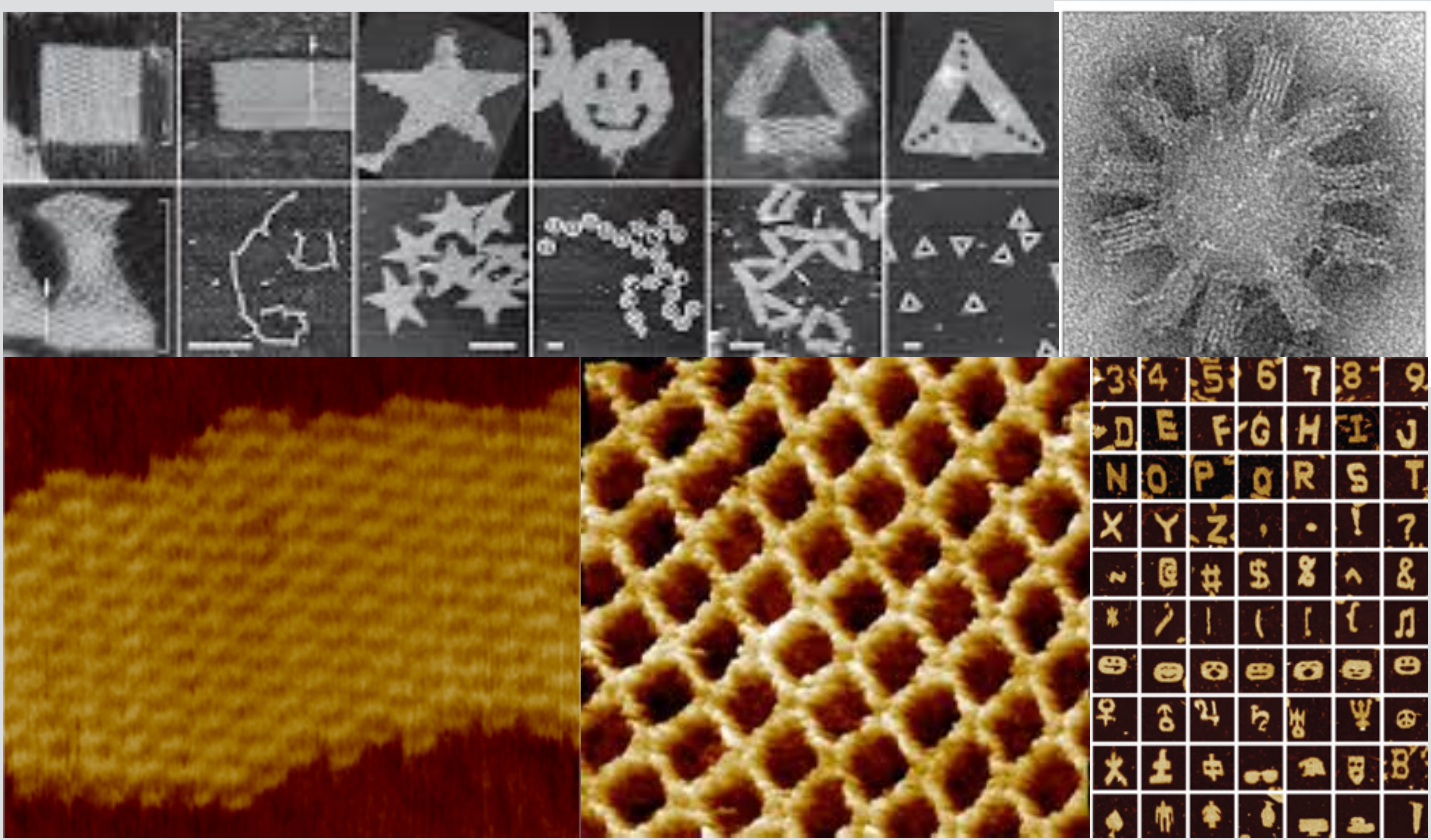
Size-Separable Tile Self-Assembly:

