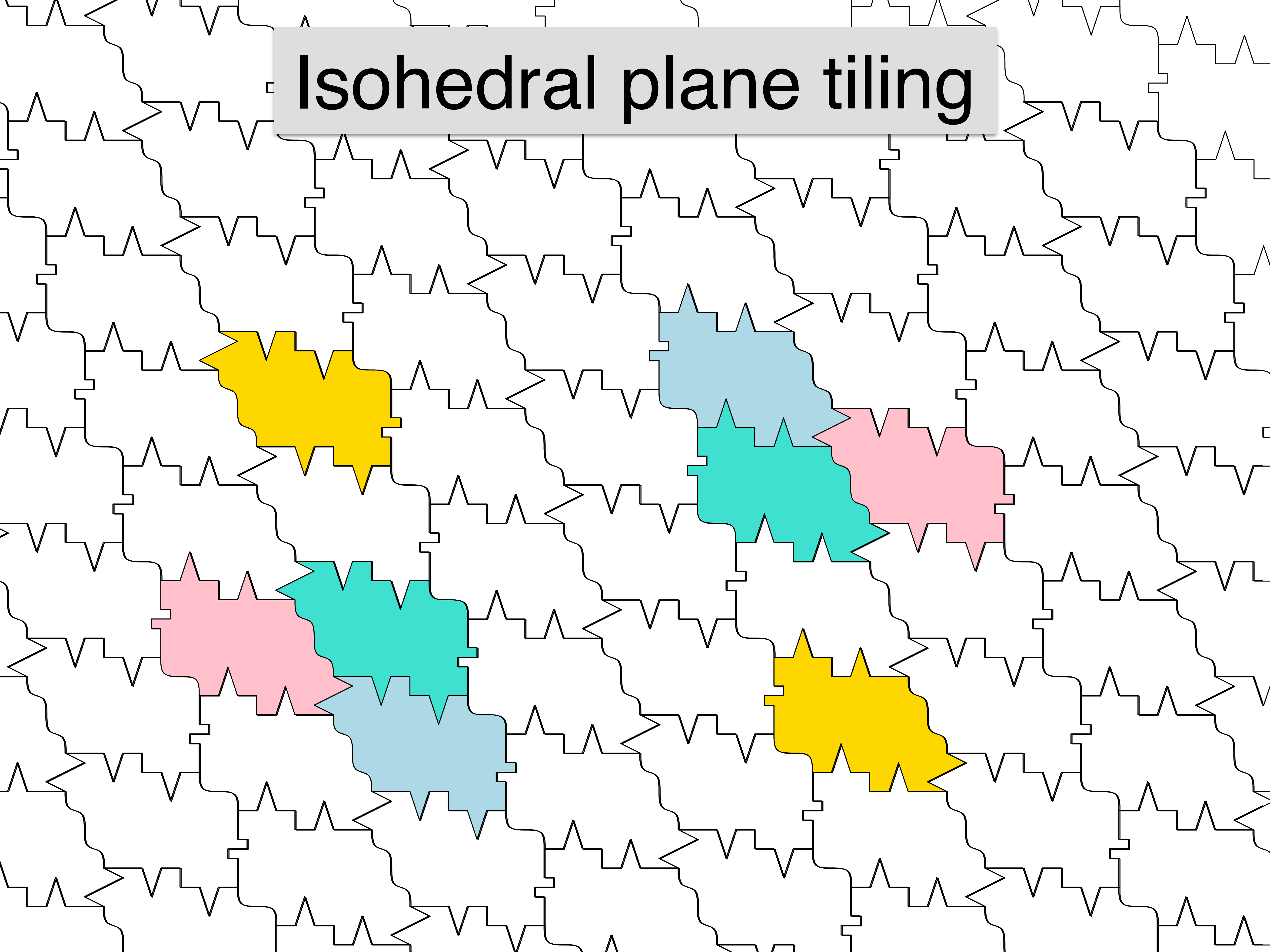
Isohedral plane tiling



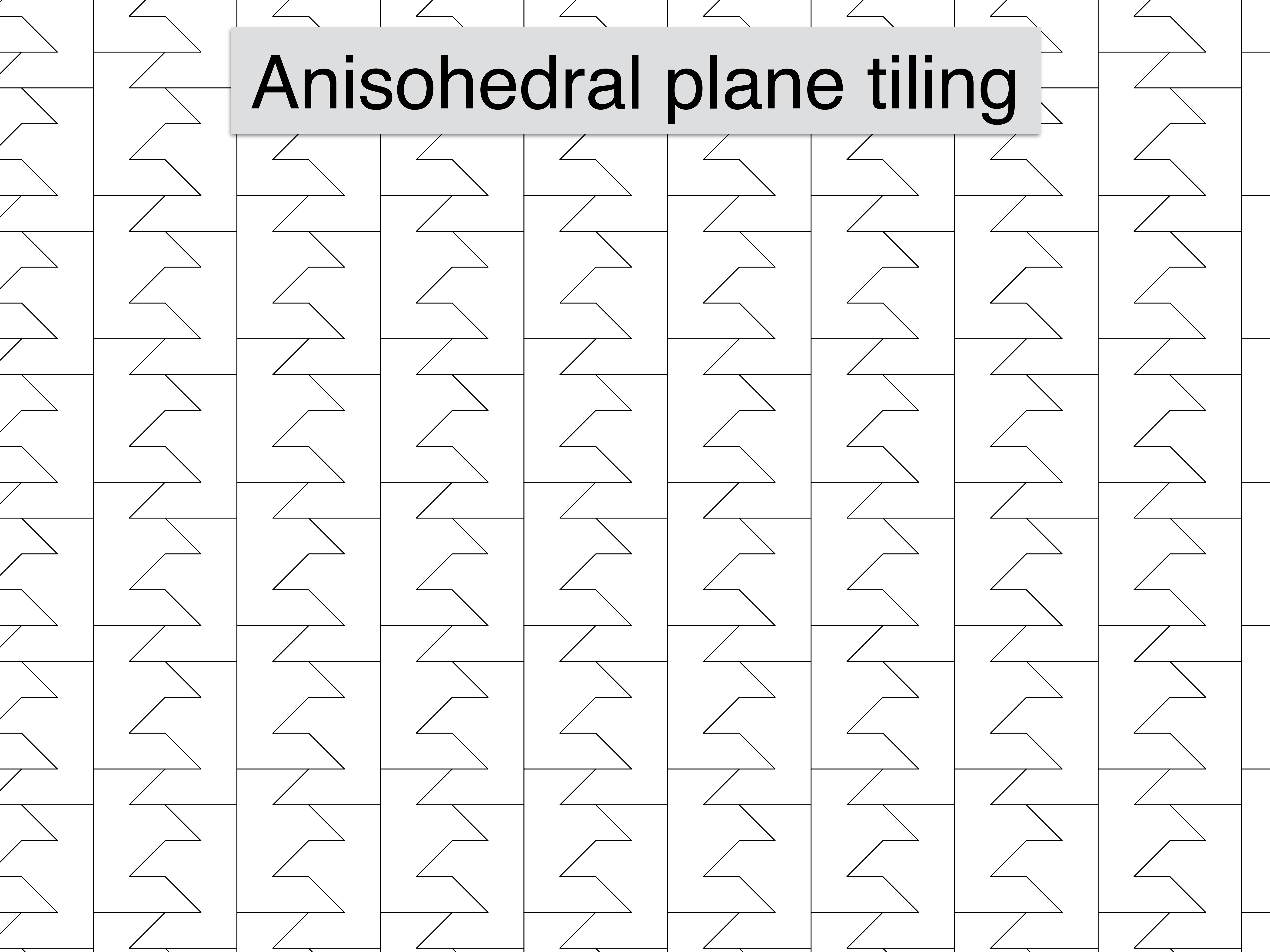
Isohedral plane tiling



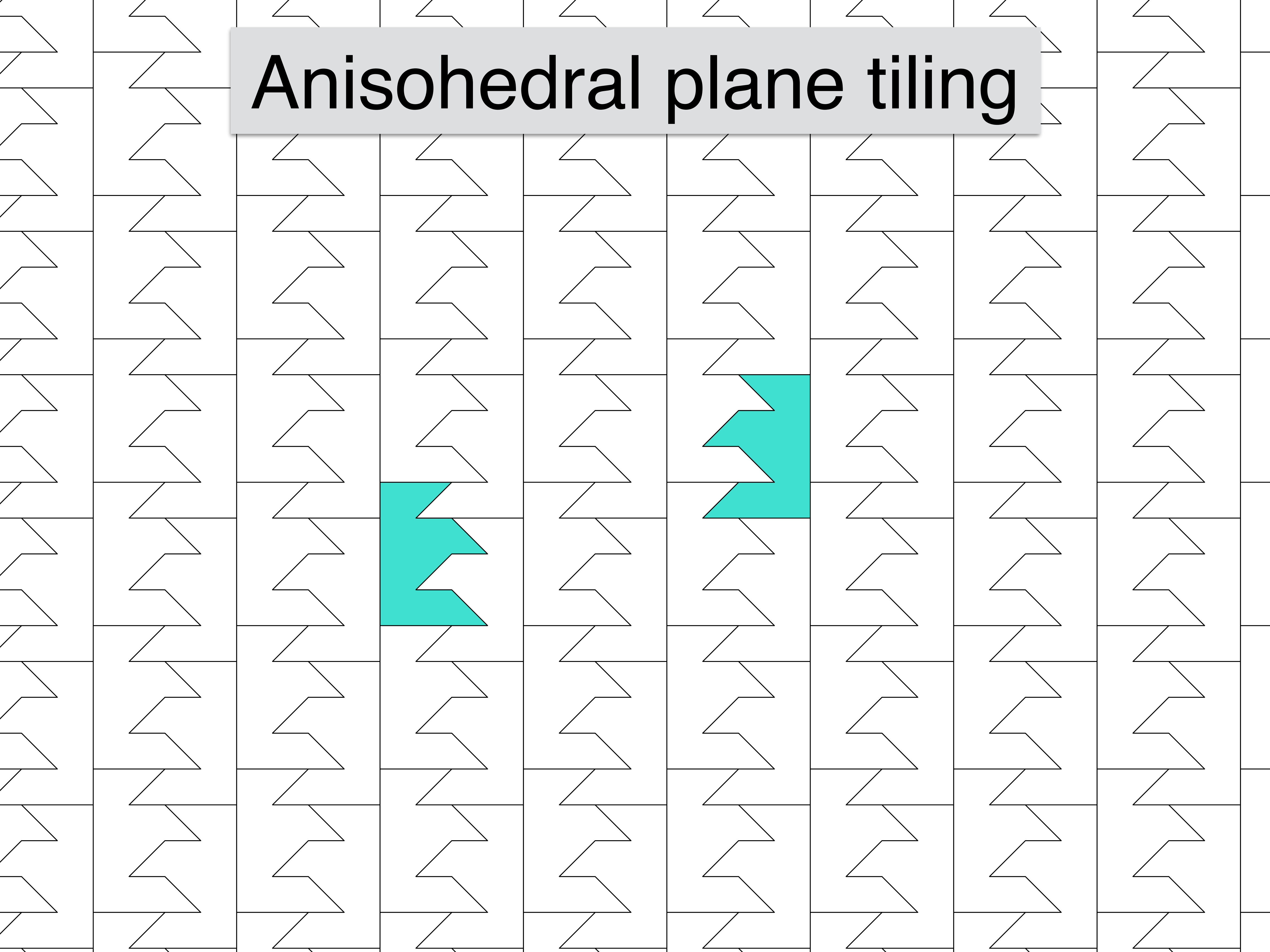
Isohedral plane tiling

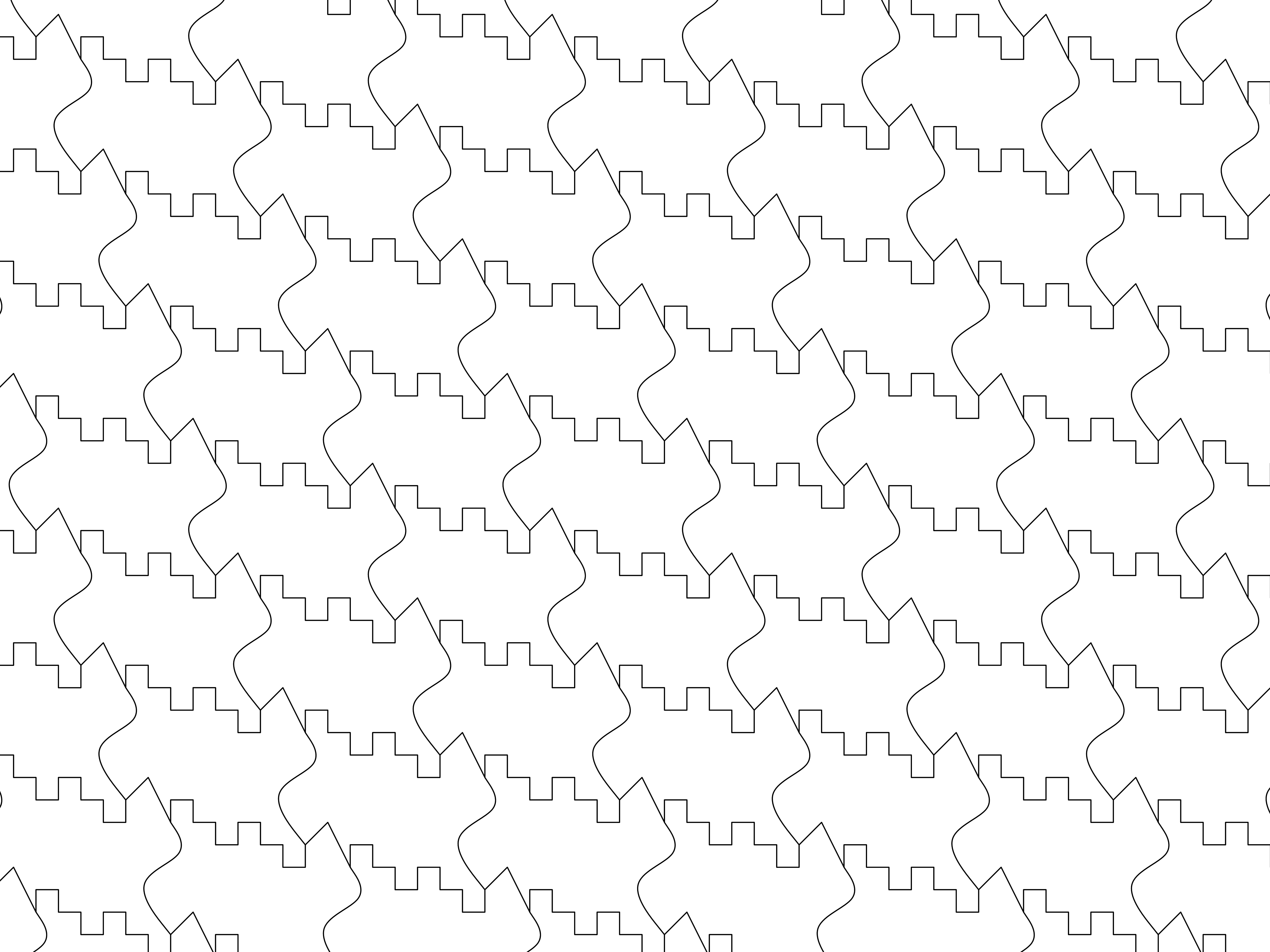


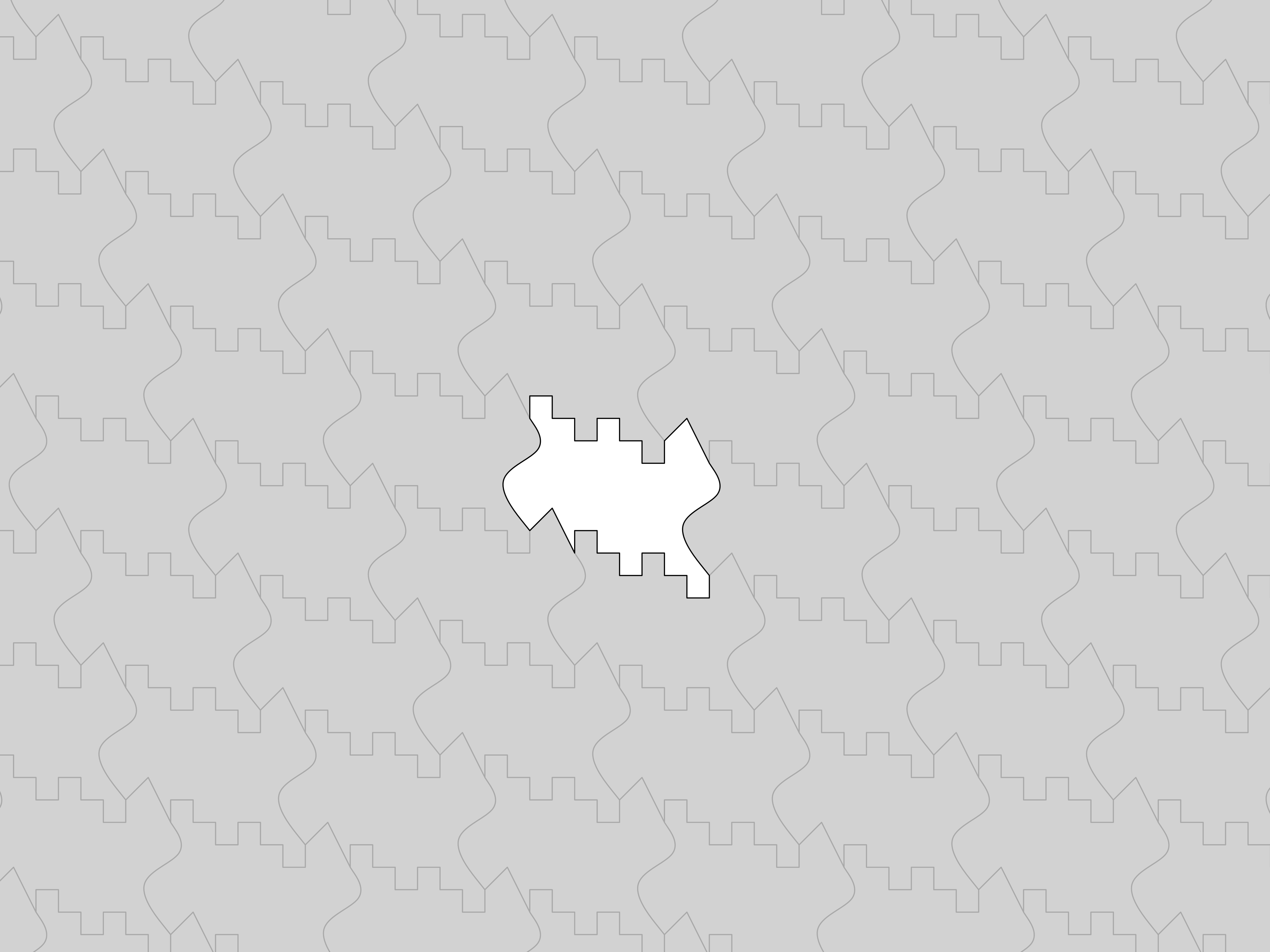
Anisohedral plane tiling

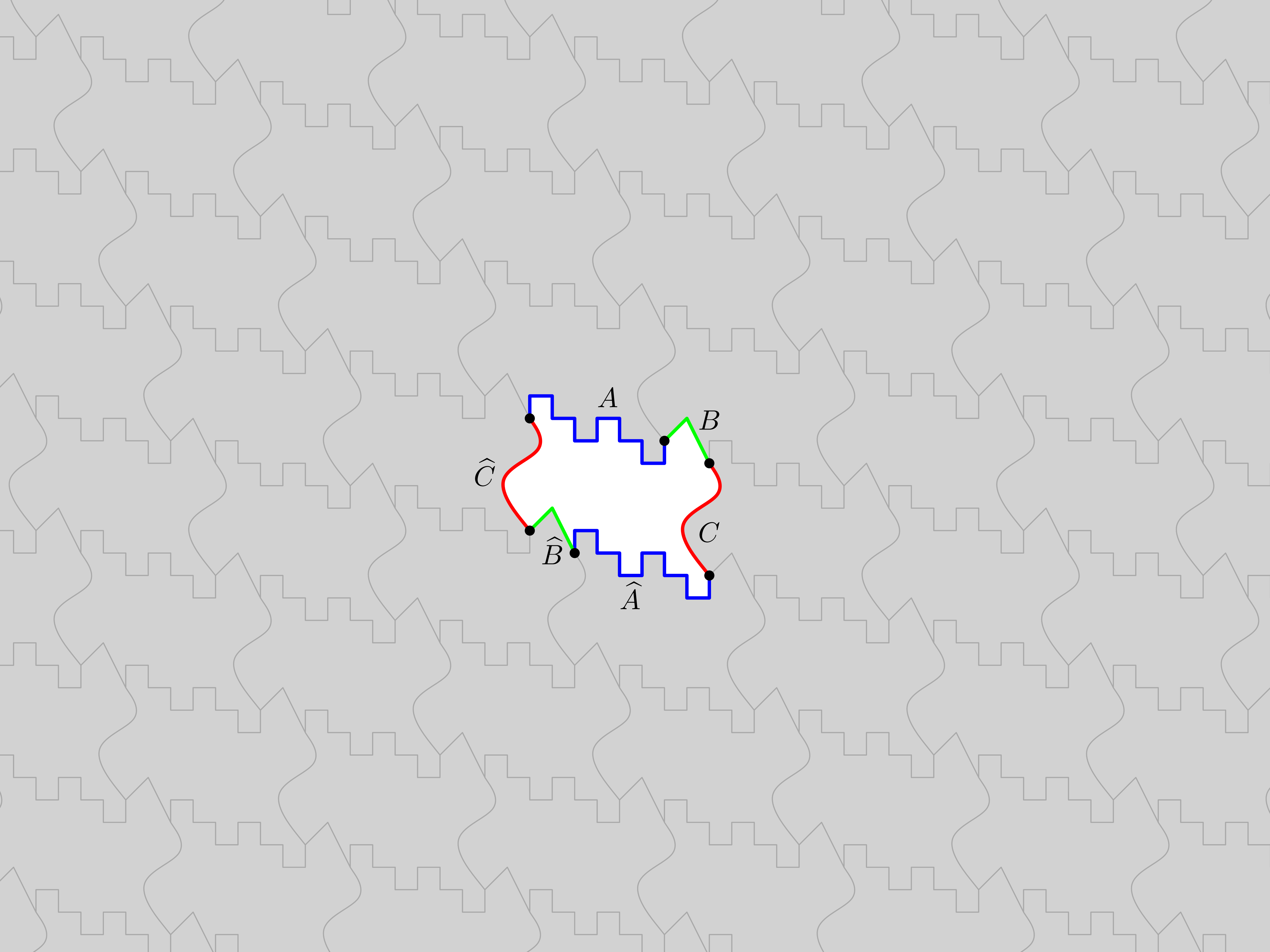


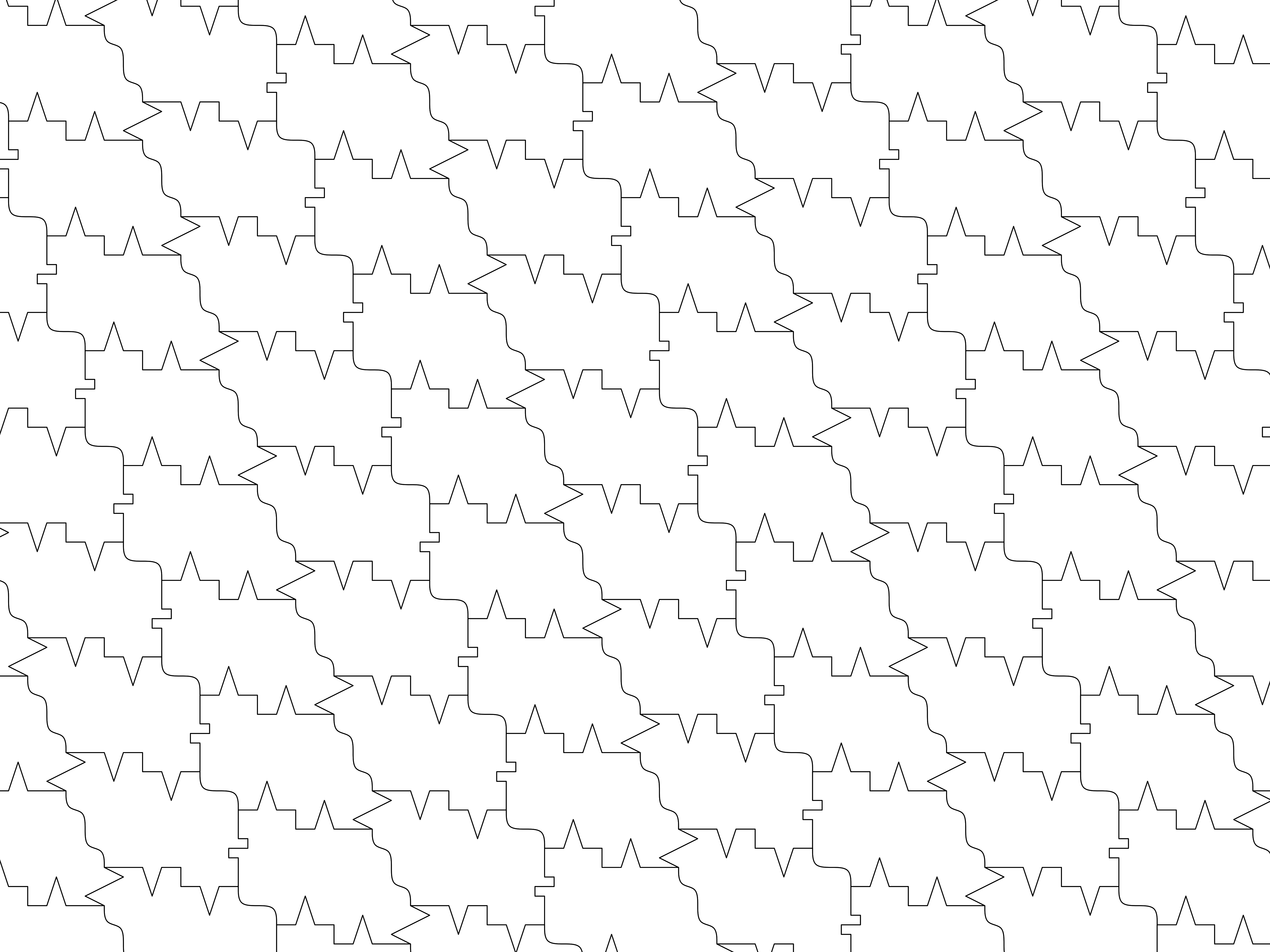
Anisohedral plane tiling