A Tight Bound for Temperature-1 Mismatch-Free Systems

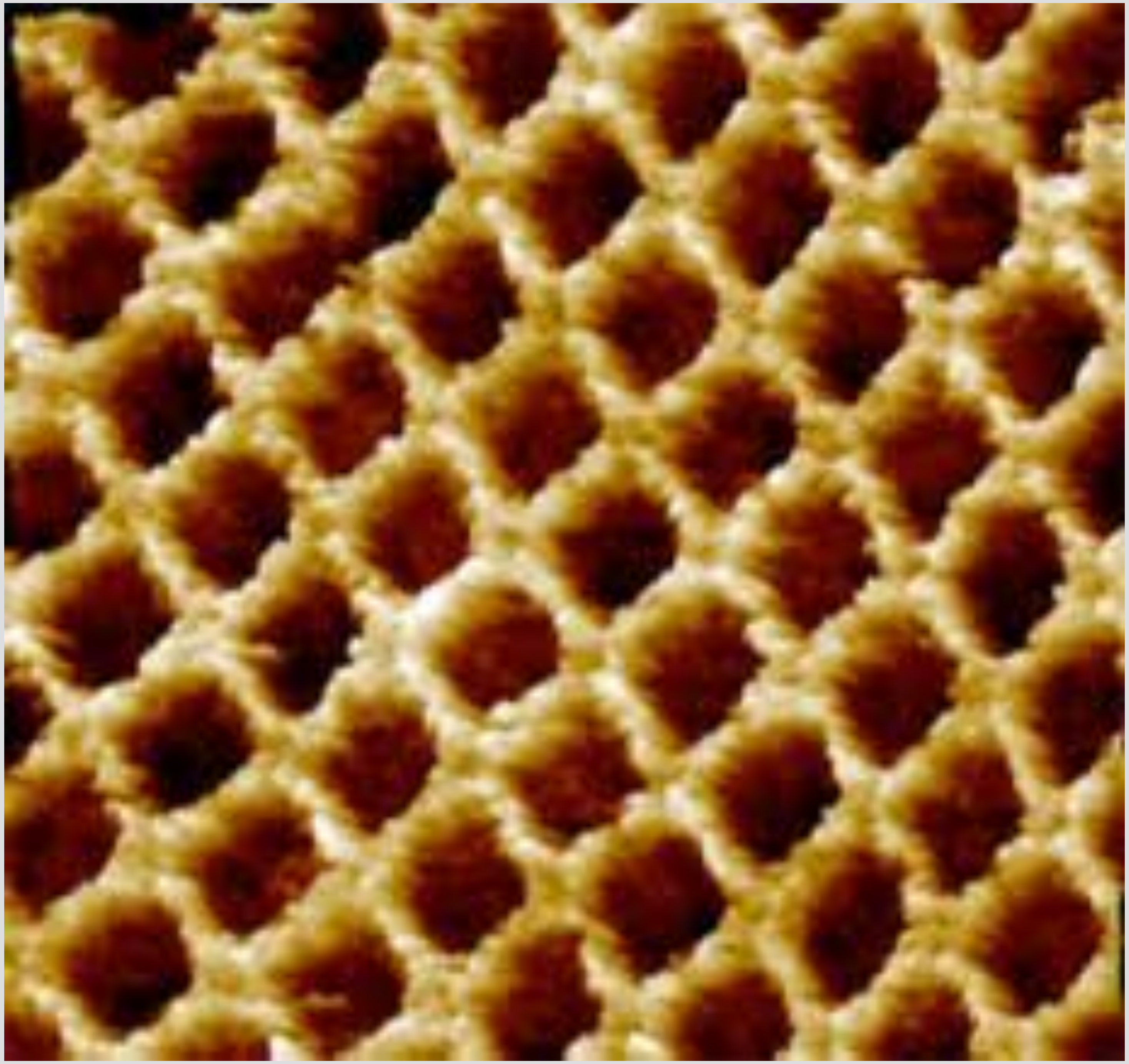
Andrew Winslow

Tufts University

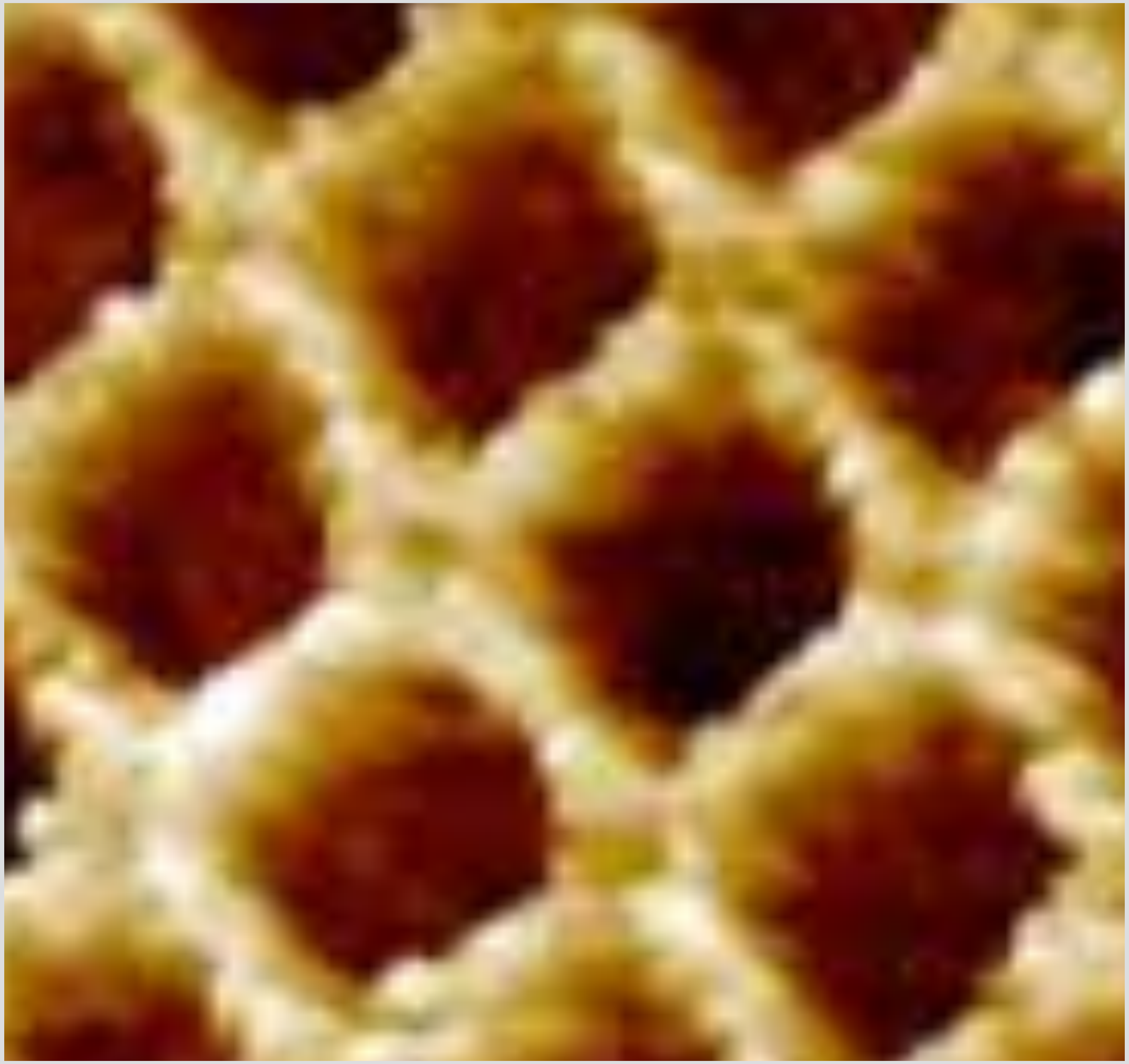
Synthetic Self-Assembly with DNA



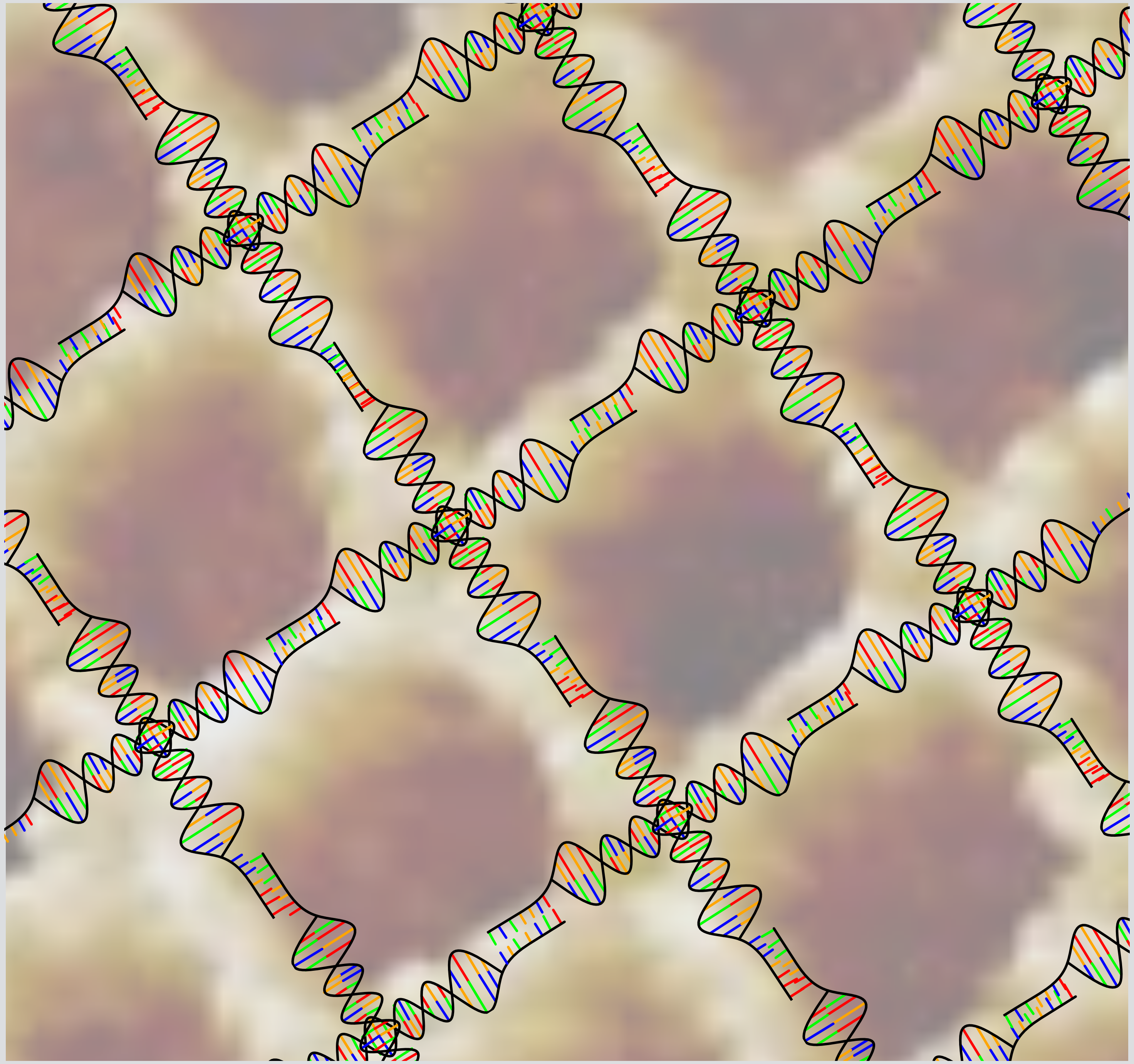
Synthetic Self-Assembly with DNA



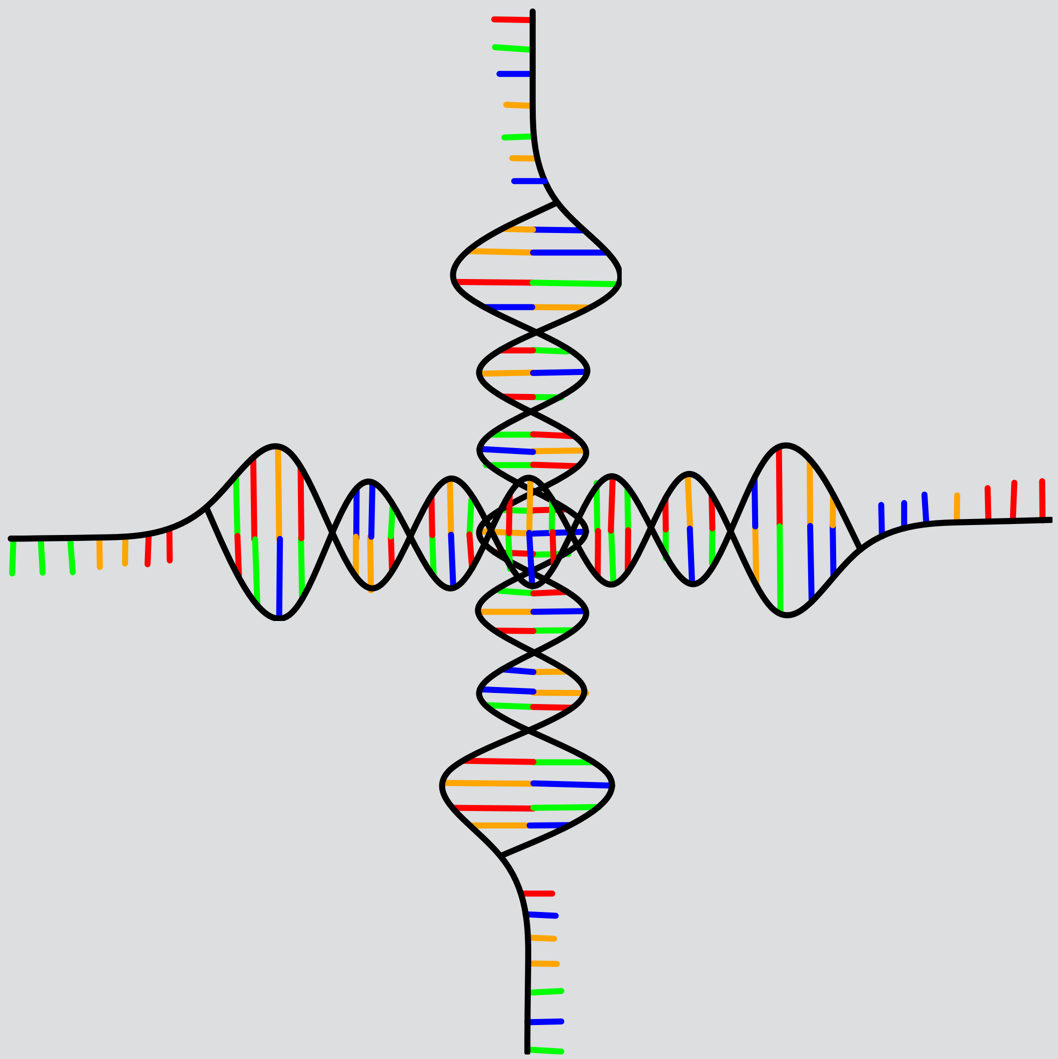
Synthetic Self-Assembly with DNA



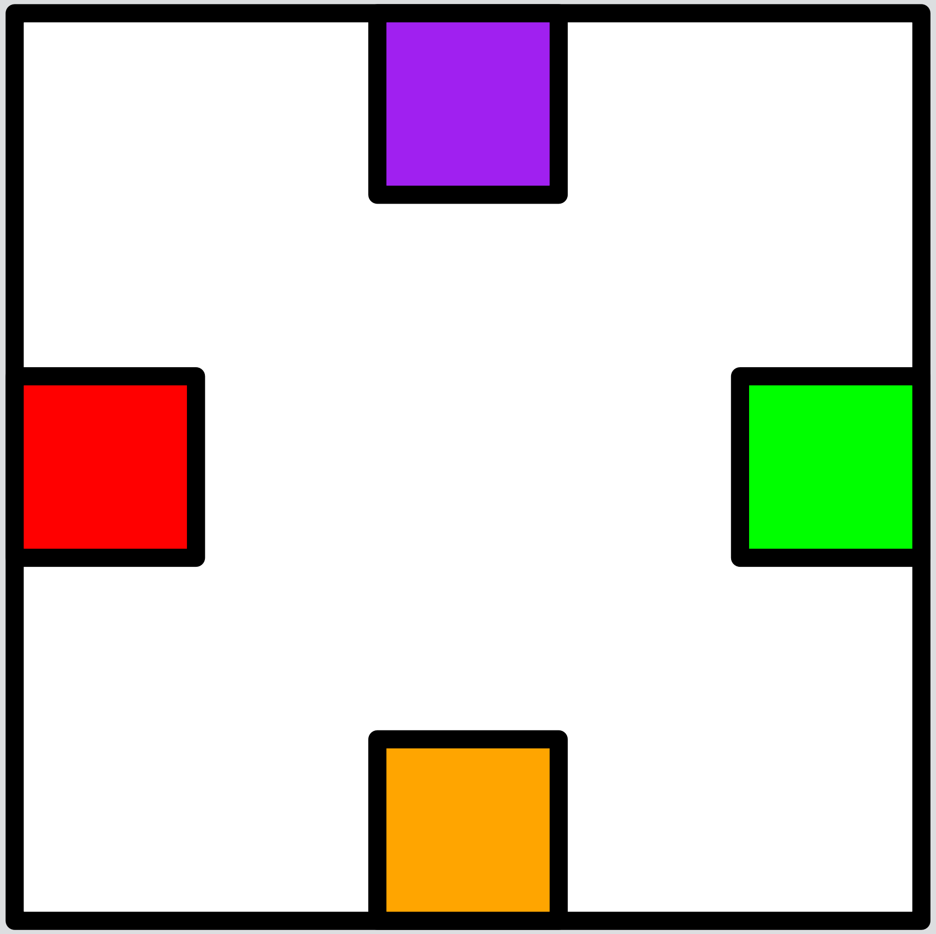
Synthetic Self-Assembly with DNA



Synthetic Self-Assembly with DNA



Tile Self-Assembly



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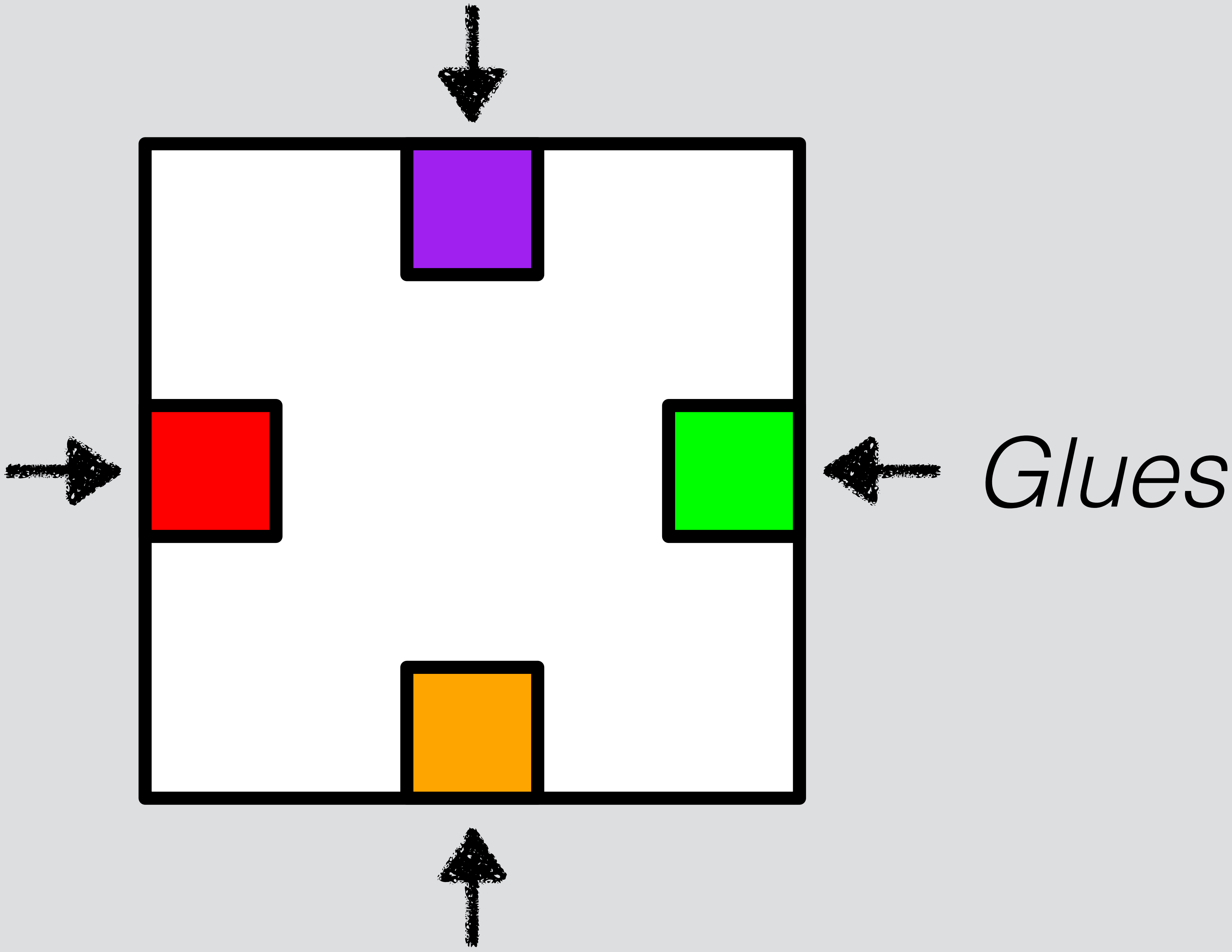

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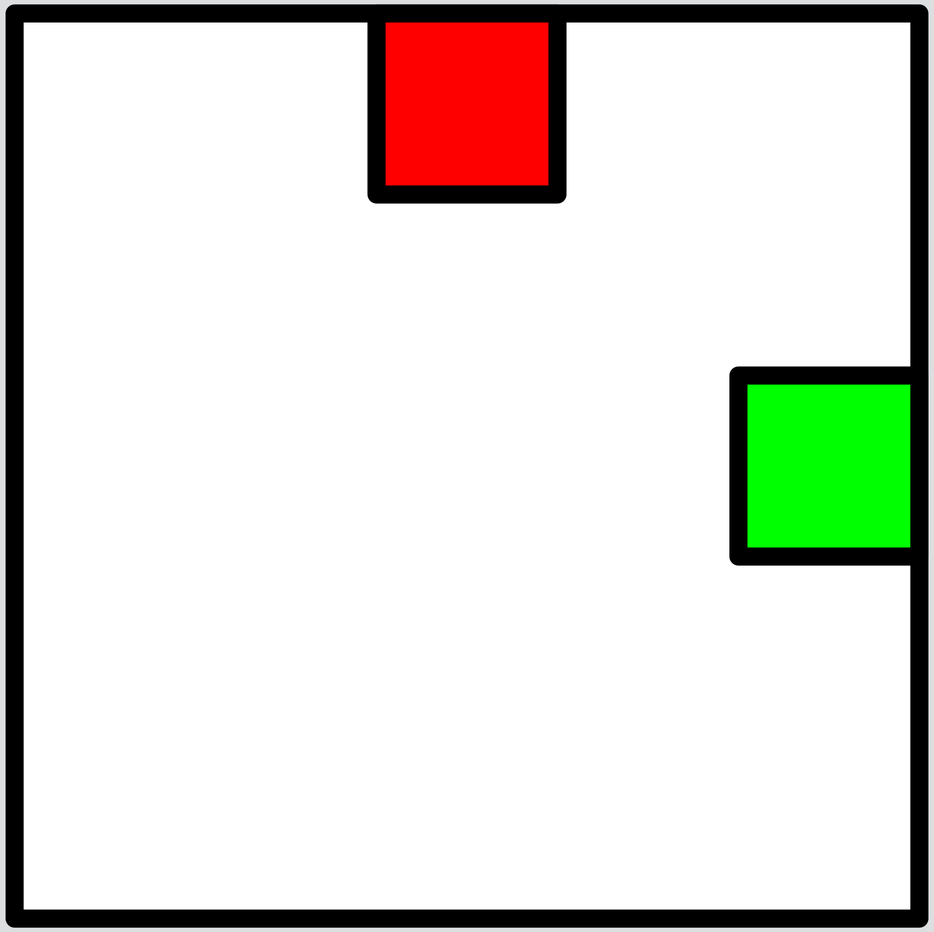