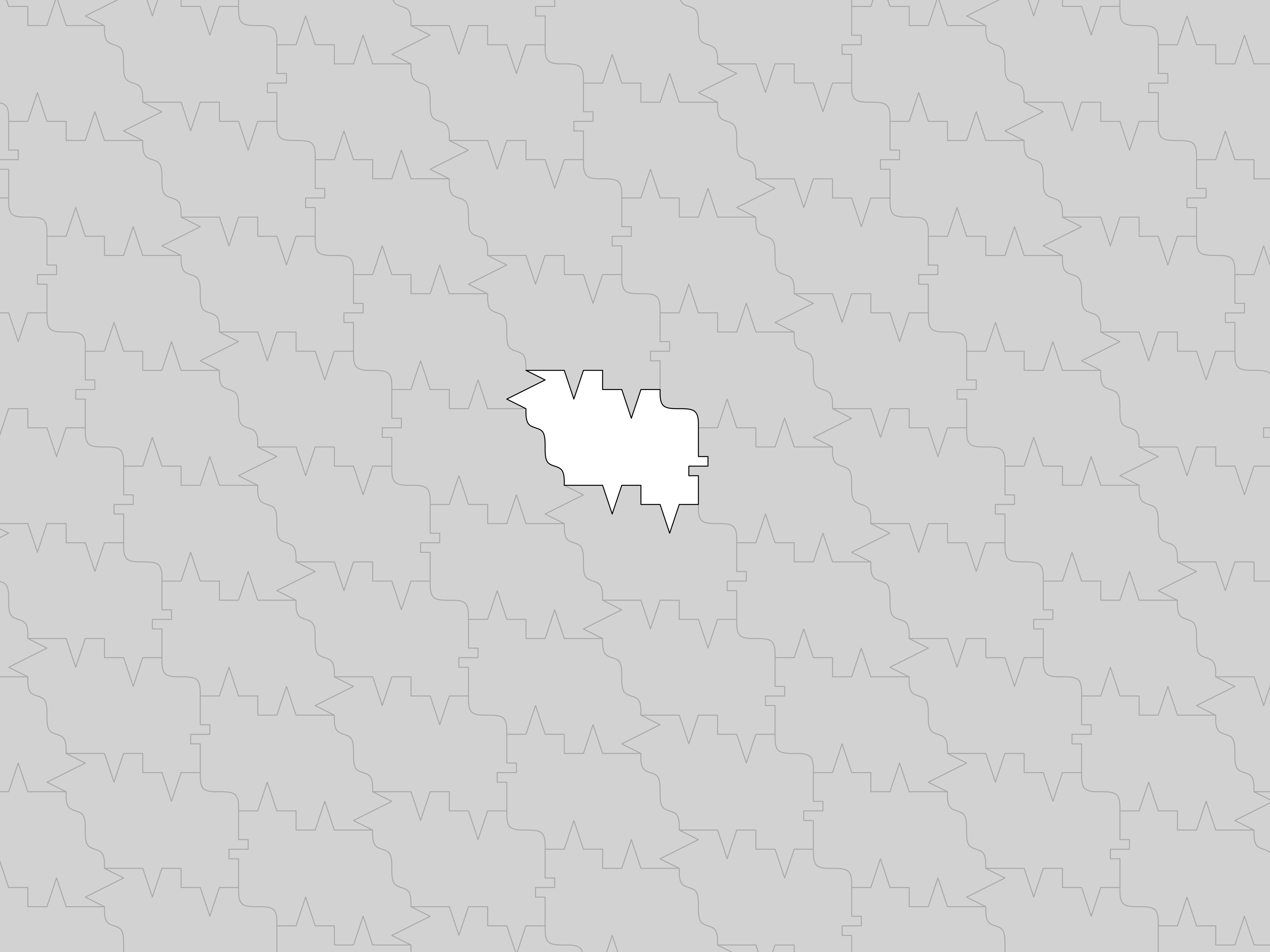


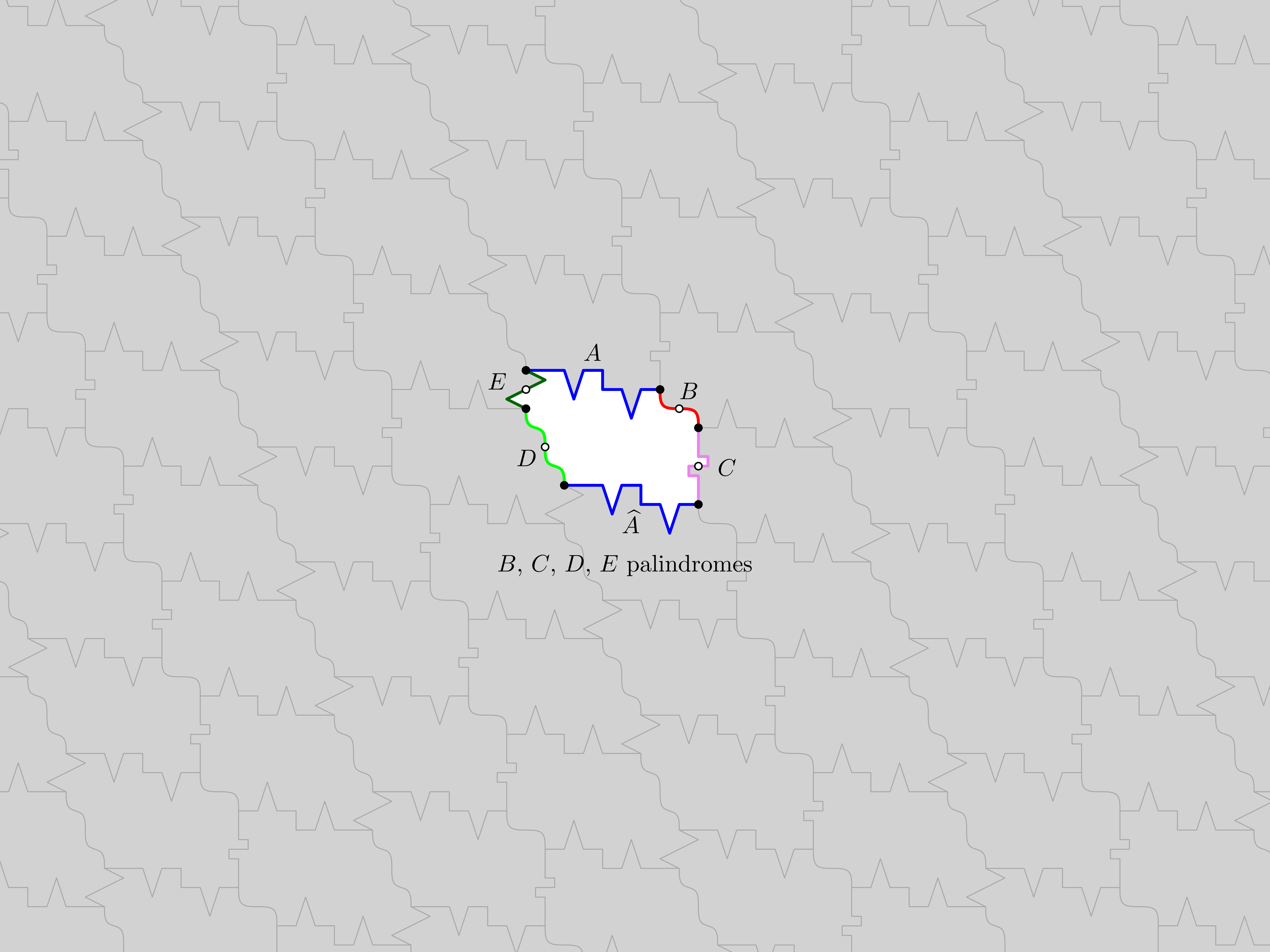





A
B
$\widehat{C}$
C
$\widehat{B}$
$\widehat{A}$

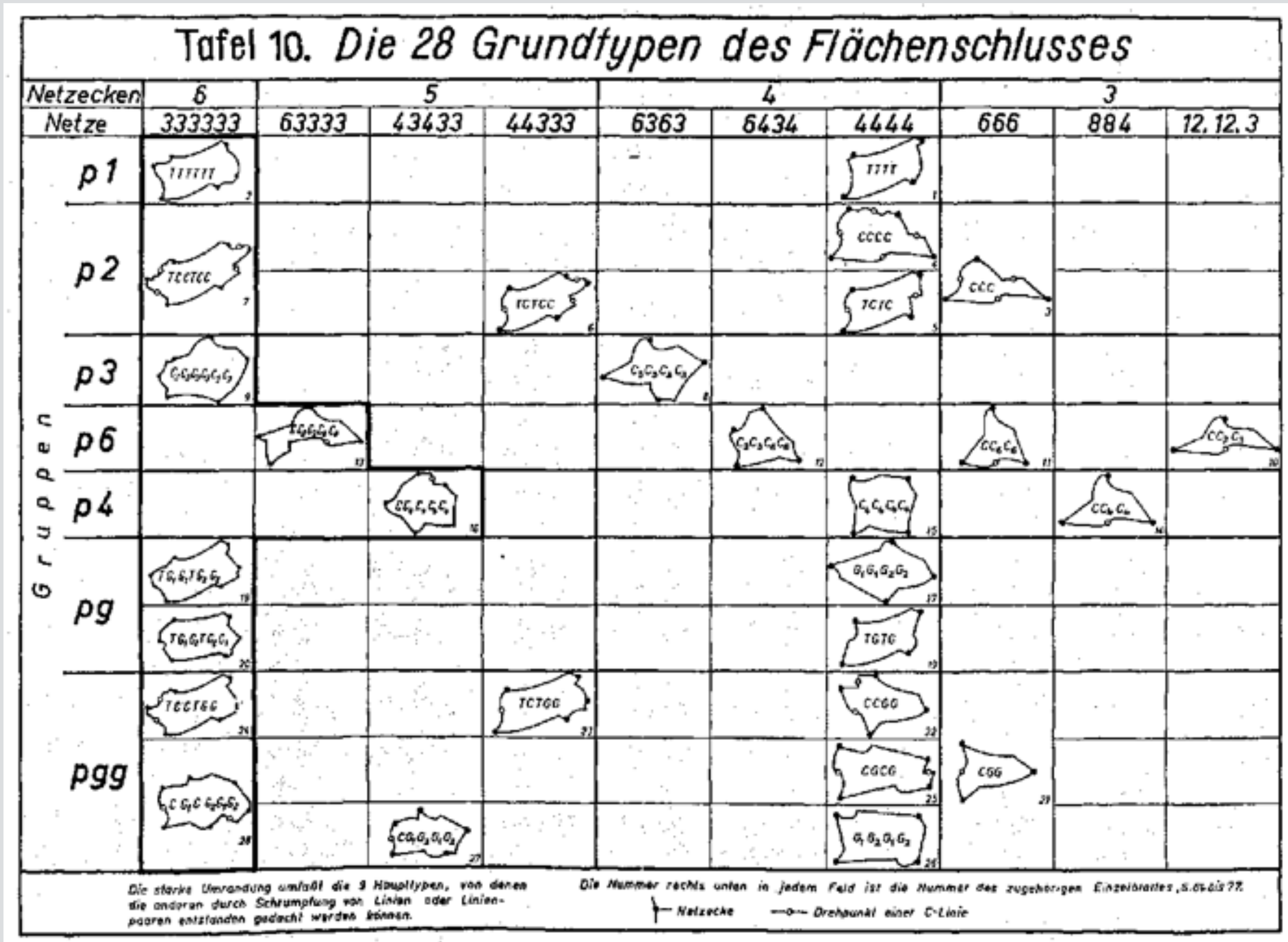




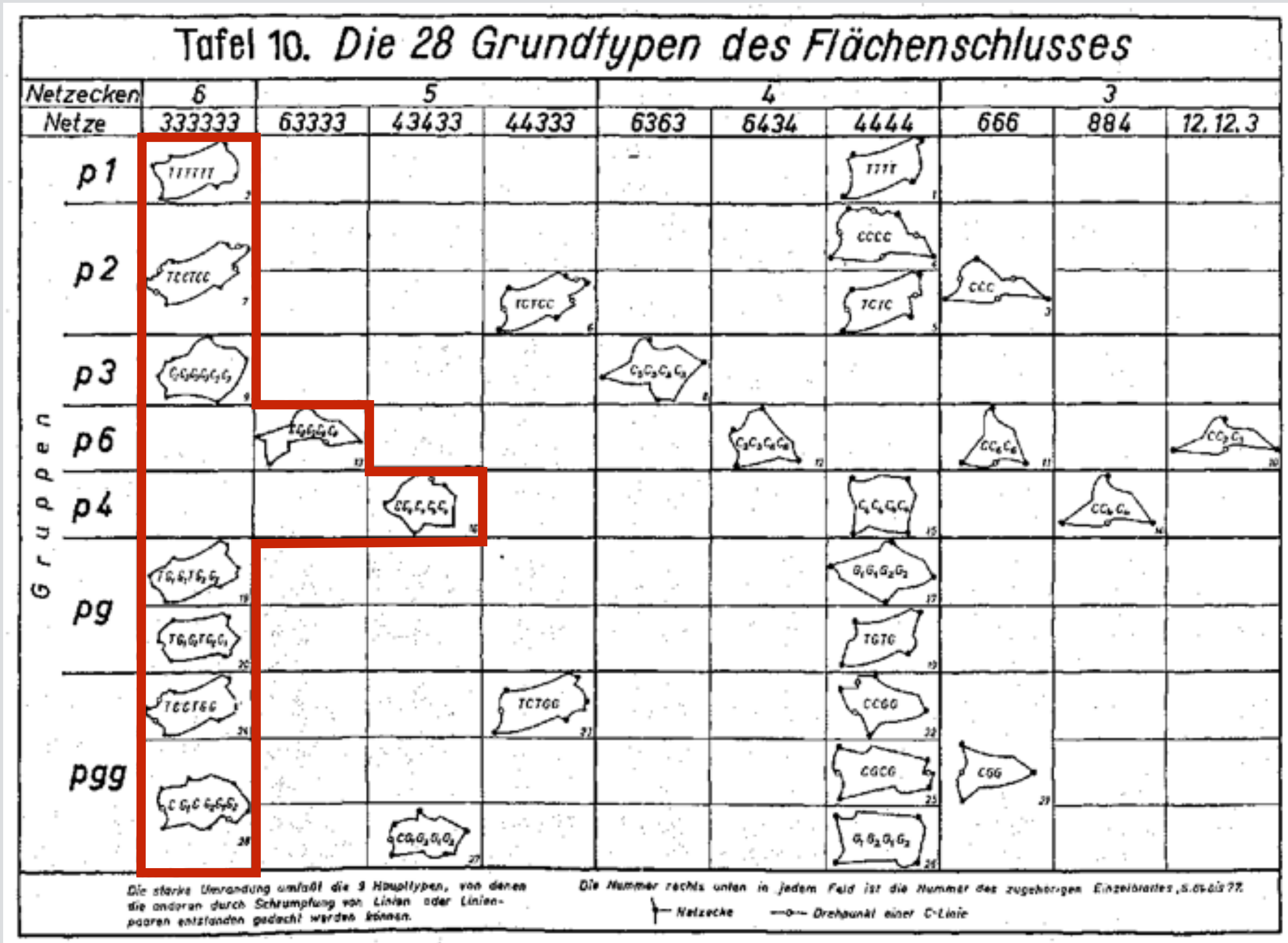


A
E
B
D
C
$\widehat{A}$
B, C, D, E palindromes

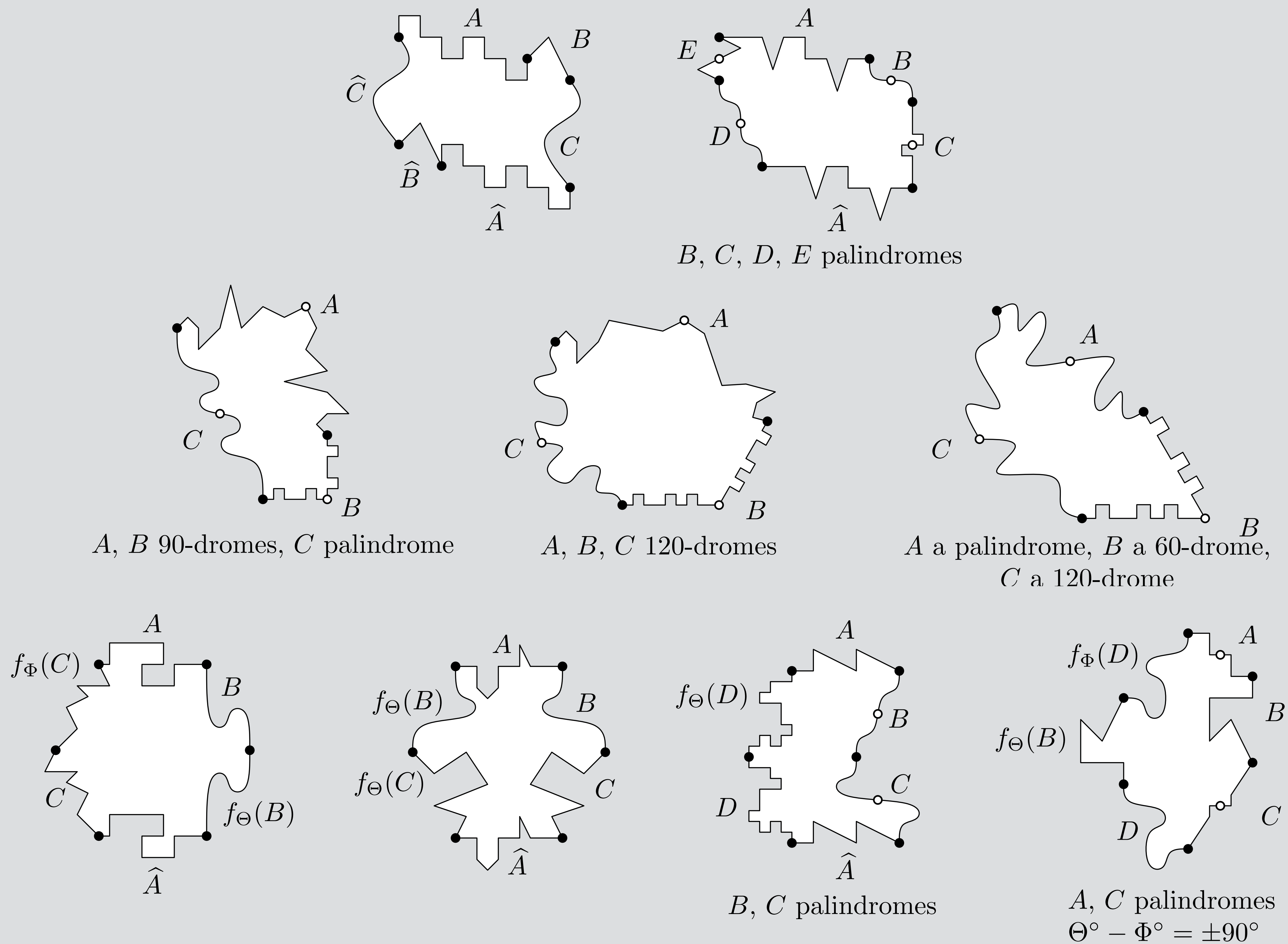
[Heesch, Kienzle 1963]



[Heesch, Kienzle 1963]

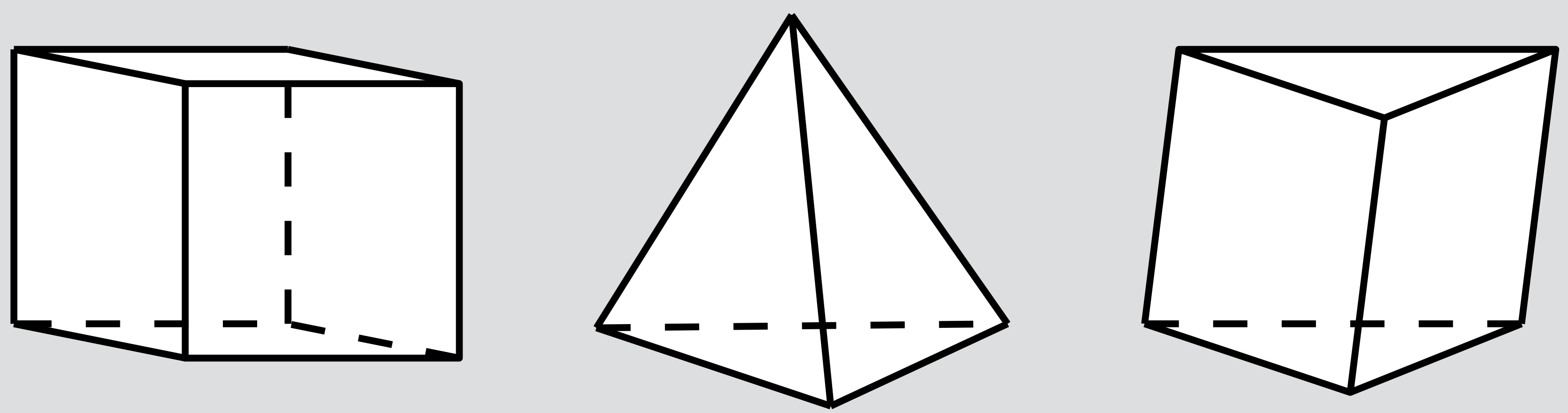


The nine isohedral tiling types



Surfaces

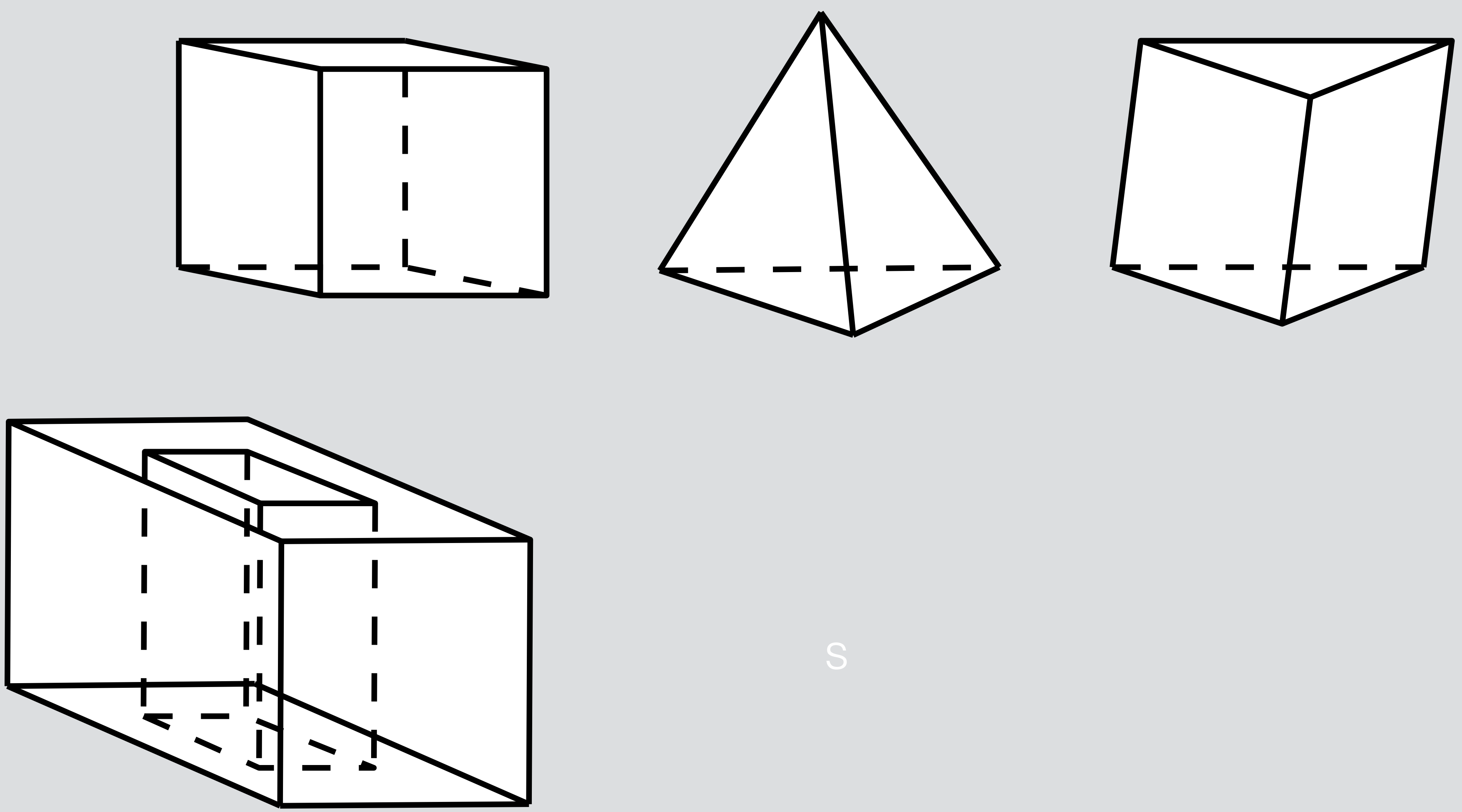
A <u>surface</u> is boundary-less, and possibly higher genus, non-orientable, or dihedral.



S

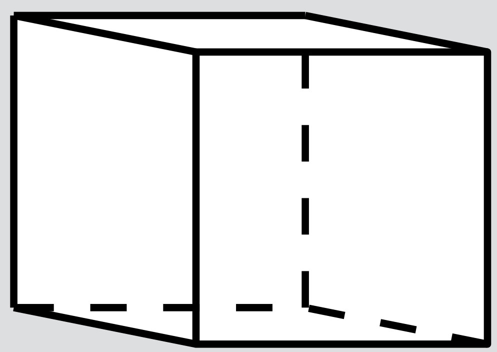
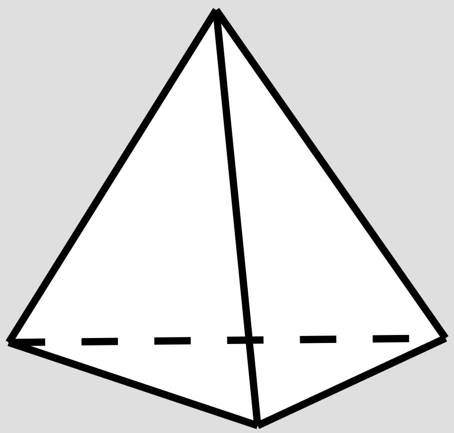
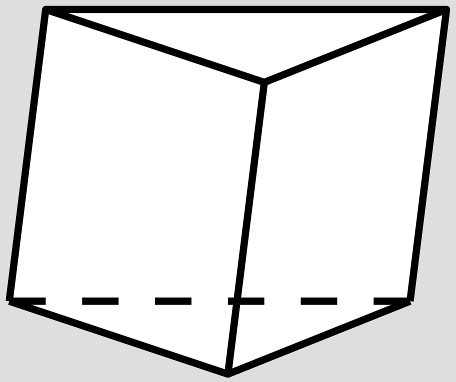
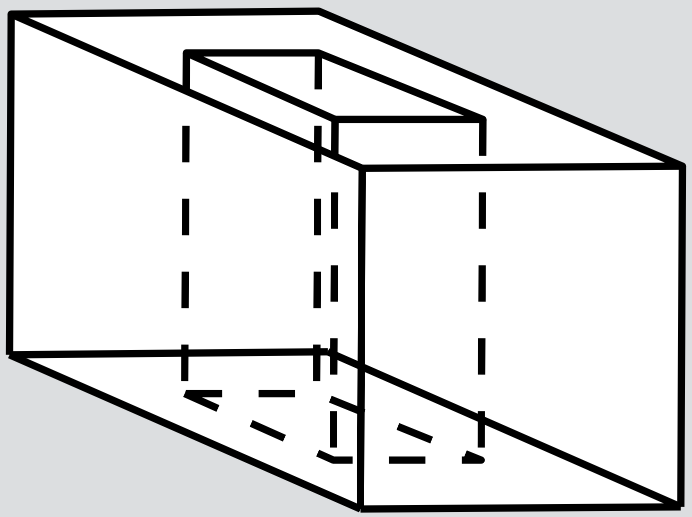
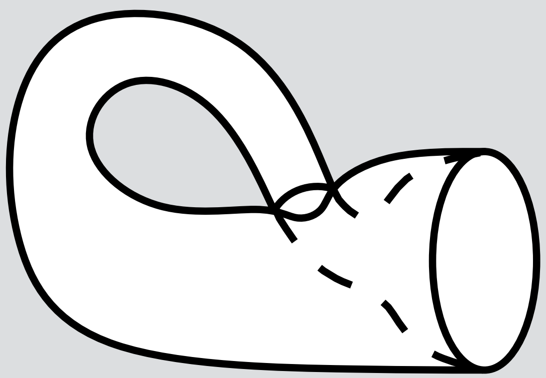
Surfaces

A surface is boundary-less, and possibly higher genus, non-orientable, or dihedral.