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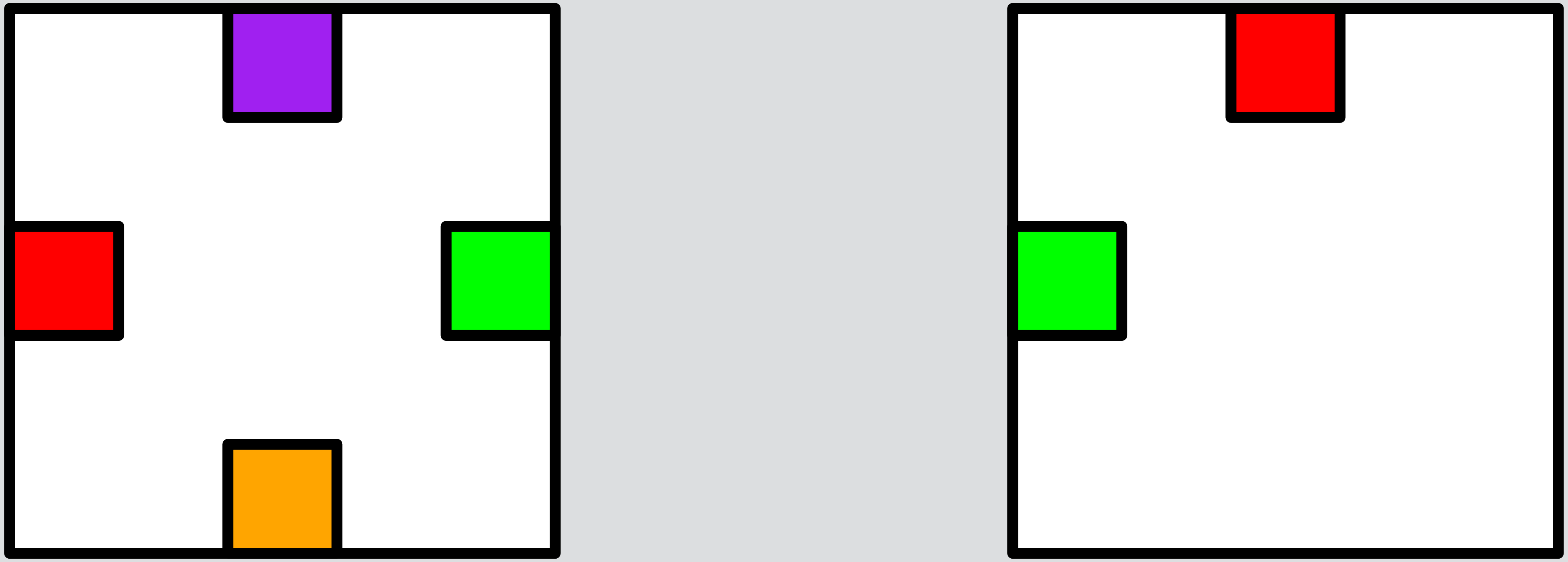

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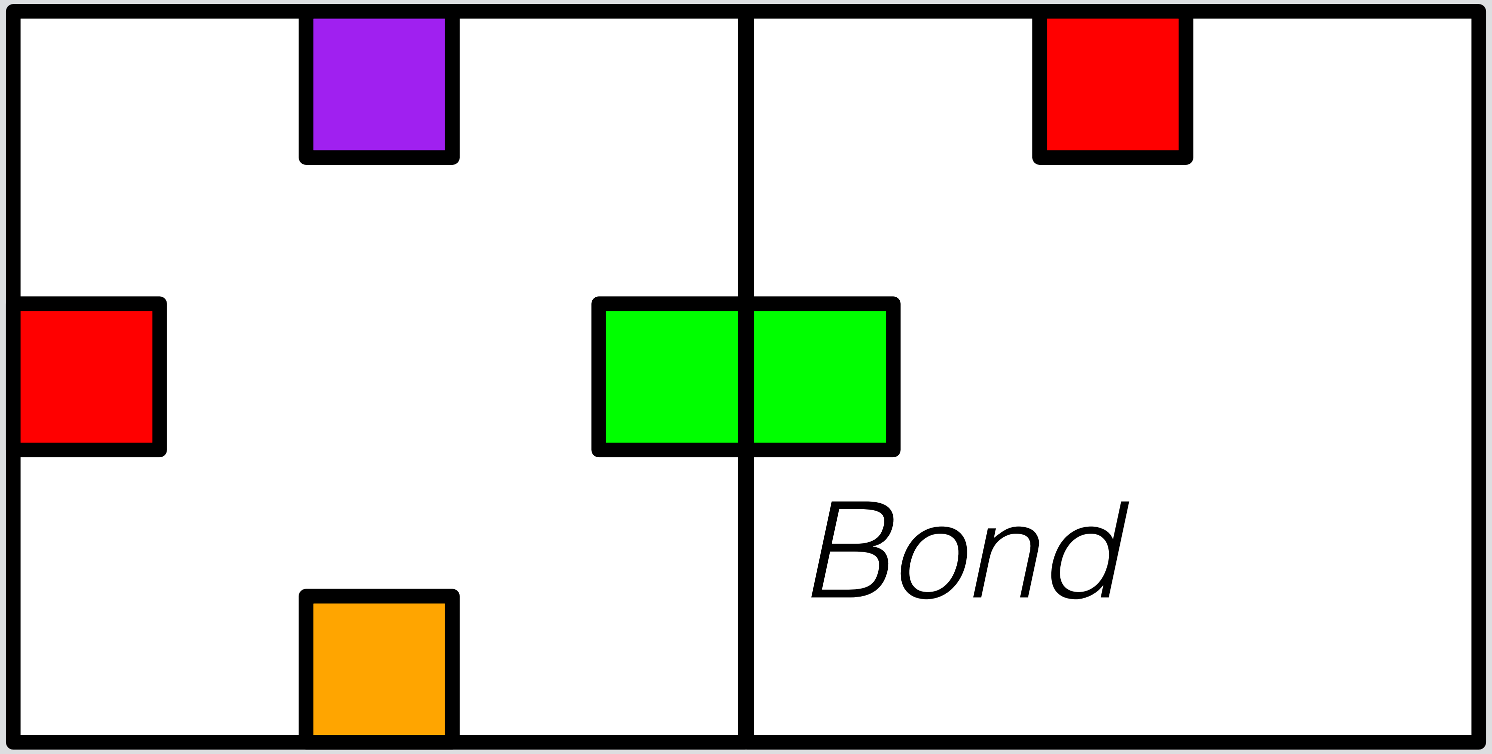
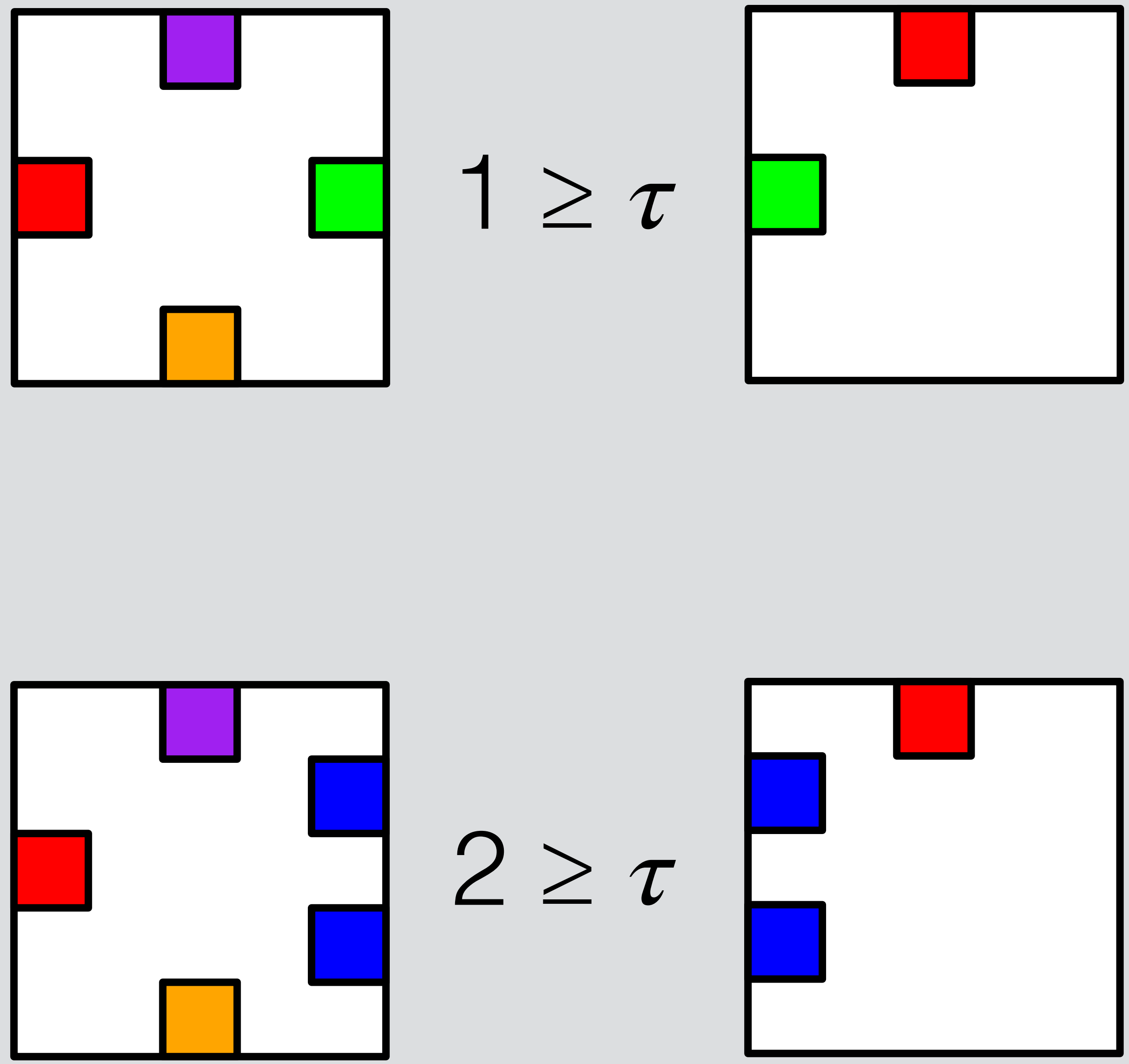