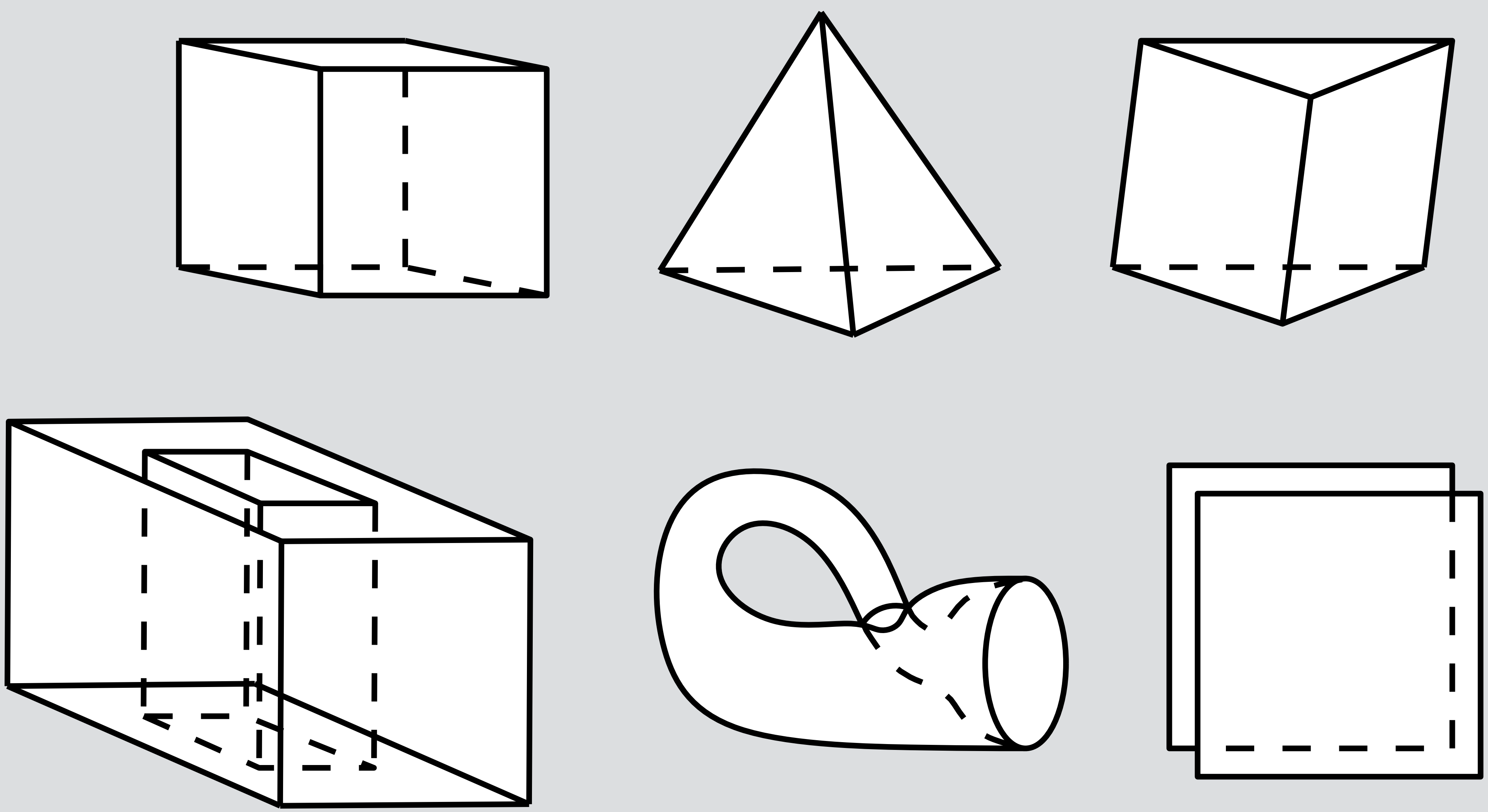
Surfaces

A <u>surface</u> is boundary-less, and possibly higher genus, non-orientable, or dihedral.

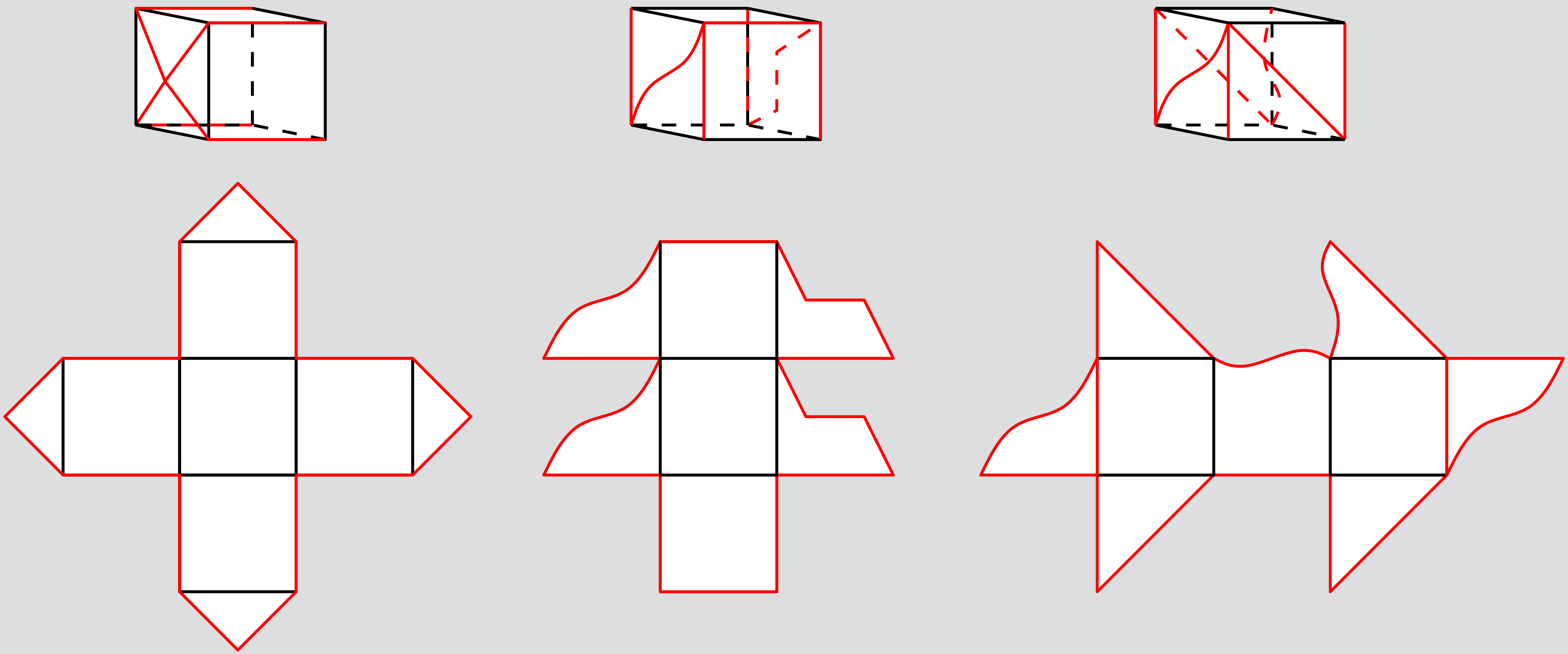
Surfaces

A <u>surface</u> is boundary-less, and possibly higher genus, non-orientable, or dihedral.



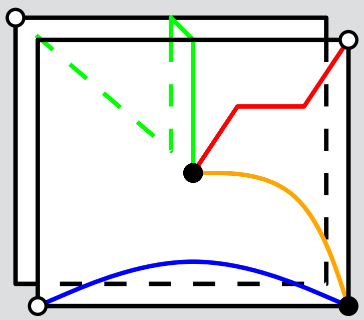
Developments

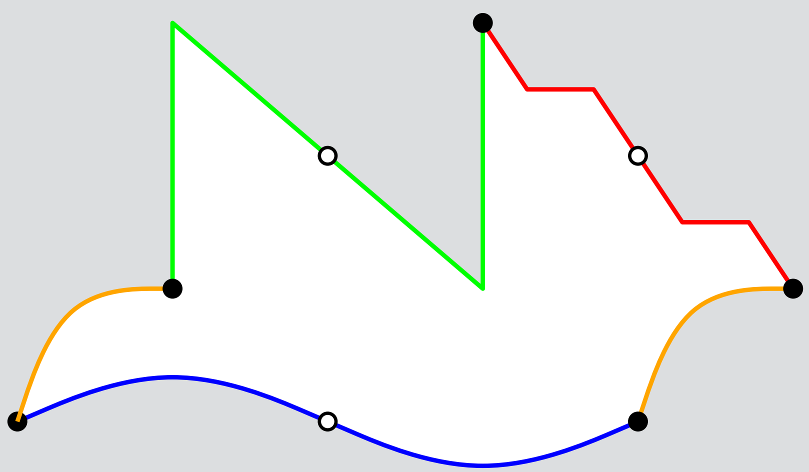
A <u>development</u> of a surface is a cutting of the surface that folds flat (possibly with overlap).

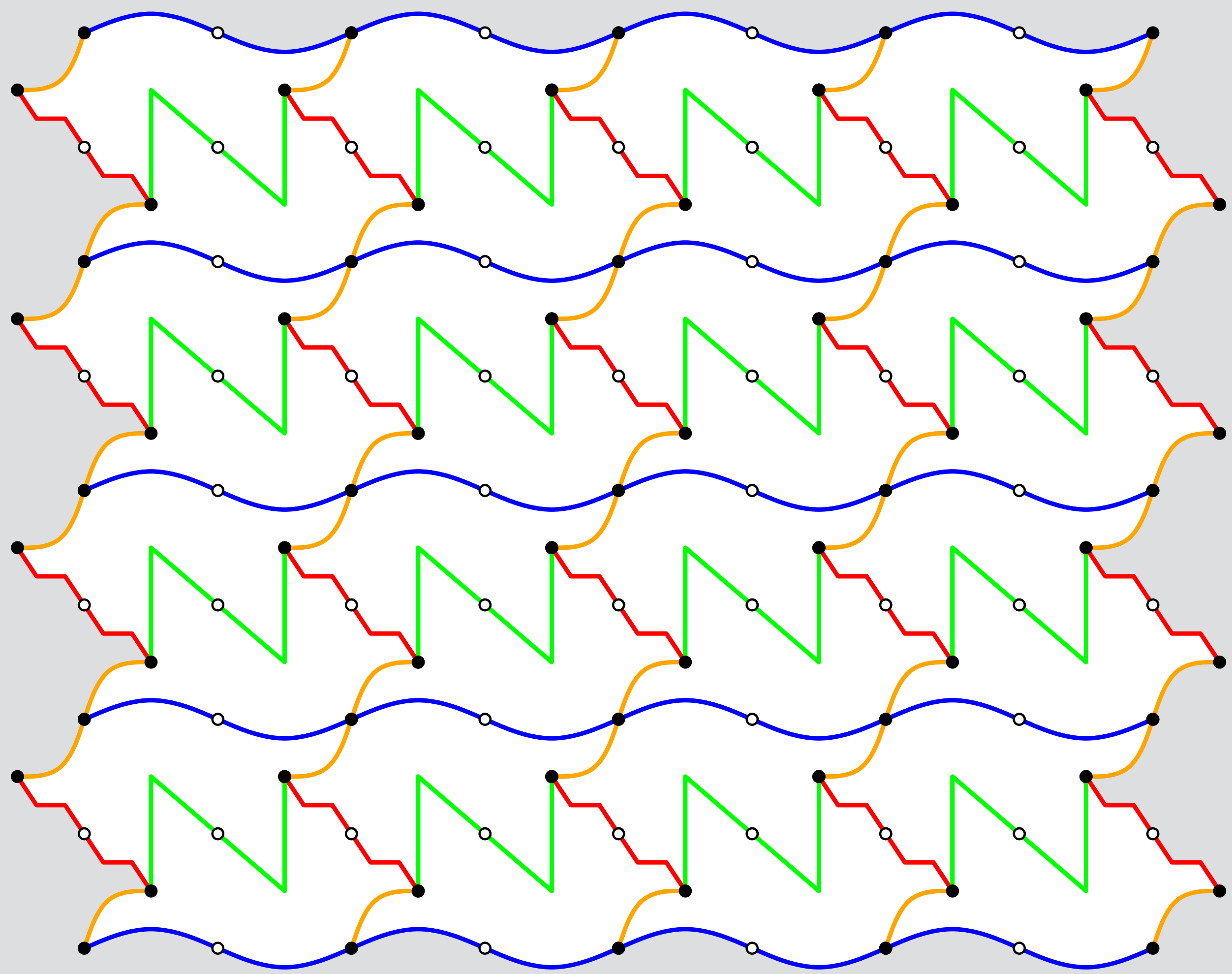


Tile-makers

A tile-maker is a surface S such that every development of S admits a tiling.

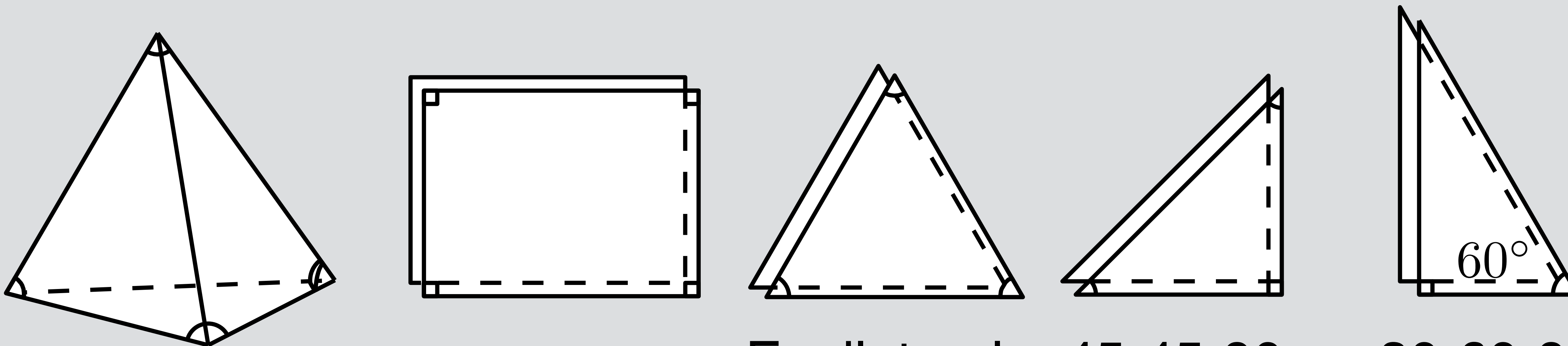






Introduced by [Akiyama 2007].

Akiyama's tile-makers



Almost-regular tetrahedra

Rectangle dihedra

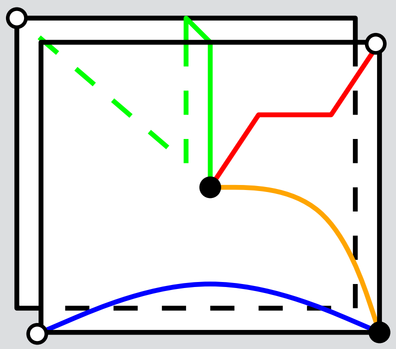
Equilateral triangle dihedra

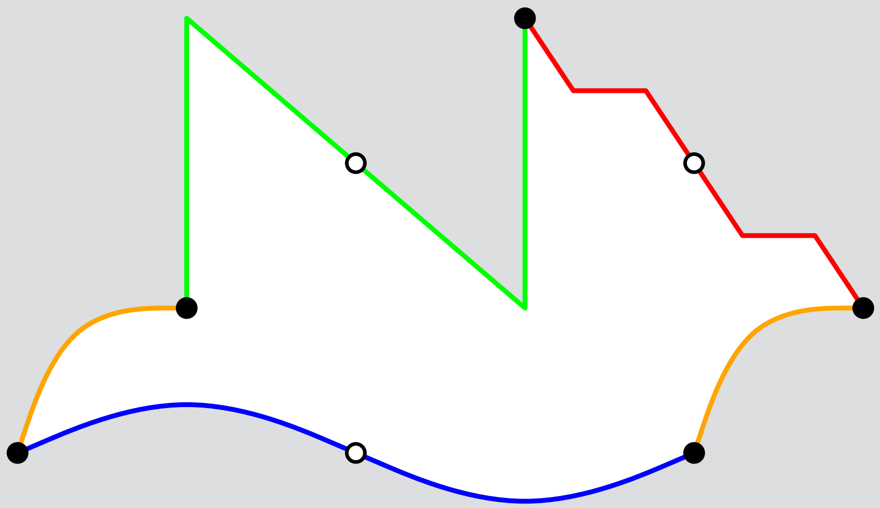
45-45-90 triangle dihedra

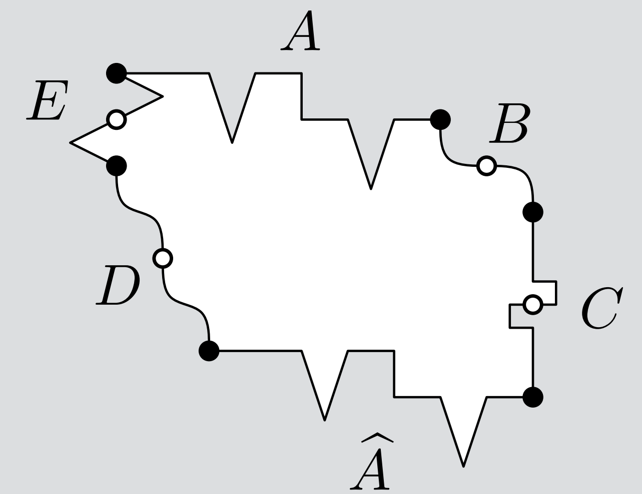
30-60-90 triangle dihedra

[Akiyama 2007]: a convex polyhedron or dihedron is a tile-maker if and only if it is one of these.

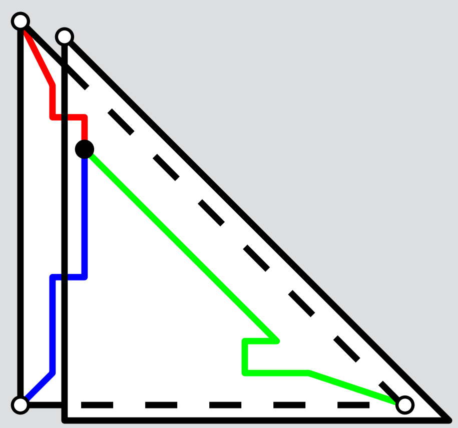
Developments and tilings

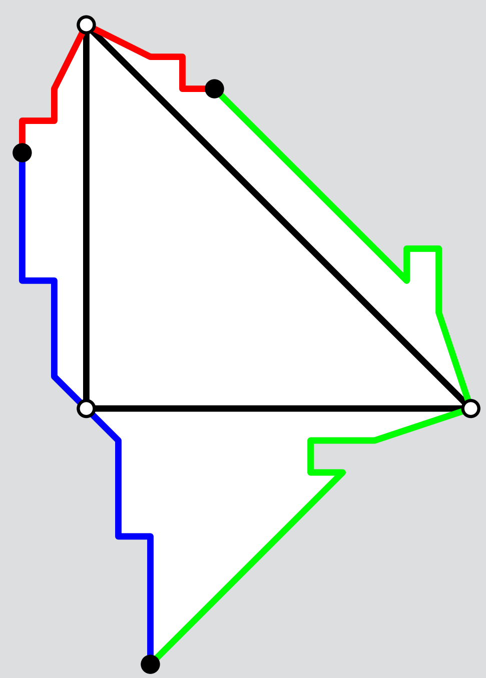


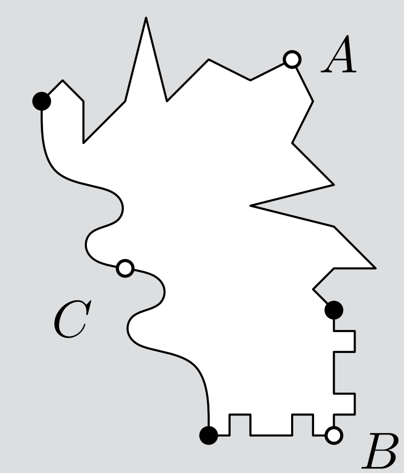




B, C, D, E palindromes

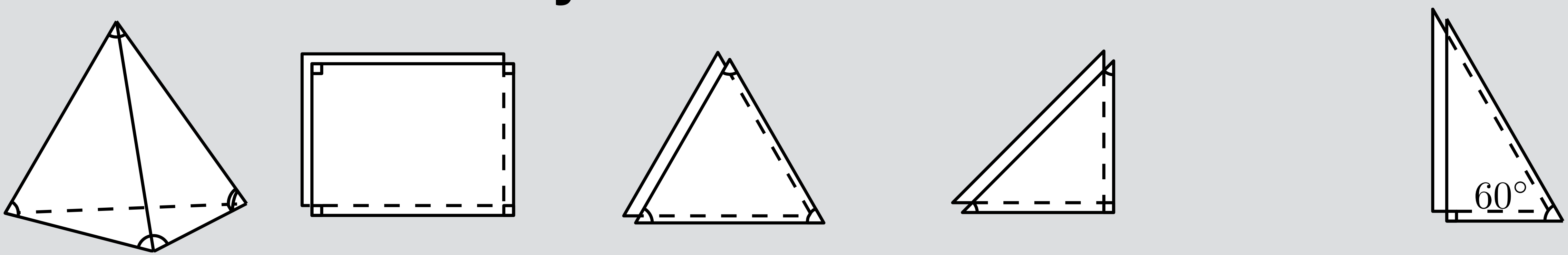


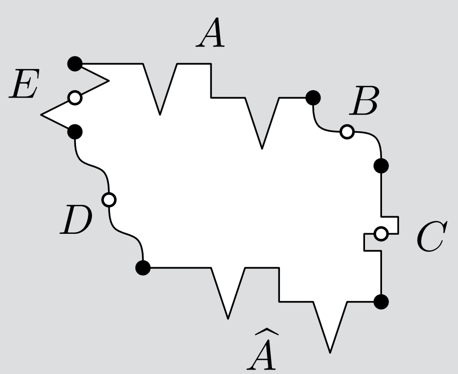




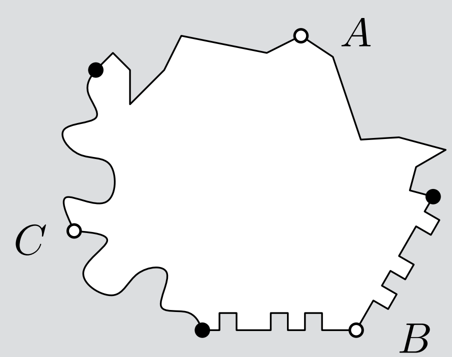
A, B 90-dromes, C palindrome

Akiyama’s tile-makers

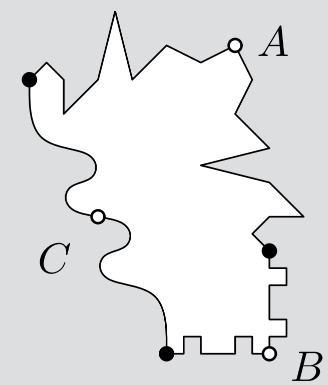




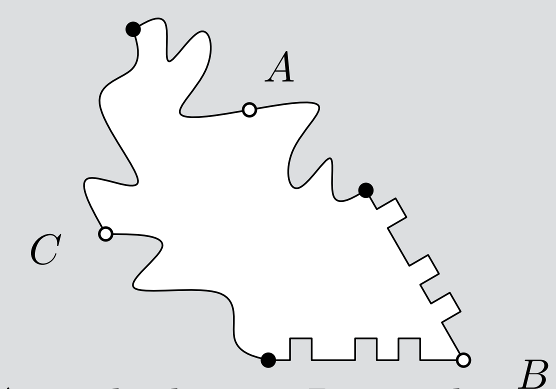
B, C, D, E palindromes



A, B, C 120-dromes



A, B 90-dromes, C palindrome

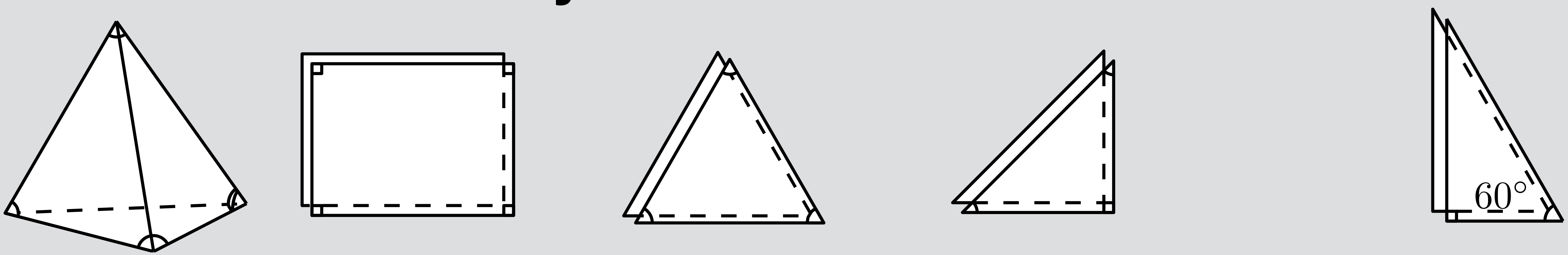


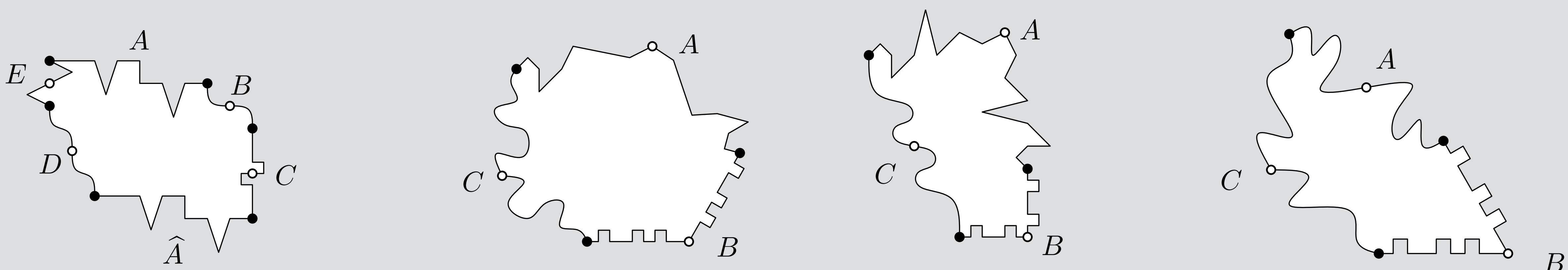
A a palindrome, B a 60-drome, C a 120-drome

Isohedral tiling types

Akiyama’s tile-makers are complete for these types!

Akiyama’s tile-makers





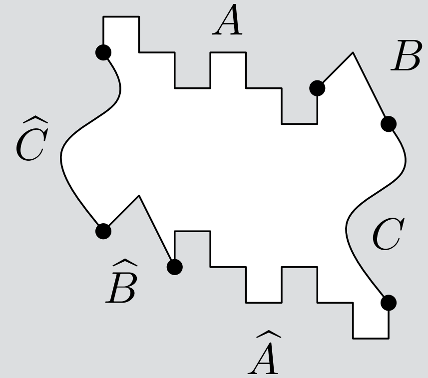
B, C, D, E palindromes

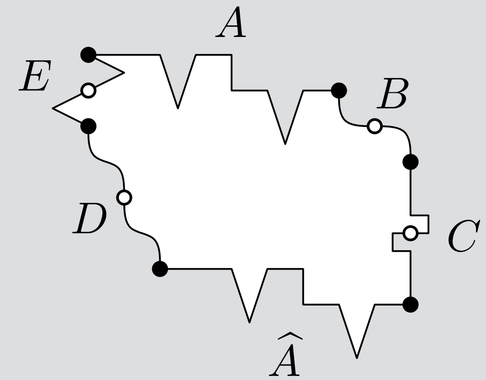
A, B, C 120-dromes

A, B 90-dromes, C palindrome

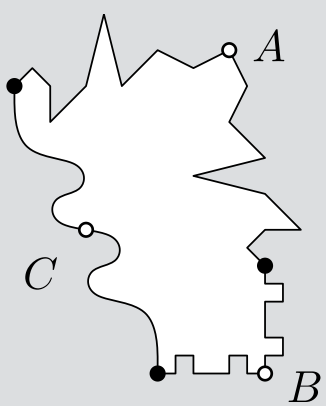
A a palindrome, B a 60-drome, C a 120-drome

Isohedral tiling types

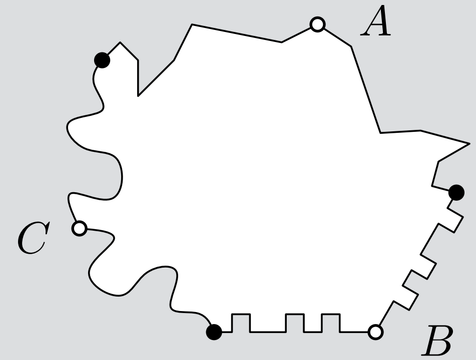




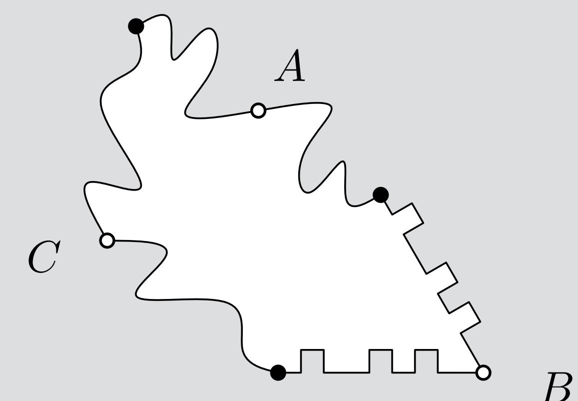
B, C, D, E palindromes



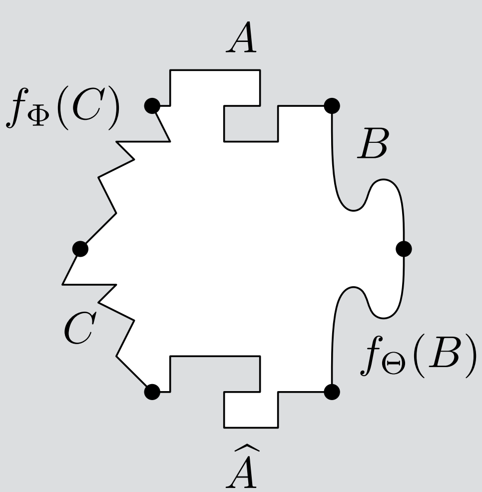
A, B 90-dromes, C palindrome

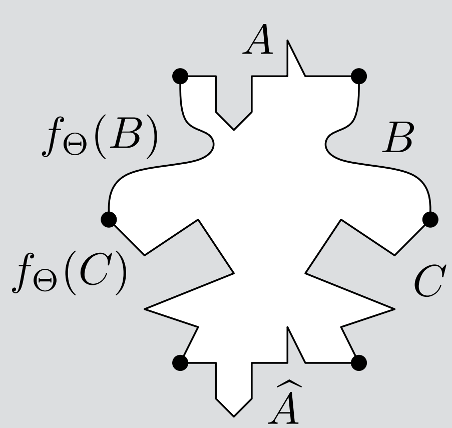


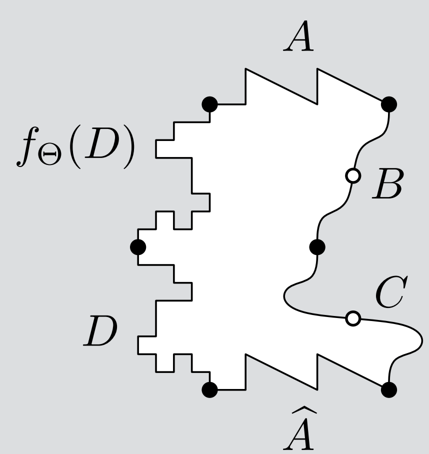
A, B, C 120-dromes



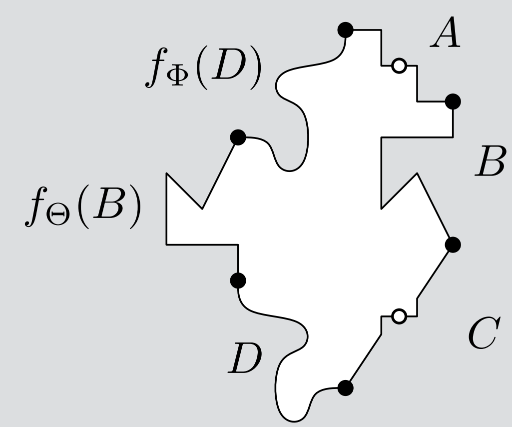
A a palindrome, B a 60-drome,
C a 120-drome