At temperature $\tau = 1$



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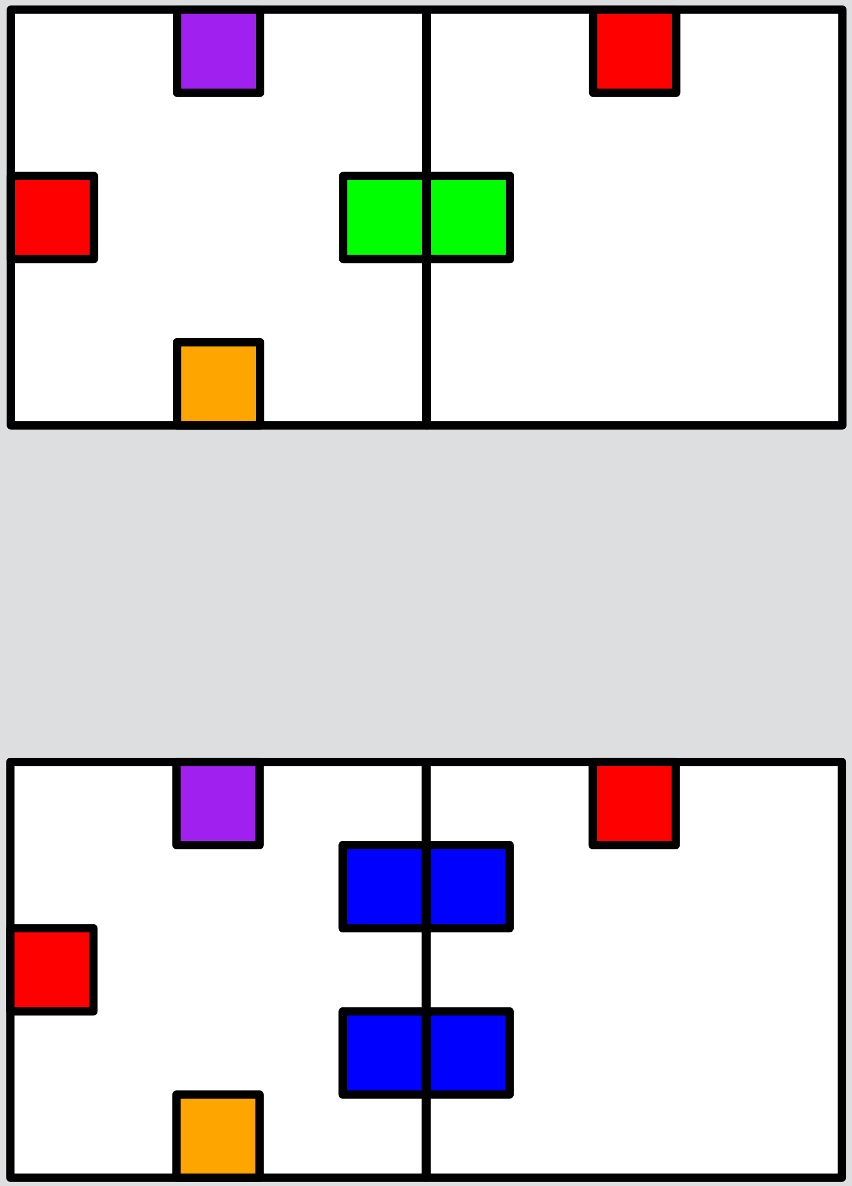
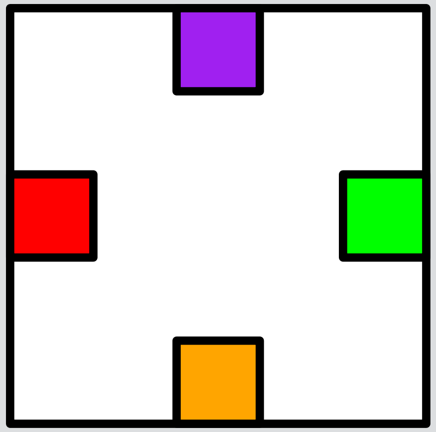
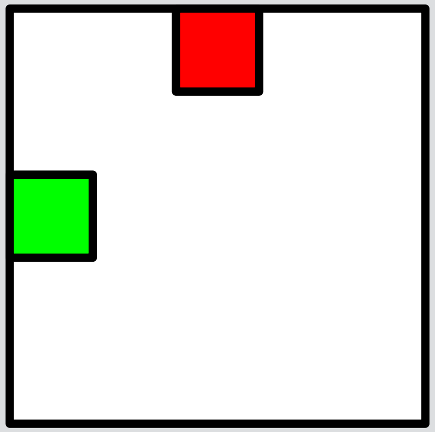
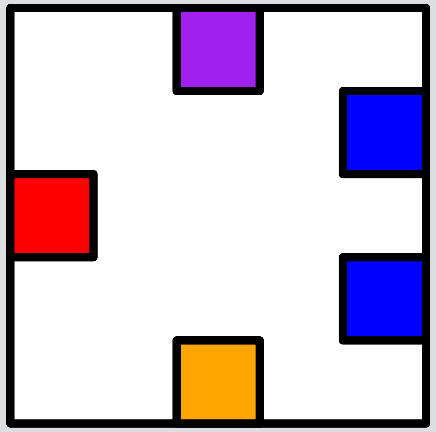


At temperature $\tau = 2$

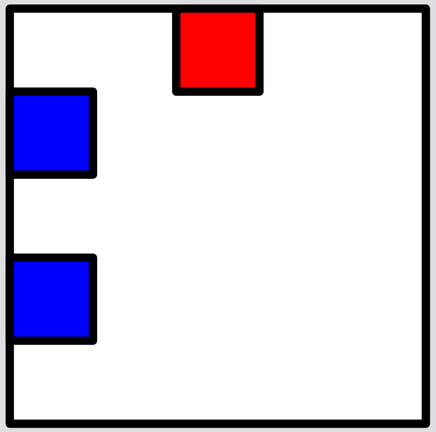


$1 \ngeq \tau$

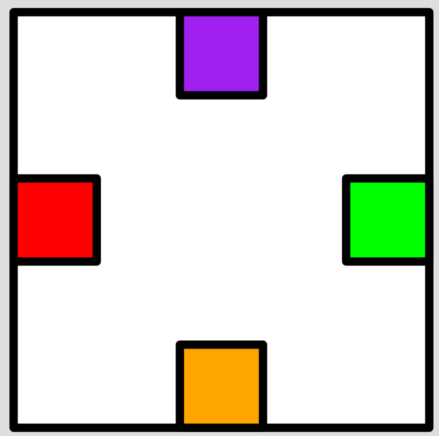




$2 \geq \tau$



At temperature $\tau = 2$



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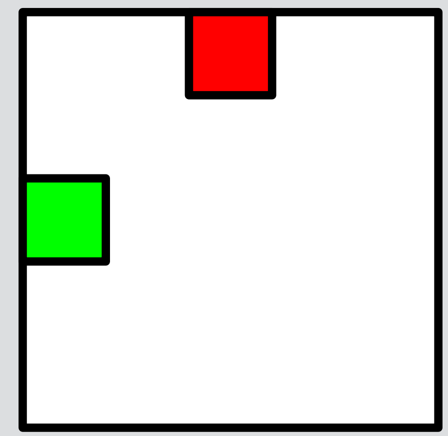
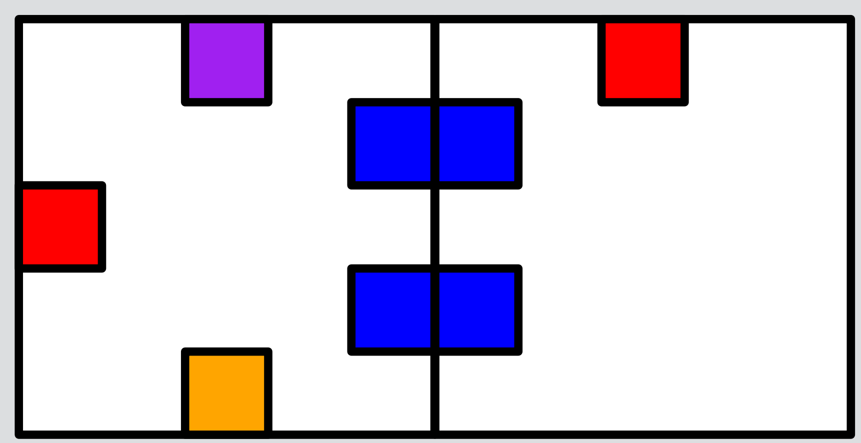
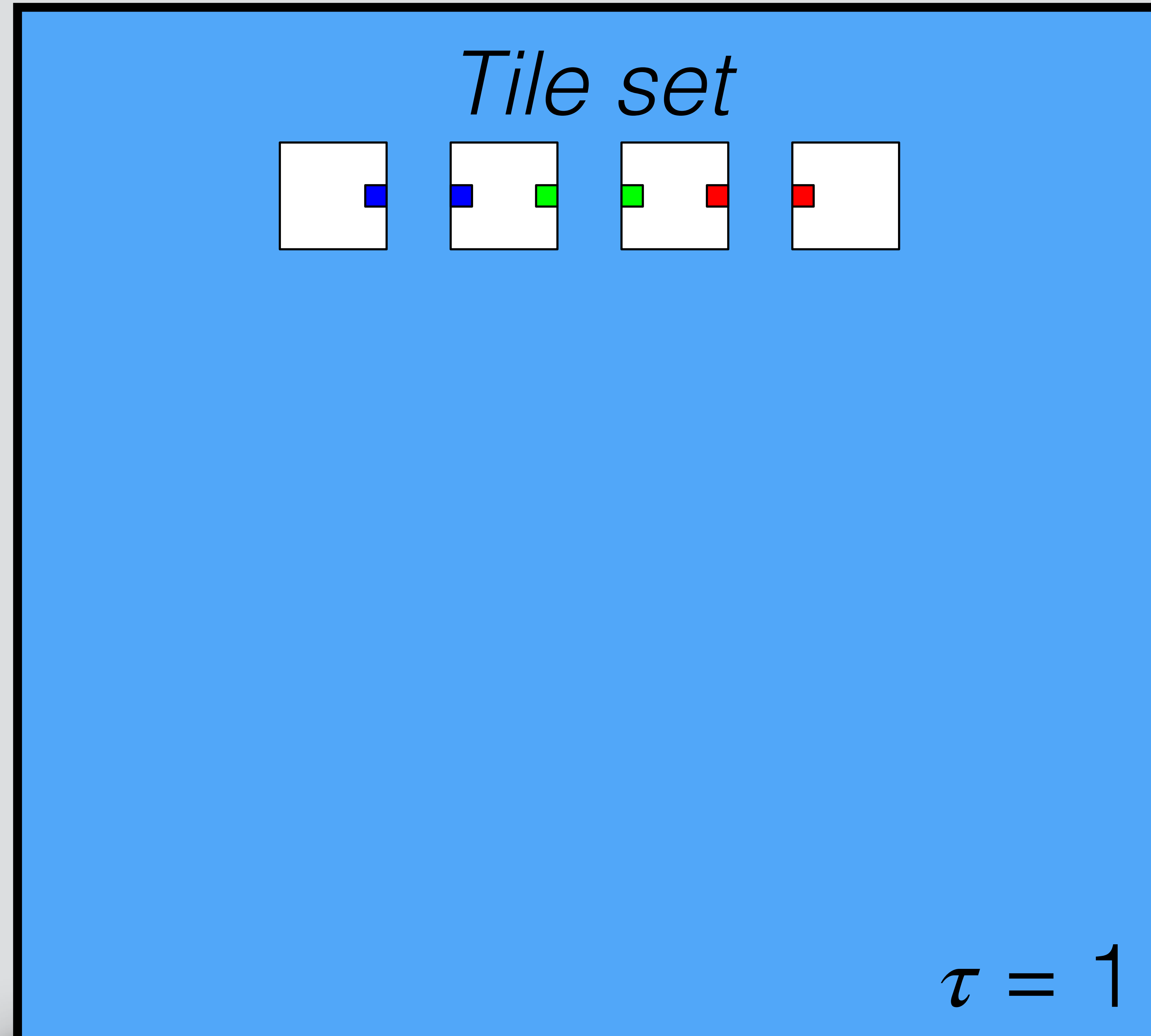