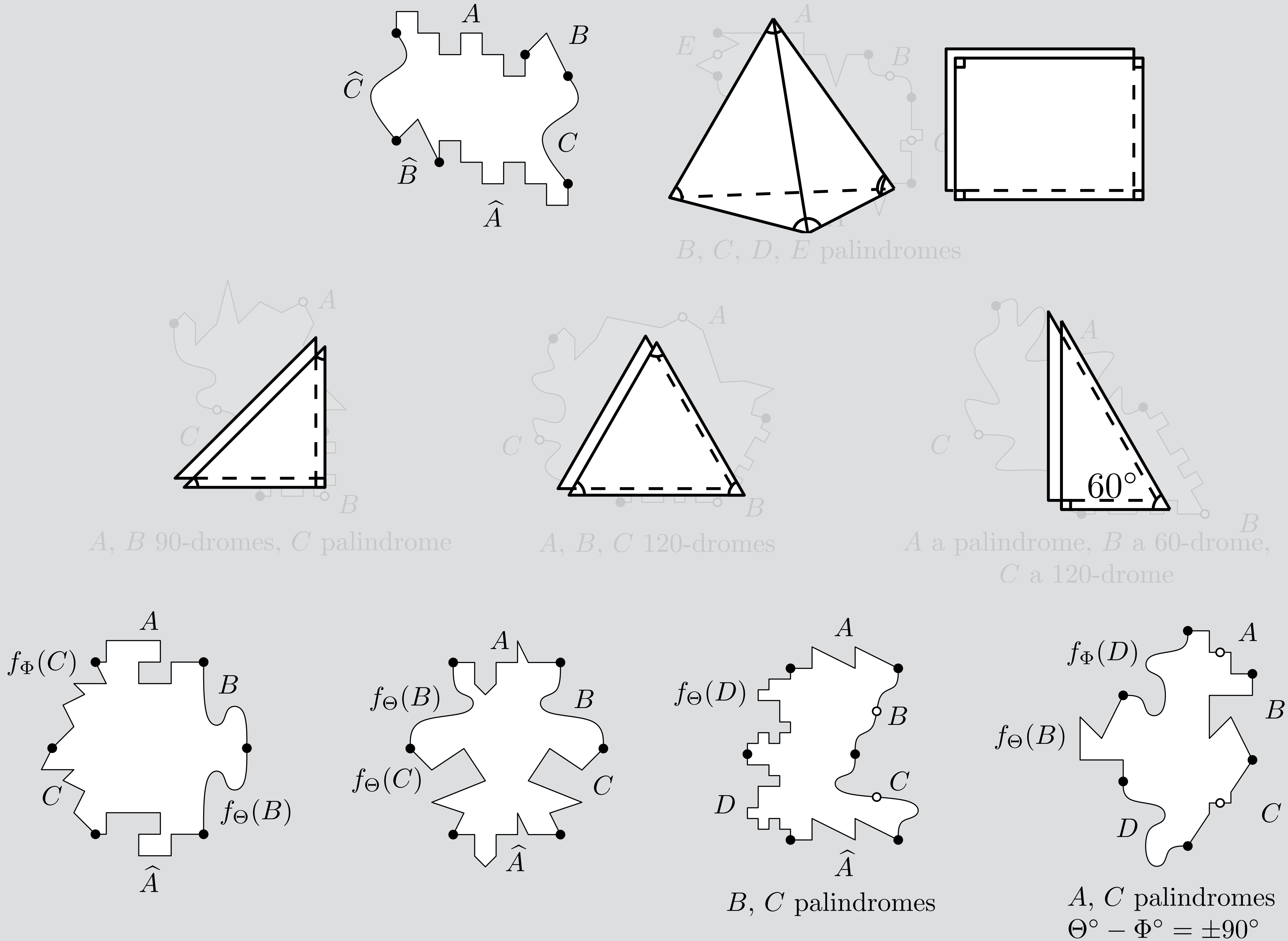




B, C palindromes



A, C palindromes
$\Theta^\circ - \Phi^\circ = \pm 90^\circ$

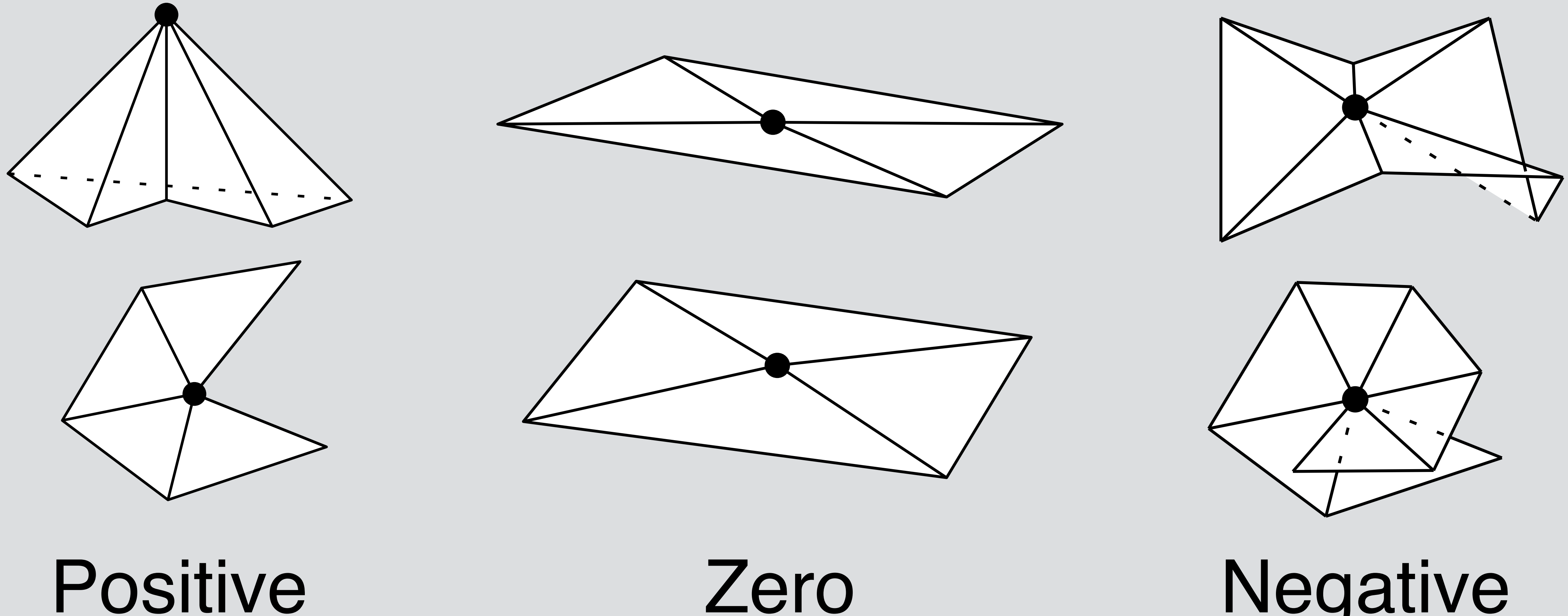

A
B
Ĉ
C
B̂
Â
B, C, D, E palindromes
A, B 90-dromes, C palindrome
A, B, C 120-dromes
60°
A a palindrome, B a 60-drome, C a 120-drome
fΦ(C)
fΘ(B)
fΘ(C)
fΘ(D)
fΦ(D)
D
B, C palindromes
A, C palindromes
Θ° − Φ° = ±90°

Are there other tile-makers?

Are they complete for other 5 isohedral tiling types?

A characterization of tile-makers

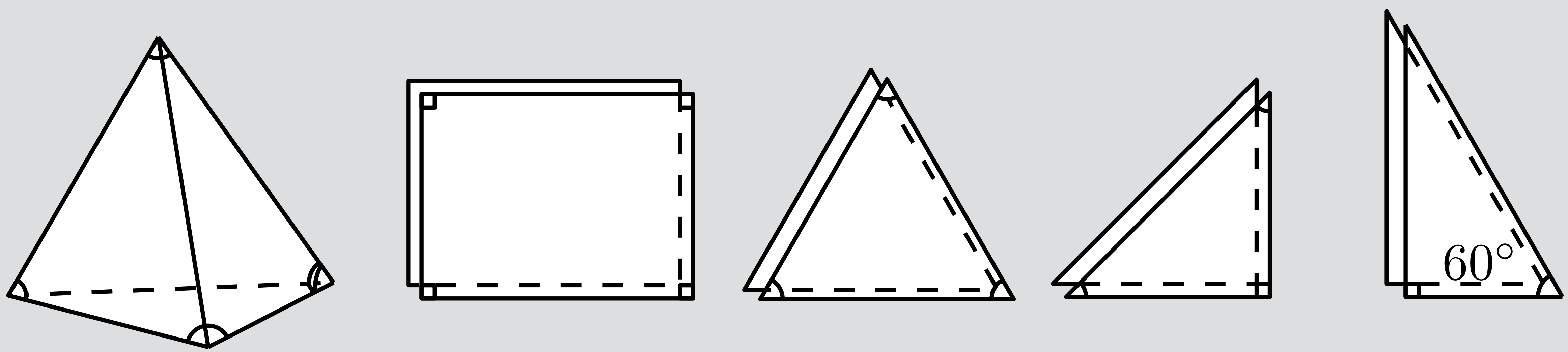
<u>Curvature</u>: 360^{o} - material (written “k(p)”).



A characterization of tile-makers

Theorem: a surface S is a tile-maker if and only if

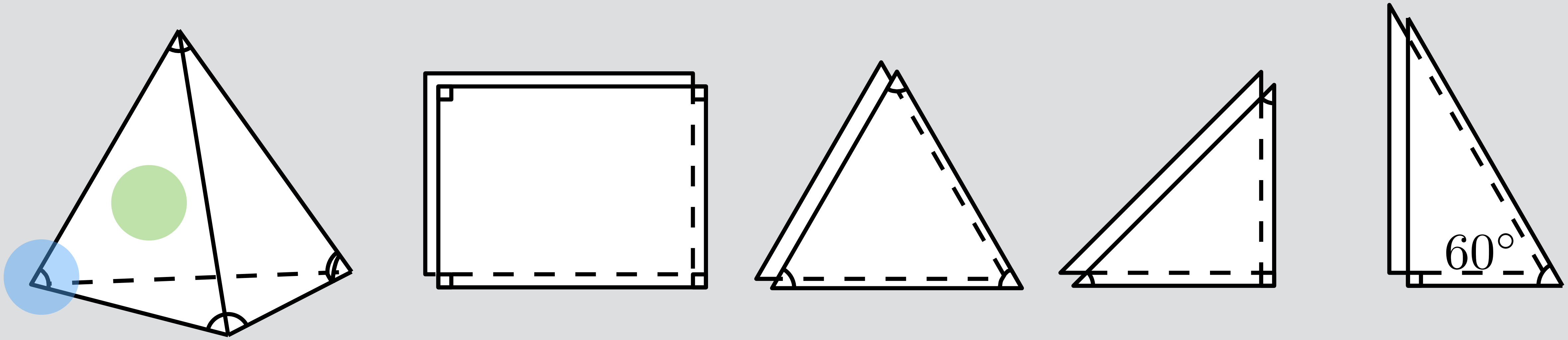
$\forall$ point p $\in$ S, k(p) $\geq$ 0 and 360^o - k(p) divides 360^o.



A characterization of tile-makers

Theorem: a surface S is a tile-maker if and only if

$\forall$ point p $\in$ S, $k(p) \geq 0$ and $360^o - k(p)$ divides 360^o.

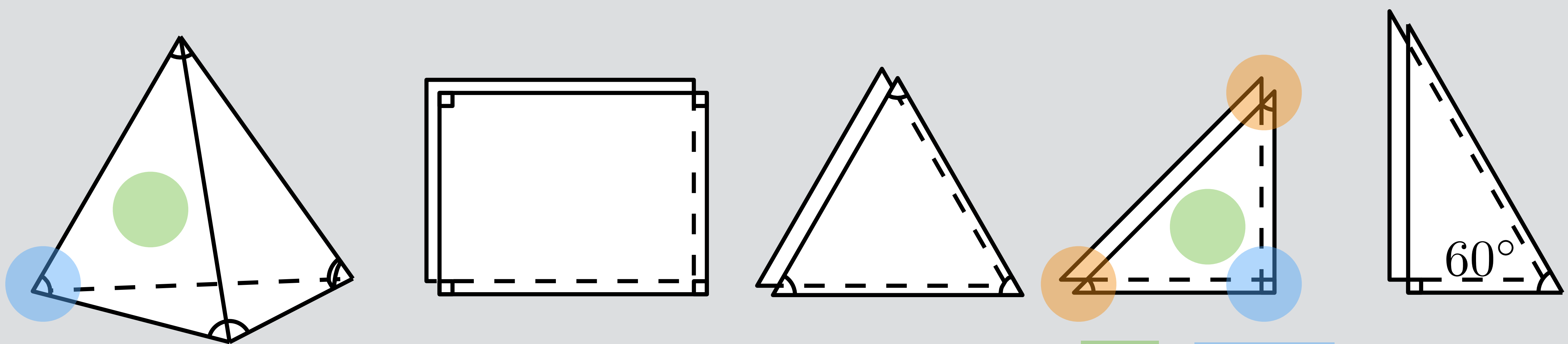


$k(p) \in \{0^o, 180^o\}$

A characterization of tile-makers

Theorem: a surface S is a tile-maker if and only if

$\forall$ point $p \in S$, $k(p) \geq 0$ and $360^o - k(p)$ divides 360^o.



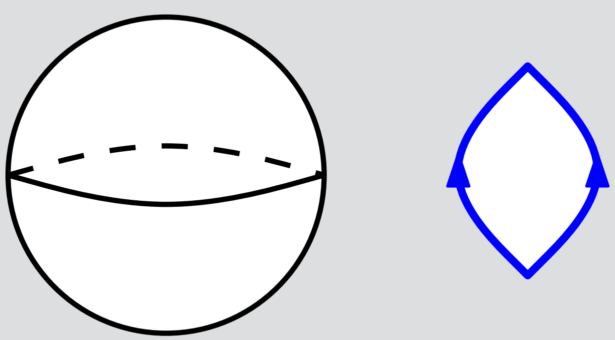
$k(p) \in \{0^o, 180^o\}$

$k(p) \in \{0^o, 180^o, 270^o\}$

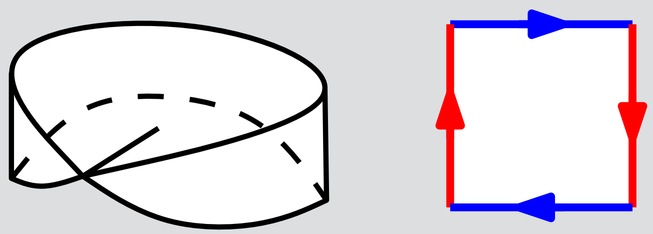
A characterization of tile-makers

Euler characteristic X of a surface S with genus g:

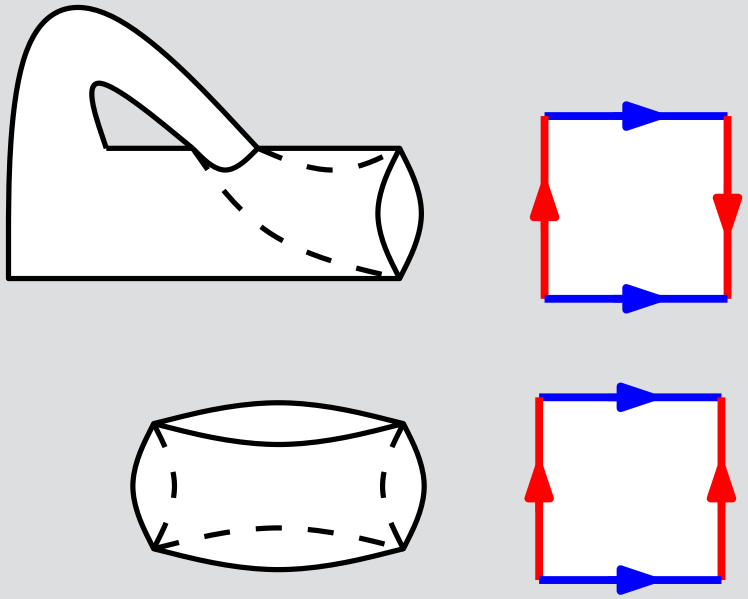
- X = 2 - 2g for orientable surfaces.
- X = 2 - g for non-orientable surfaces.



X = 2



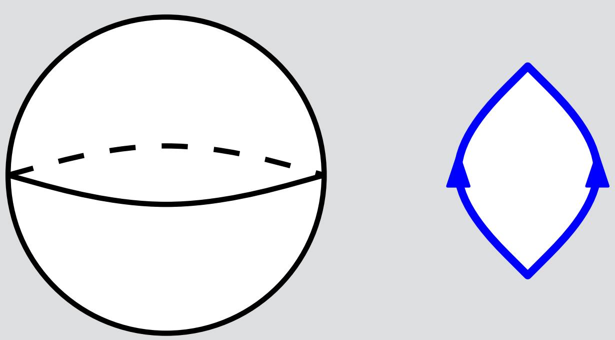
X = 1



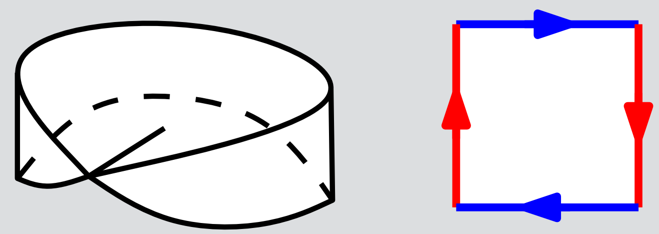
X = 0

A characterization of tile-makers

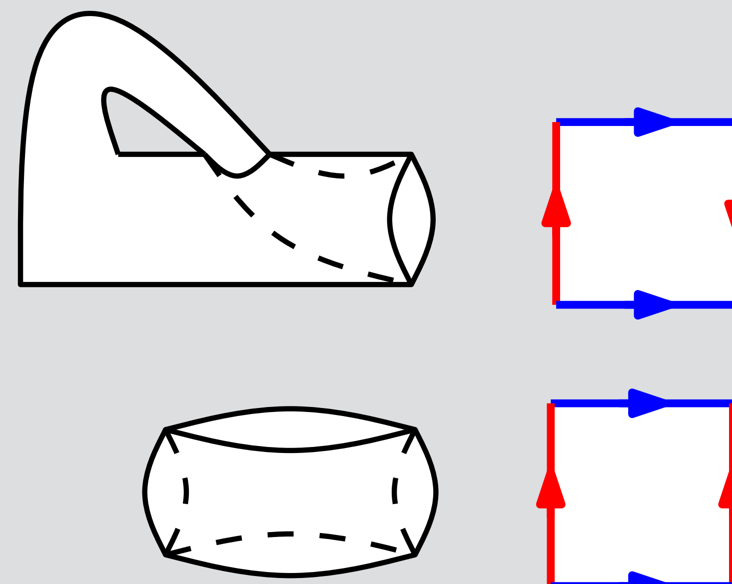
Gauss-Bonnet Theorem: sum of a surface's curvature is 360^{o} X, where X is Euler characteristic.