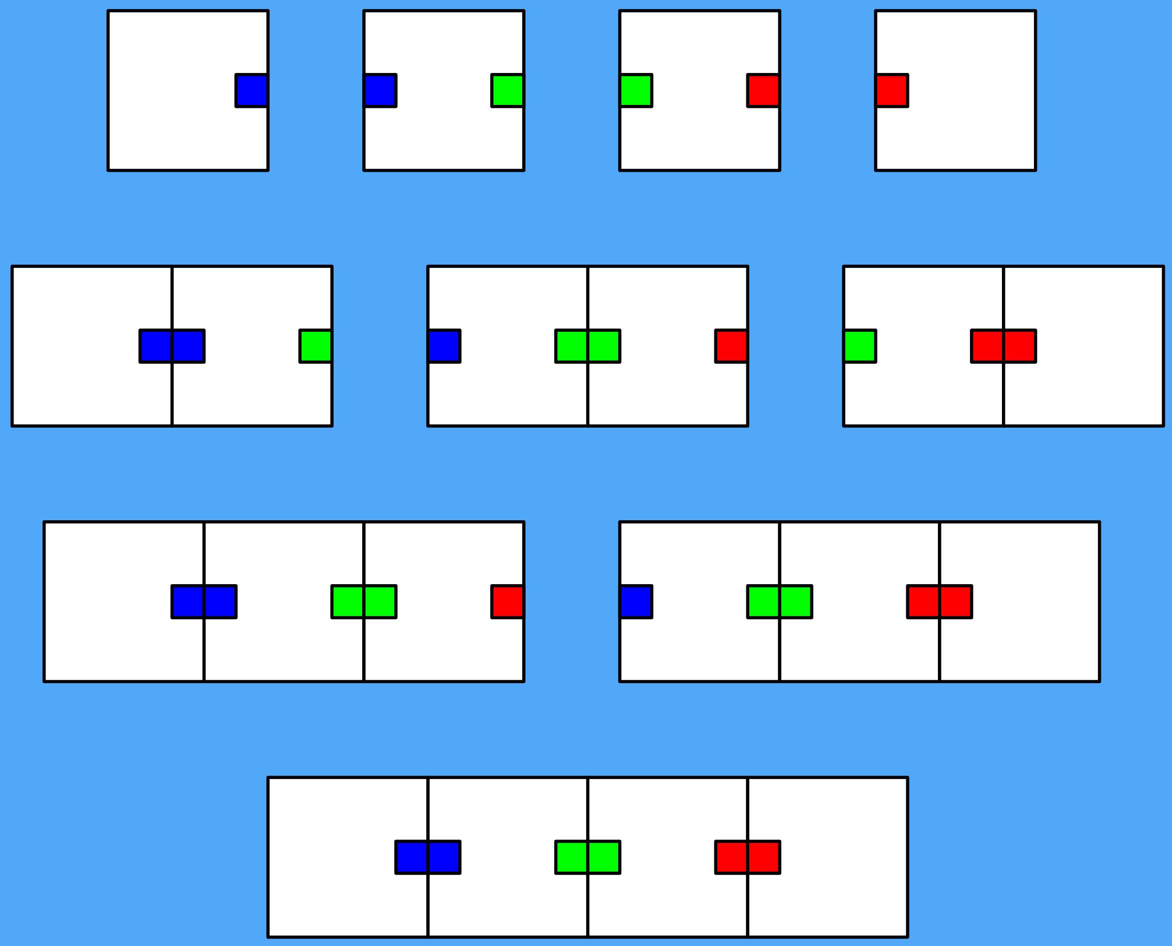


Two-handed assembly



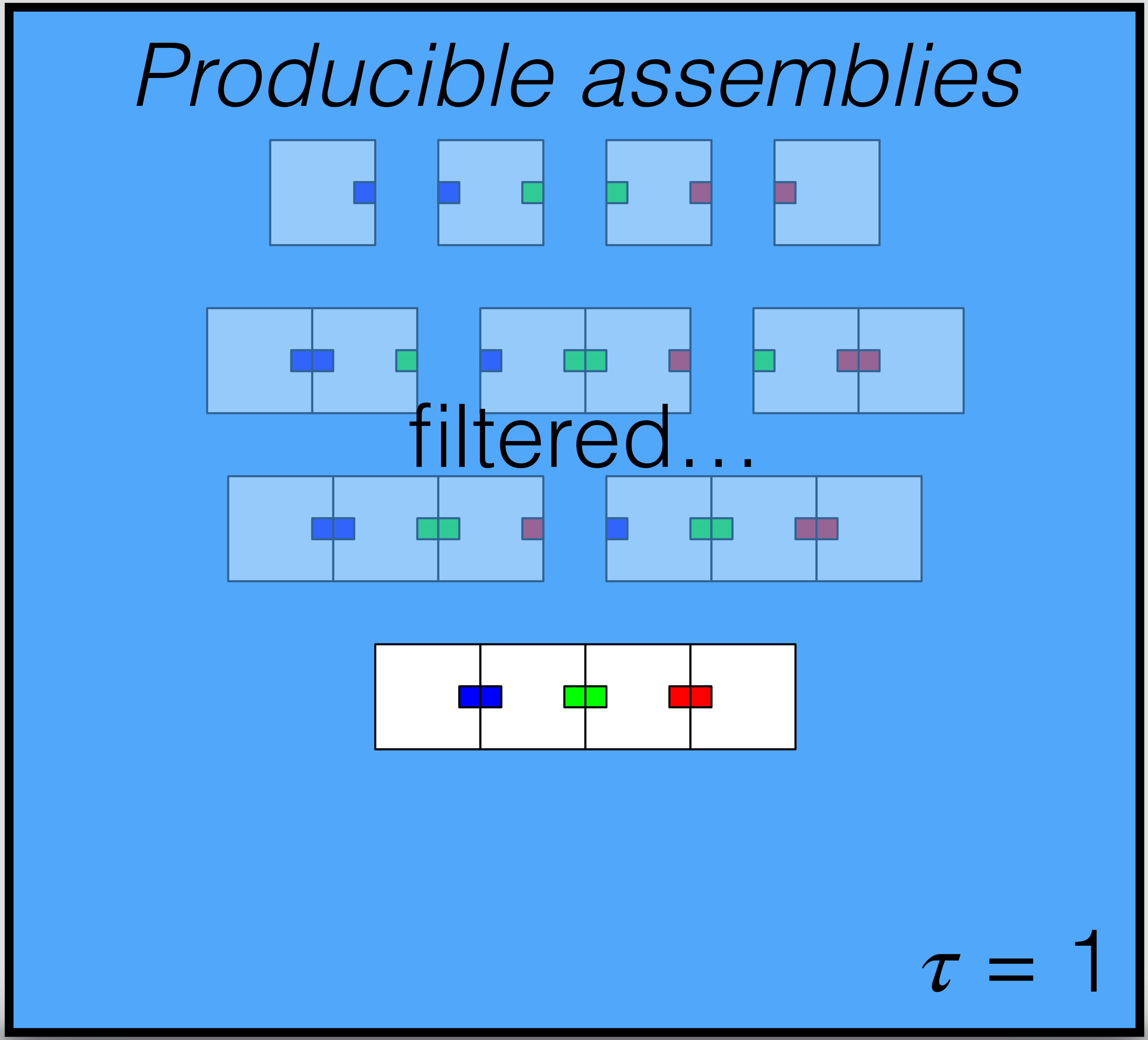
Two-handed assembly

Producible assemblies

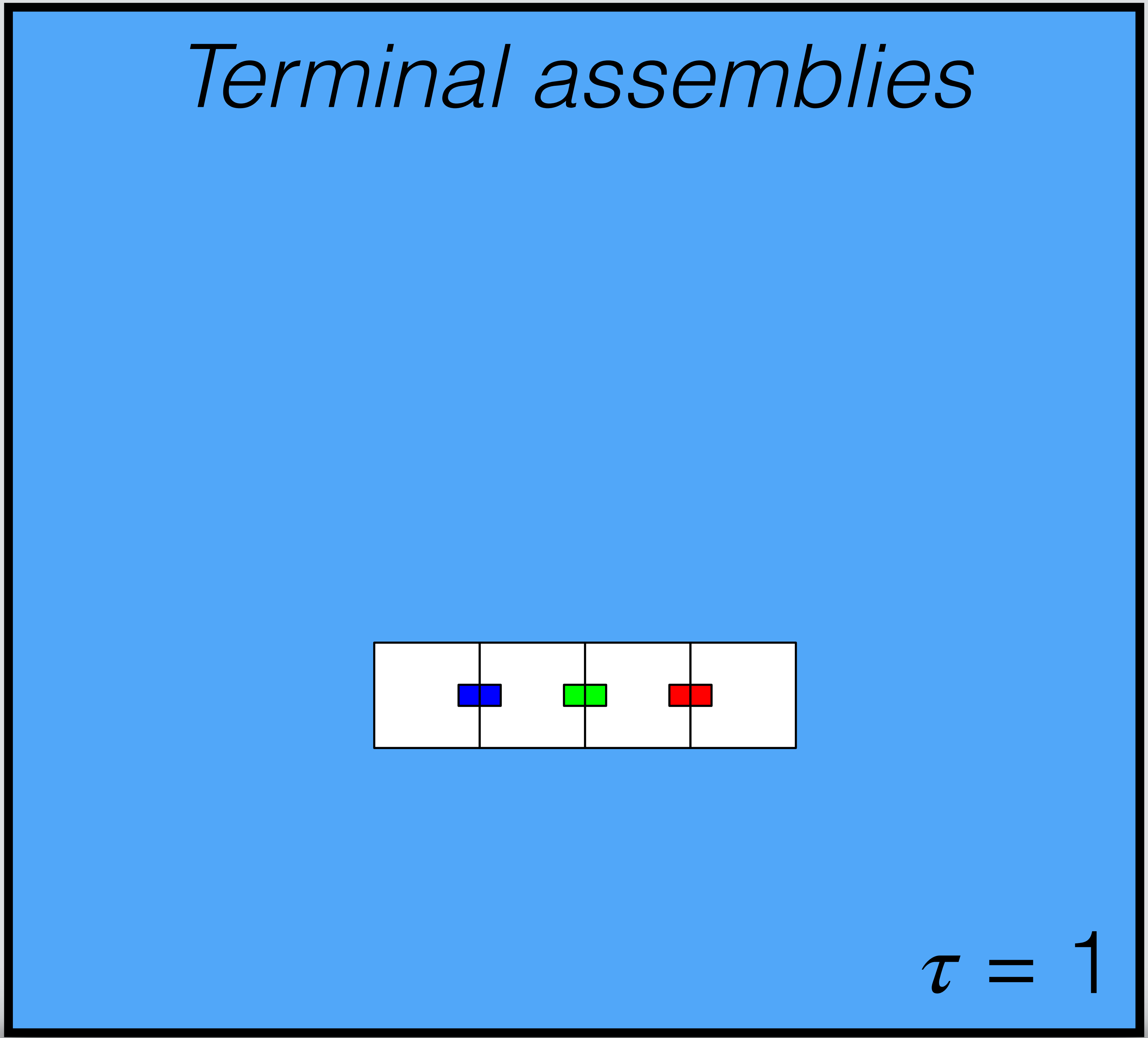


$\tau = 1$

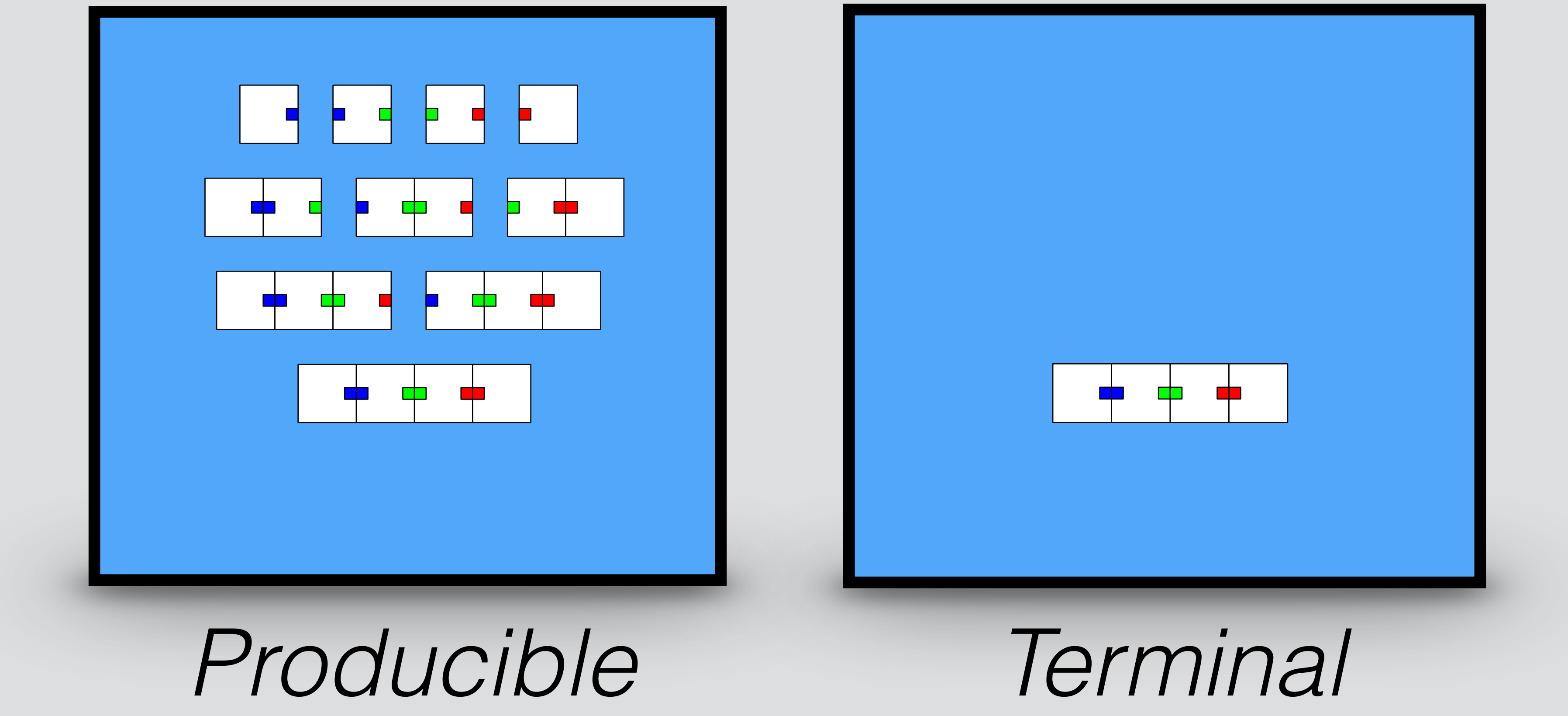
Two-handed assembly