X = 2



X = 1

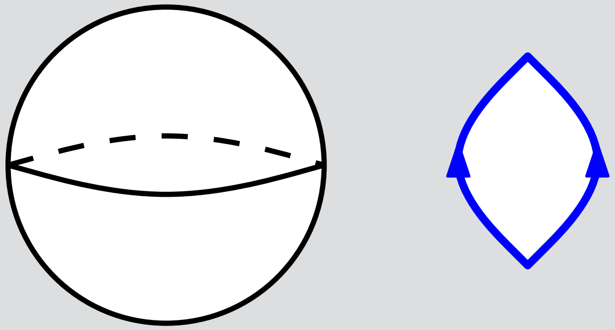


X = 0

A characterization of tile-makers

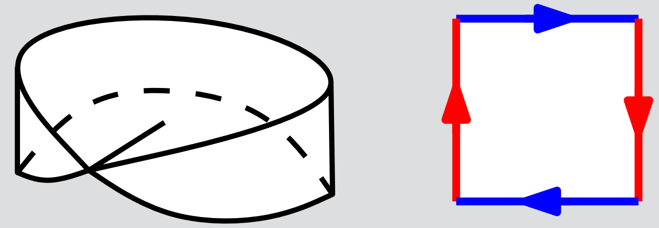
Gauss-Bonnet Theorem: sum of a surface's curvature is 360° X, where X is Euler characteristic.

720° curvature



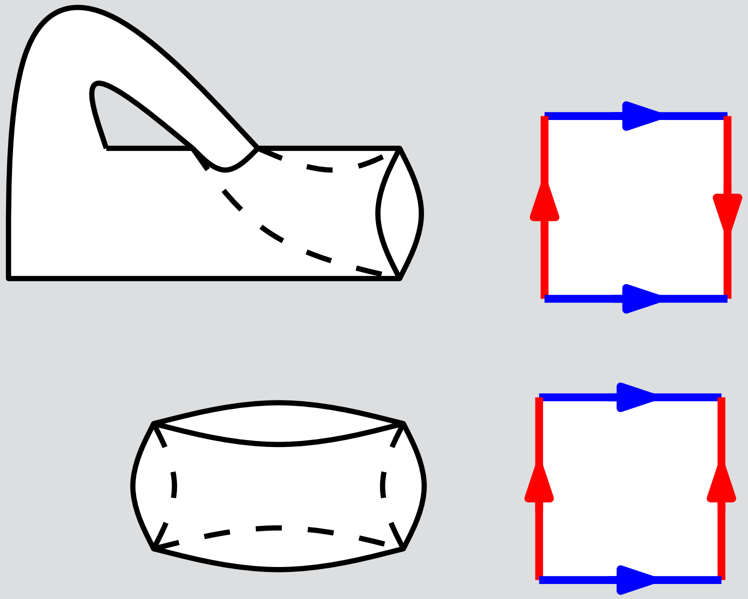
X = 2

360° curvature



X = 1

0° curvature



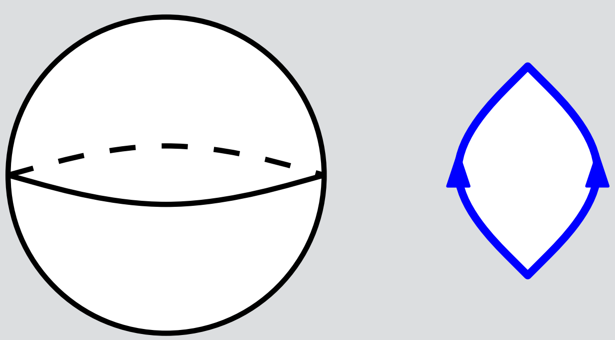
X = 0

A characterization of tile-makers

Theorem: a surface S is a tile-maker if and only if

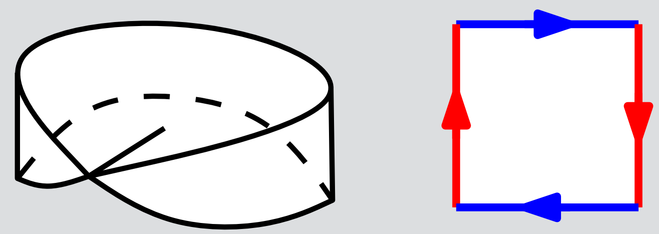
$\forall$ point $p \in S$, $k(p) \geq 0$ and $360^o - k(p)$ divides 360^o.

720° curvature



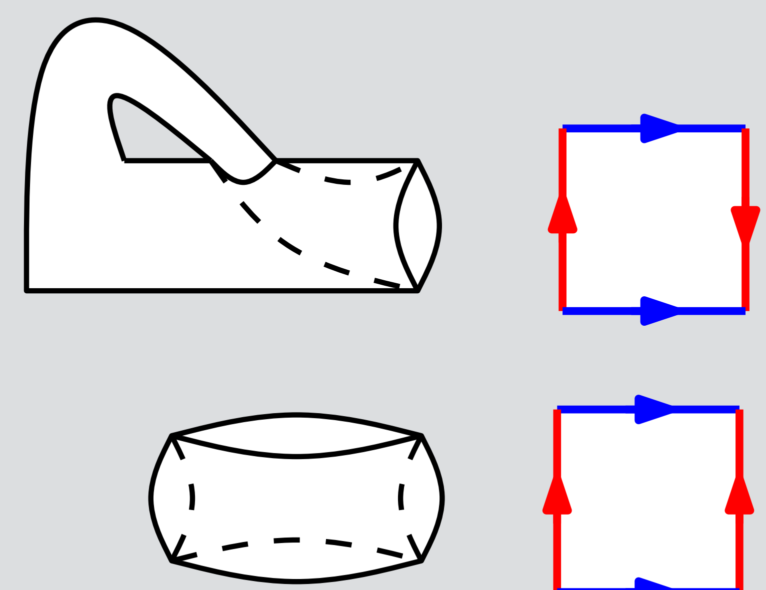
$X = 2$

360° curvature



$X = 1$

0° curvature



$X = 0$

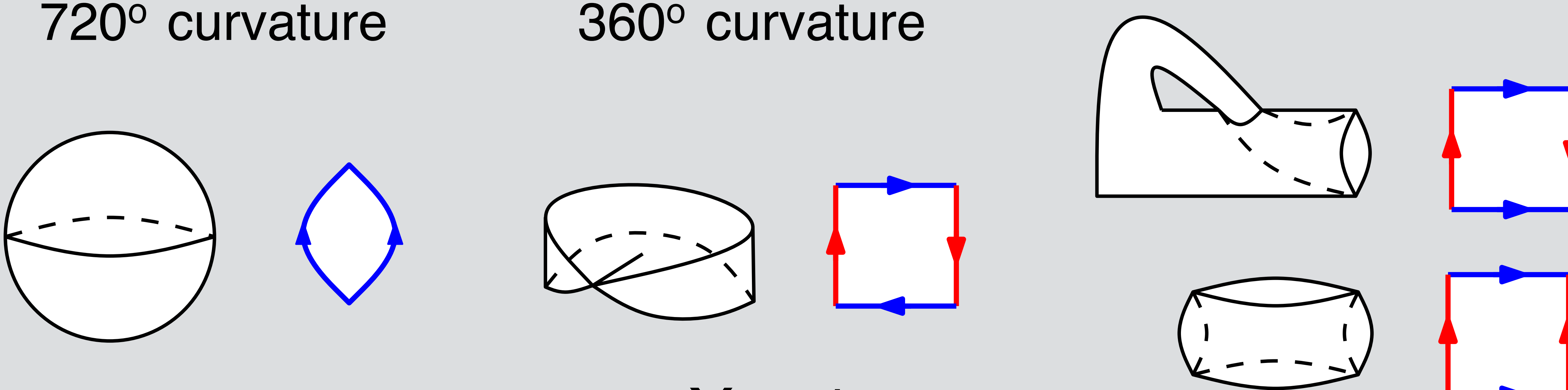
A characterization of tile-makers

Theorem: a surface S is a tile-maker if and only if

$\forall$ point $p \in S$, $k(p) \geq 0$ and $360^o - k(p)$ divides 360^o.

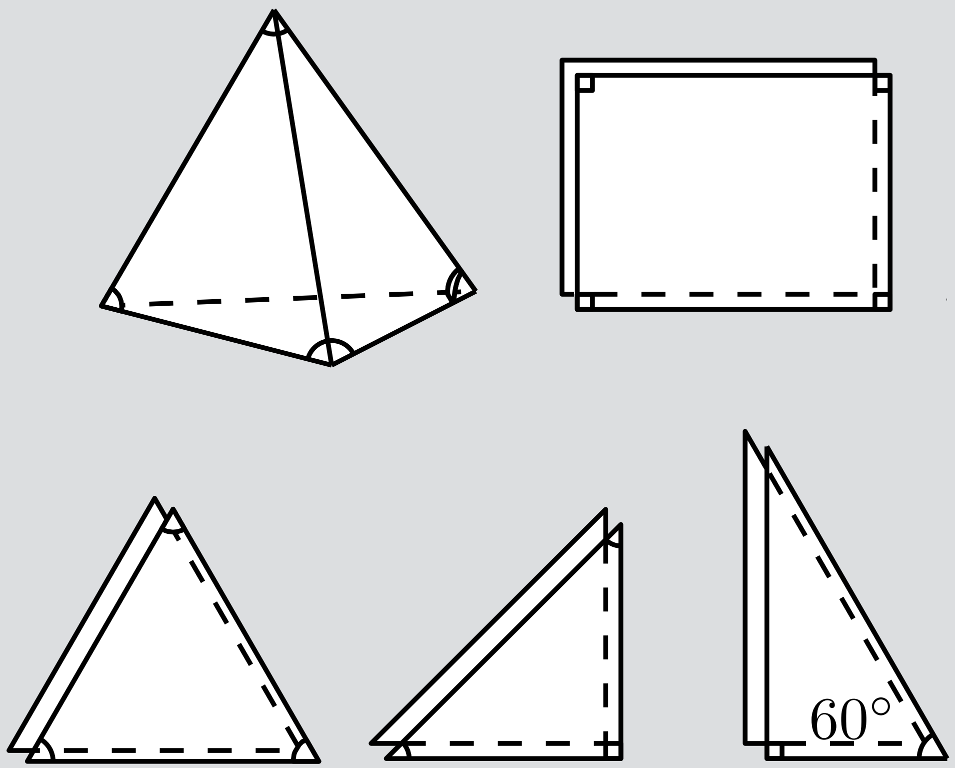
implies $k(p) \in \{0^o, 180^o, 240^o, 270^o, \ldots\}$

720° curvature — 360° curvature — 0° curvature



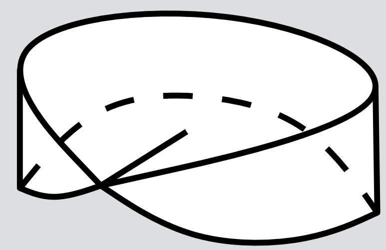
X = 2 — X = 1 — X = 0

All tile-makers



X = 2

720^o curvature

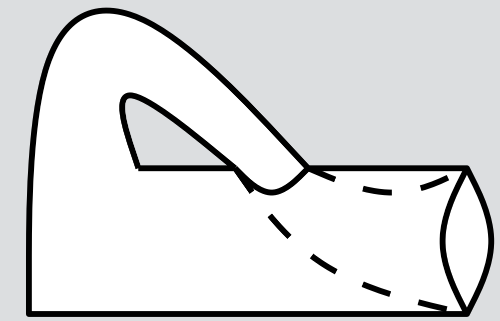


with $p_1, p_2 \in S$,

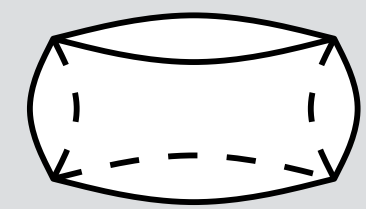
$k(p_1,)\ k(p_2) = 180^o$

X = 1

360^o curvature



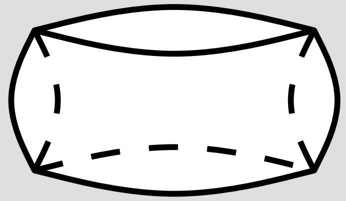
flat everywhere



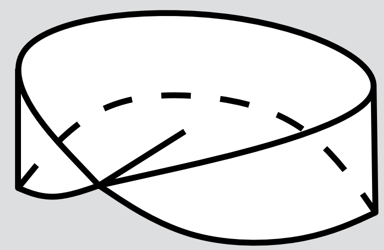
X = 0

0^o curvature

New tile-makers

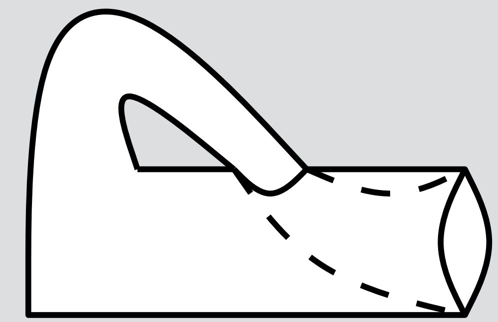


flat everywhere

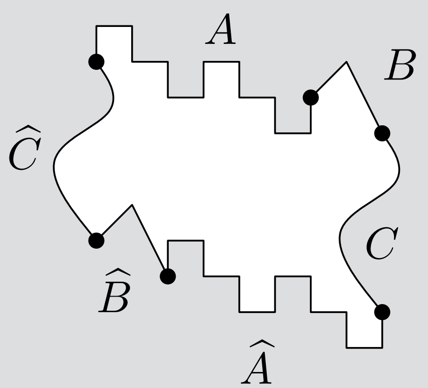


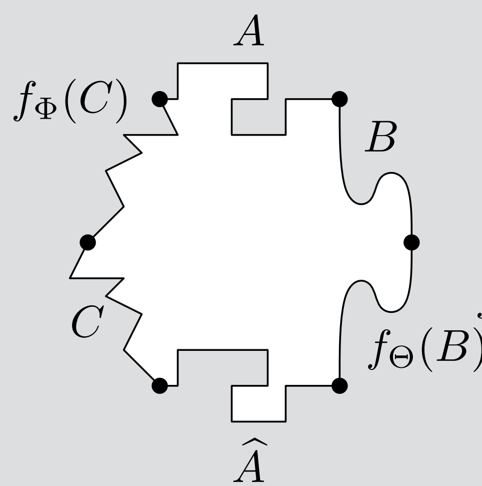
with p_1, $p_2 \in S$,

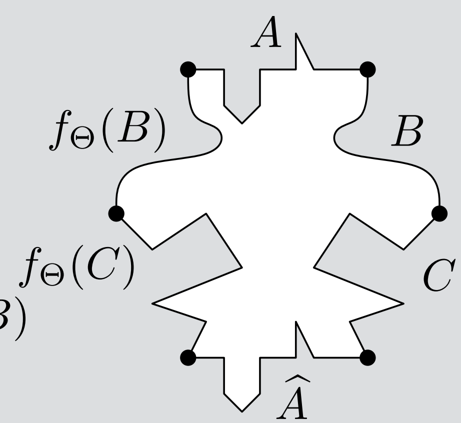
$k(p_1,)\ k(p_2) = 180^o$

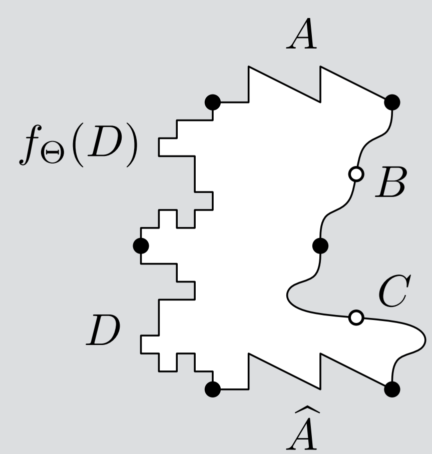


flat everywhere

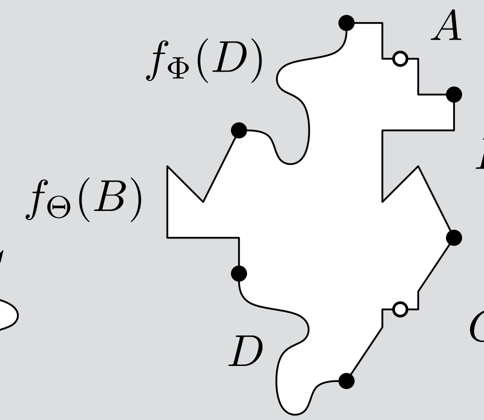






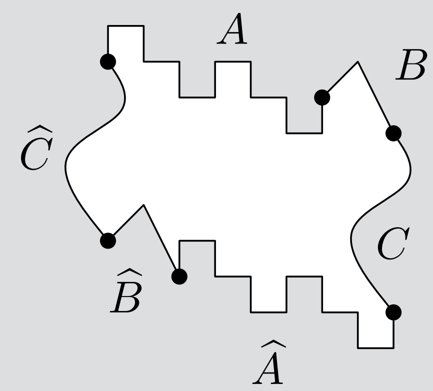


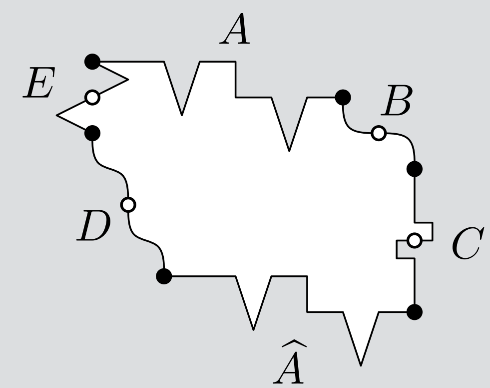
B, C palindromes



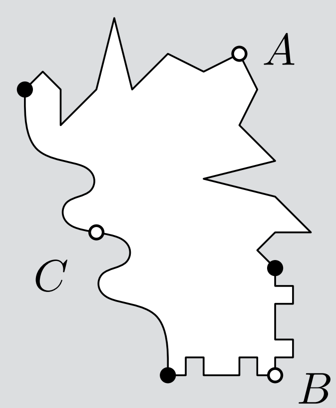
A, C palindromes
$\Theta^\circ - \Phi^\circ = \pm 90^\circ$