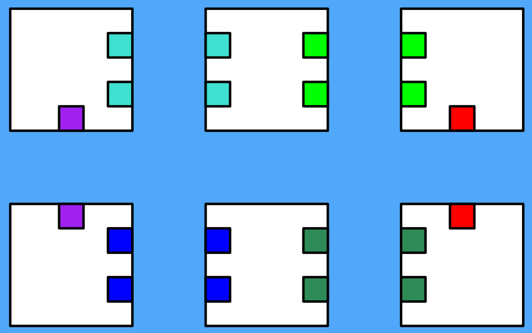


Two-handed assembly



How practical is this filtering?





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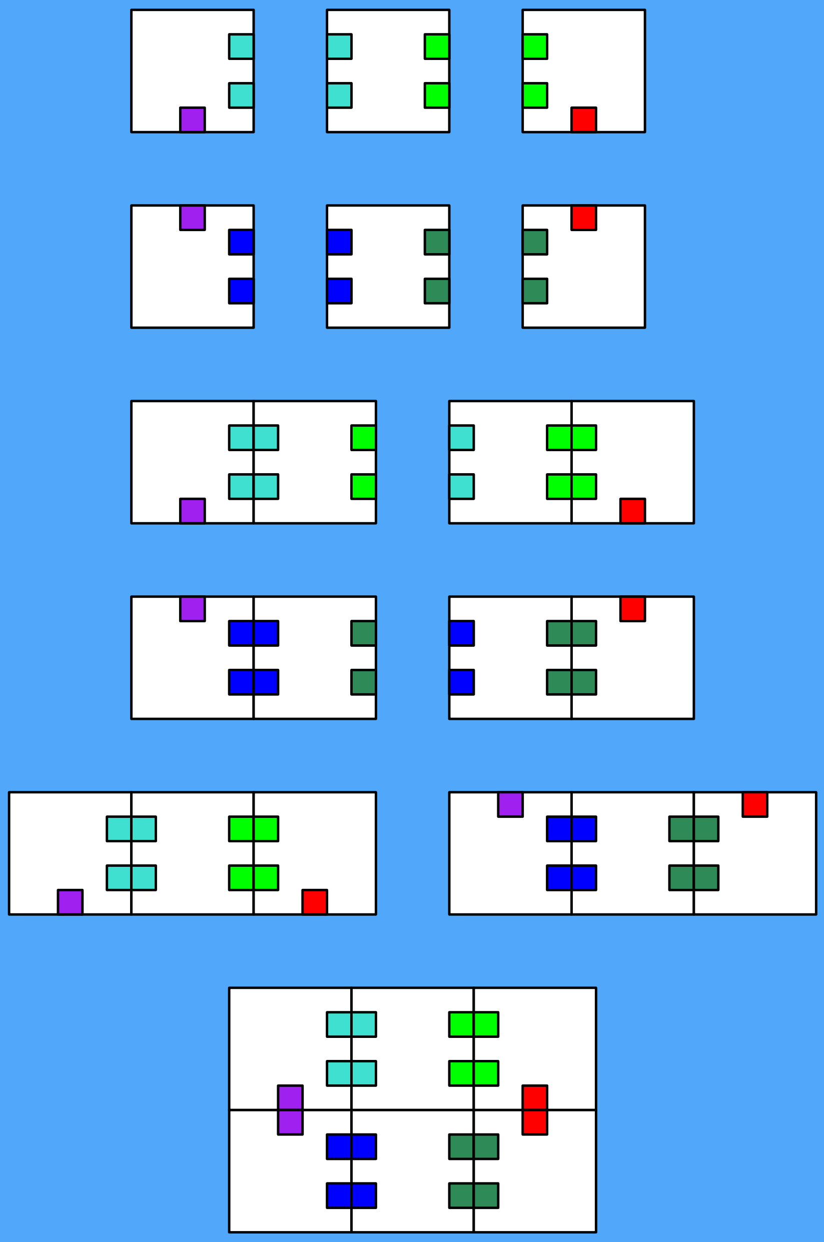
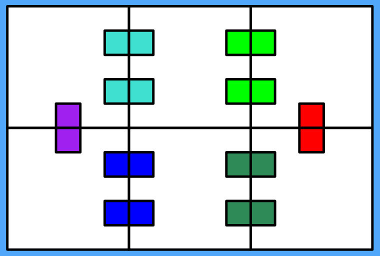
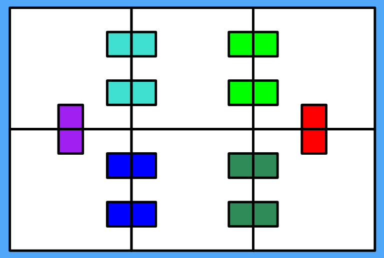