Isohedral tiling types

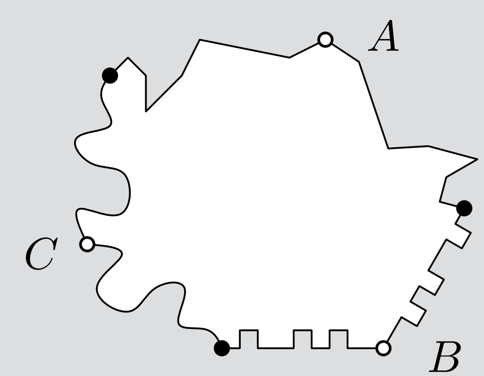




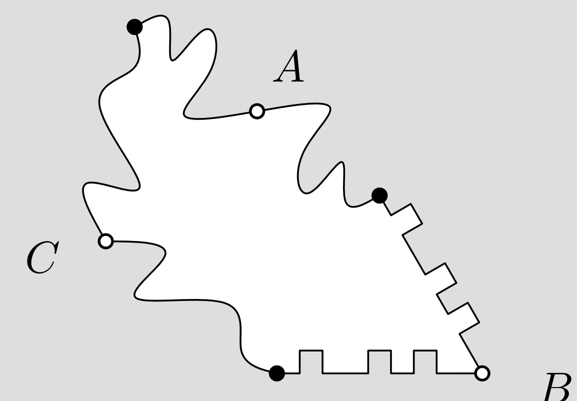
B, C, D, E palindromes



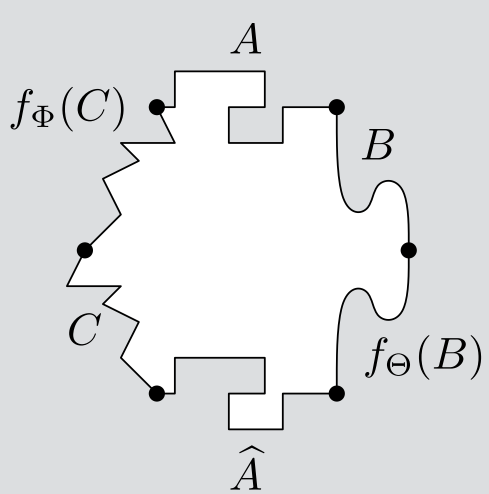
A, B 90-dromes, C palindrome

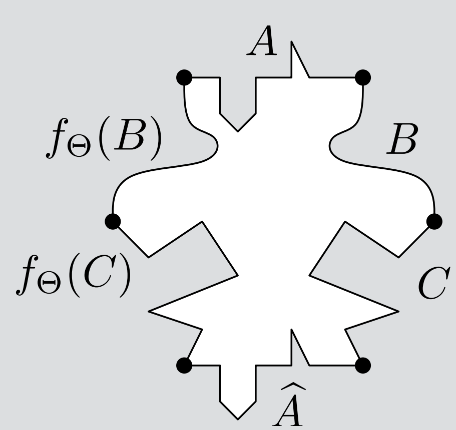


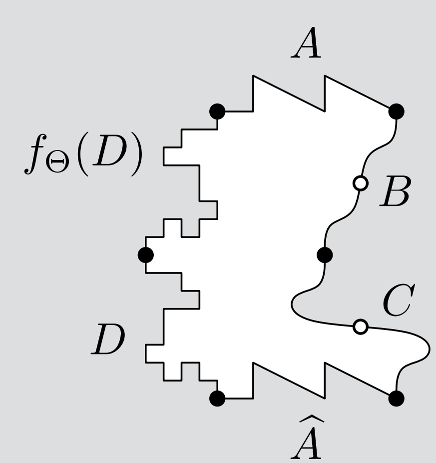
A, B, C 120-dromes



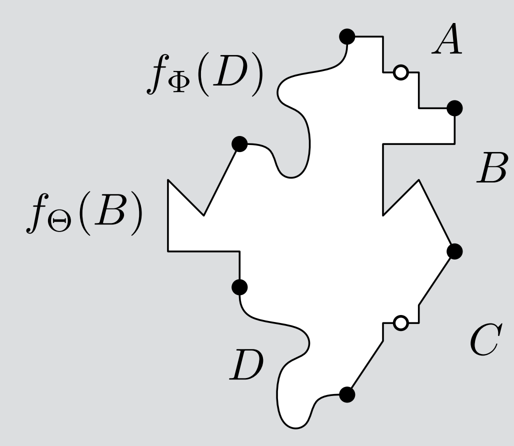
A a palindrome, B a 60-drome,
C a 120-drome



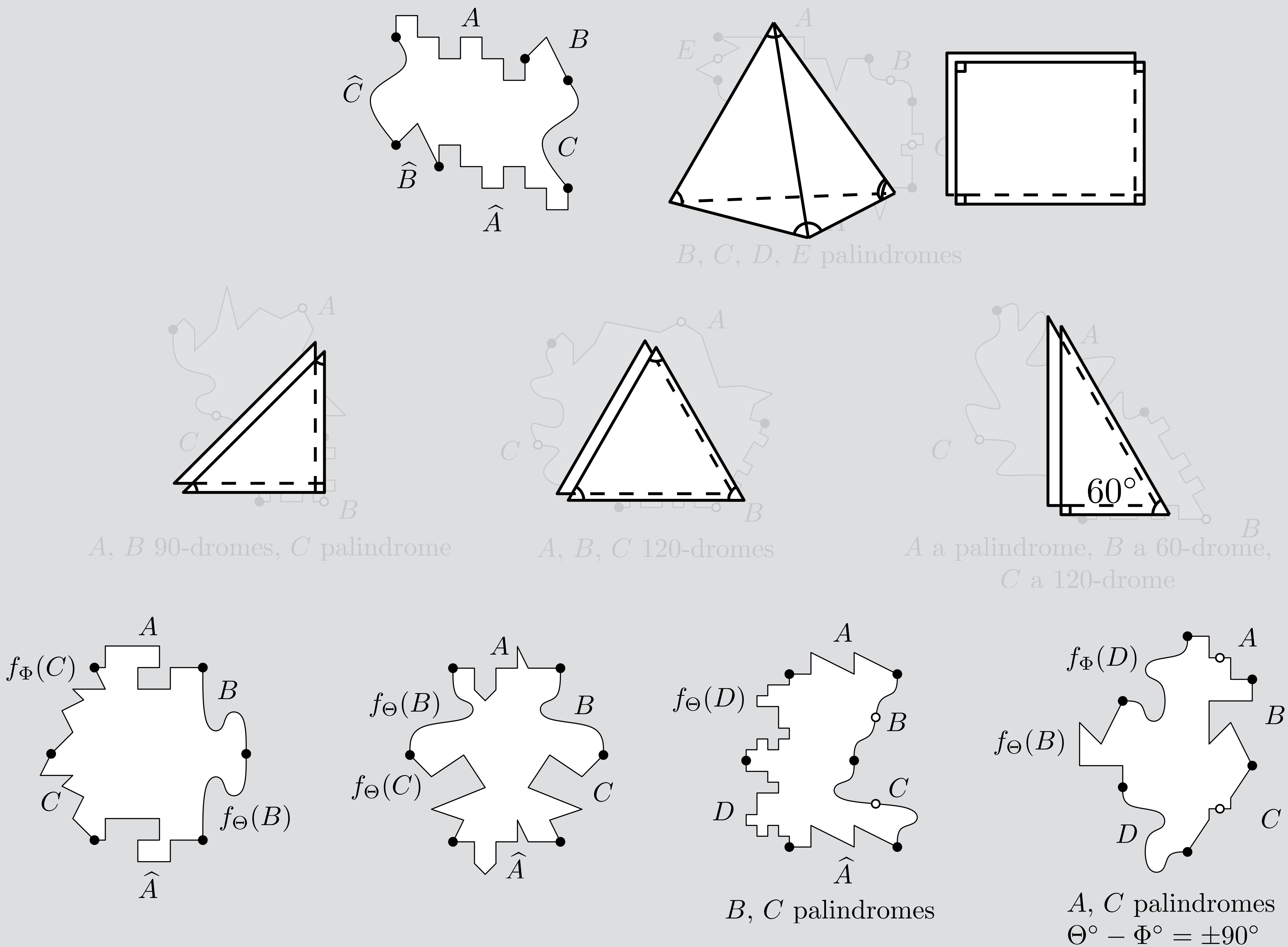




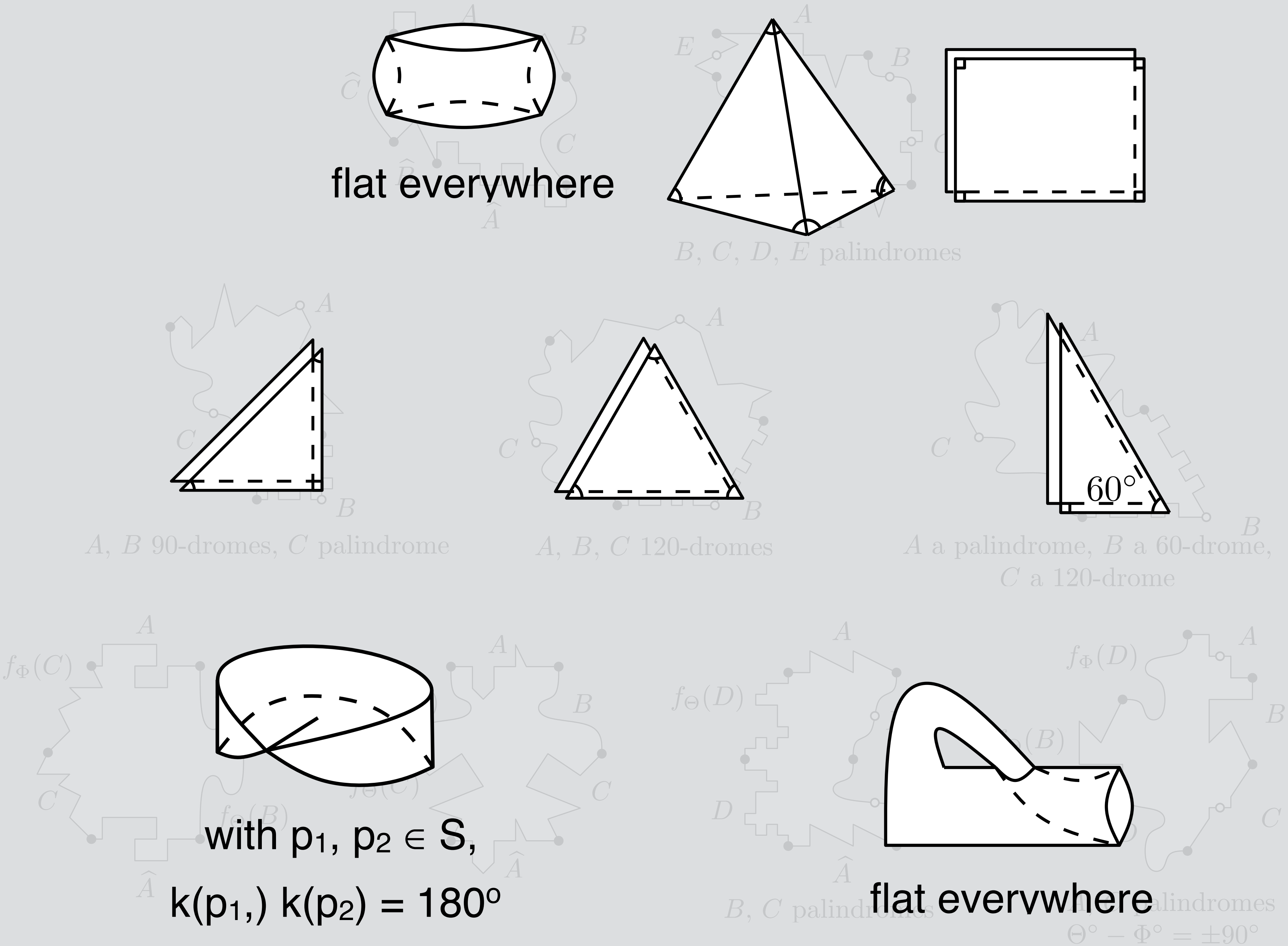
B, C palindromes



A, C palindromes
$\Theta^\circ - \Phi^\circ = \pm 90^\circ$

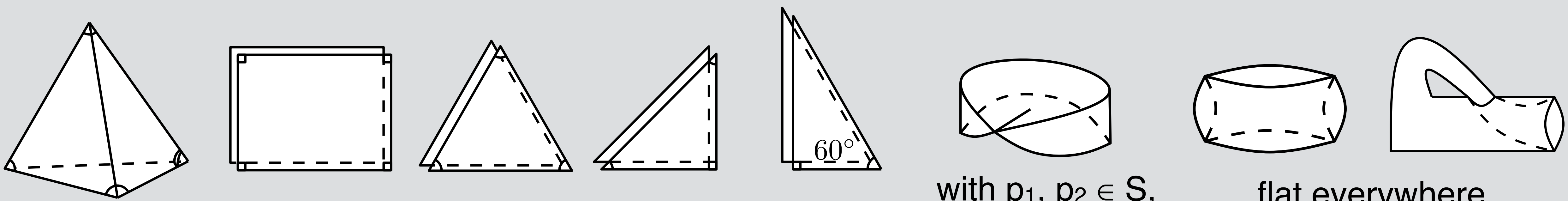


A
B
Ĉ
C
B̂
Â
B, C, D, E palindromes
A, B 90-dromes, C palindrome
A, B, C 120-dromes
60°
A a palindrome, B a 60-drome, C a 120-drome
fΦ(C)
fΘ(B)
fΘ(C)
fΘ(D)
D
fΦ(D)
B, C palindromes
A, C palindromes
Θ° − Φ° = ±90°


flat everywhere
B, C, D, E palindromes
A, B 90-dromes, C palindrome
A, B, C 120-dromes
60°
A a palindrome, B a 60-drome,
C a 120-drome
with p_1, $p_2 \in S$,
$k(p_1,)\ k(p_2) = 180^o$
B, C palindromes
flat everywhere
$\Theta^\circ - \Phi^\circ = \pm 90^\circ$

Results

Theorem: the set of all tile-makers is



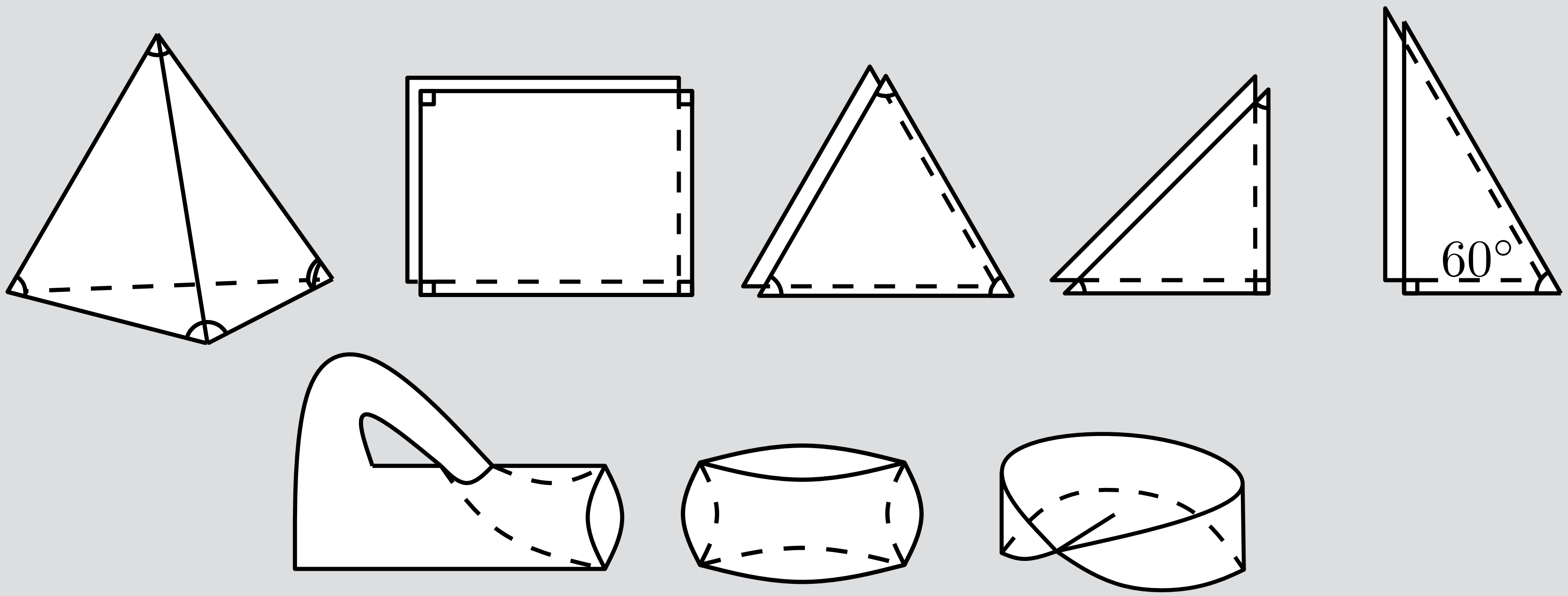
with p_1, $p_2 \in S$,

$k(p_1,)\ k(p_2) = 180^o$

flat everywhere

Theorem: the developments of tile-makers are exactly the set of all isohedral tilings.

Some Results on Tile-makers



Stefan Langerman, Andrew Winslow

UNIVERSITE LIBRE DE BRUXELLES