$\tau = 2$

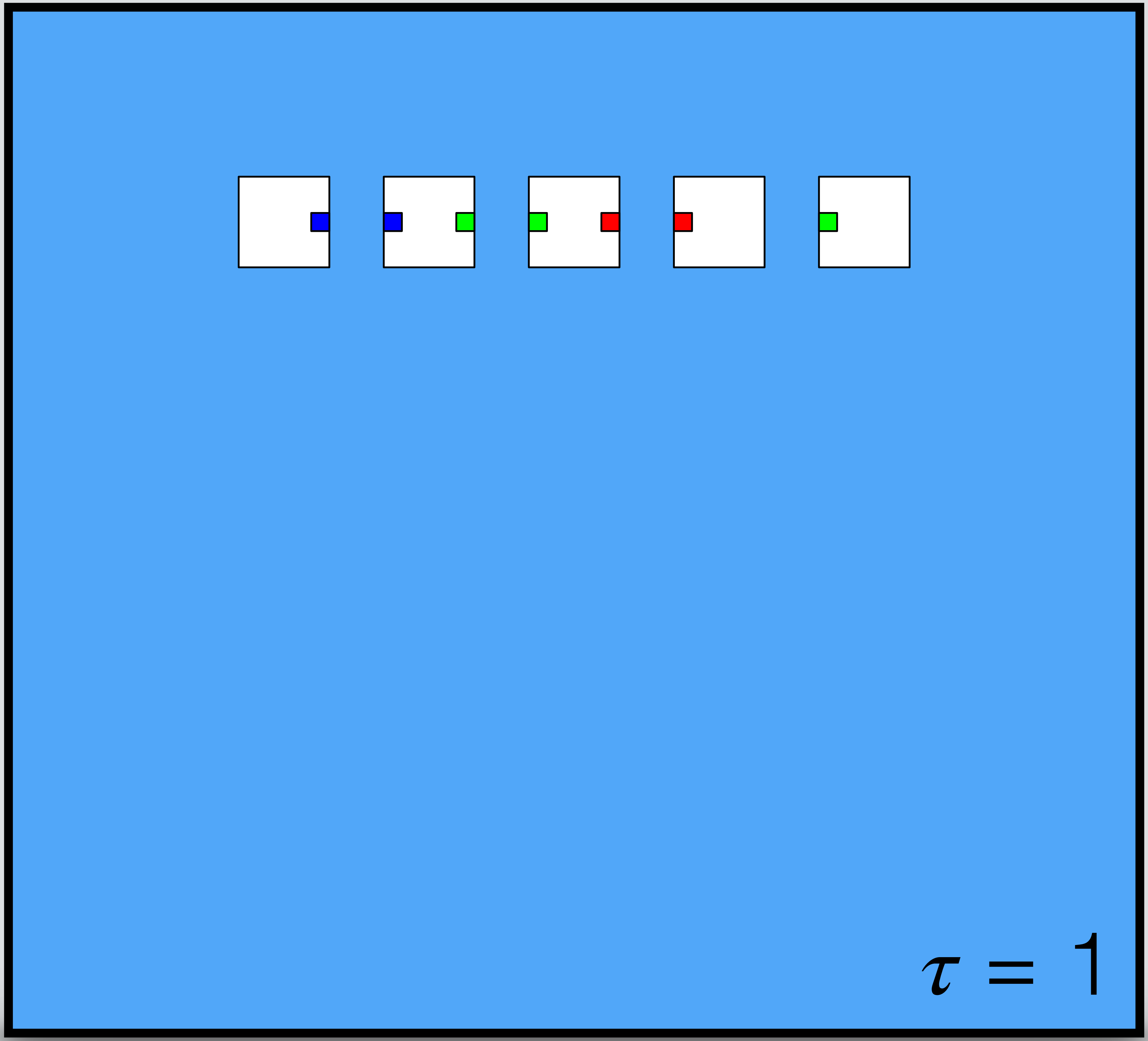


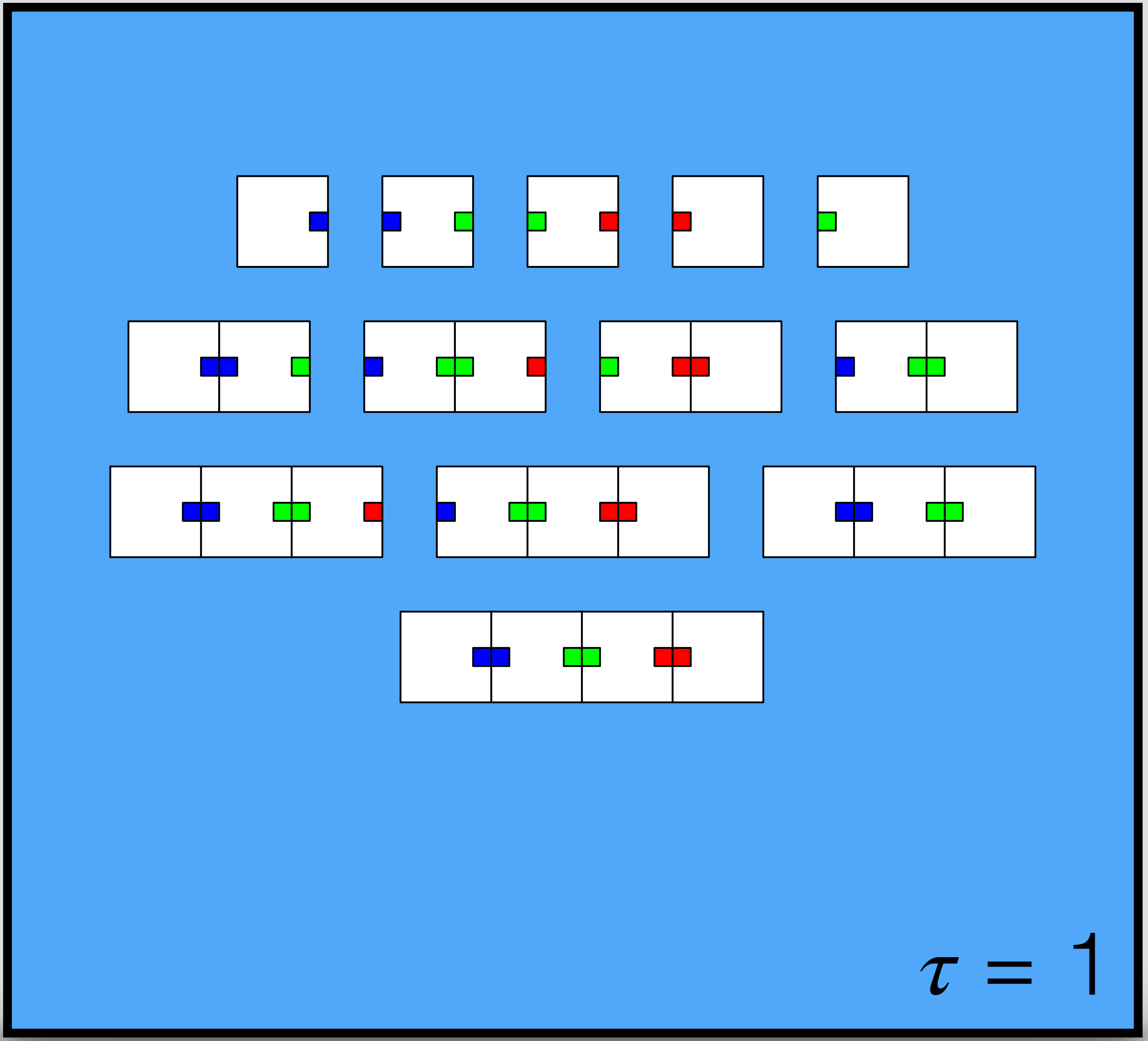
$\tau = 2$

Terminal assembly twice the size of any other producible assemblies.

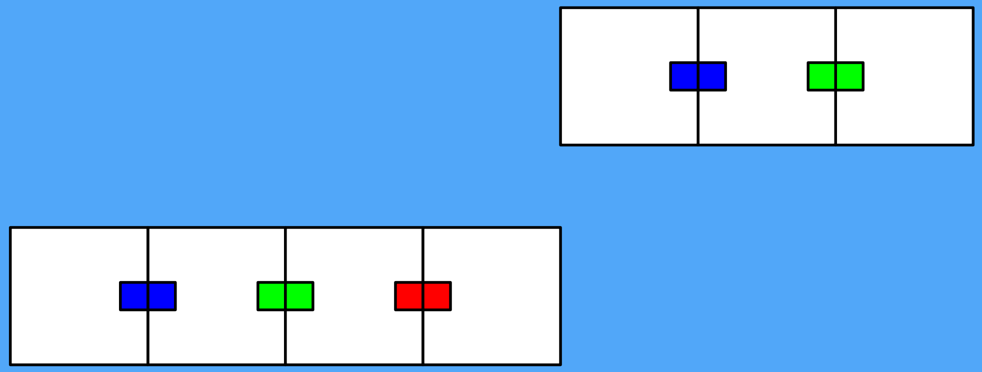


$\tau = 2$

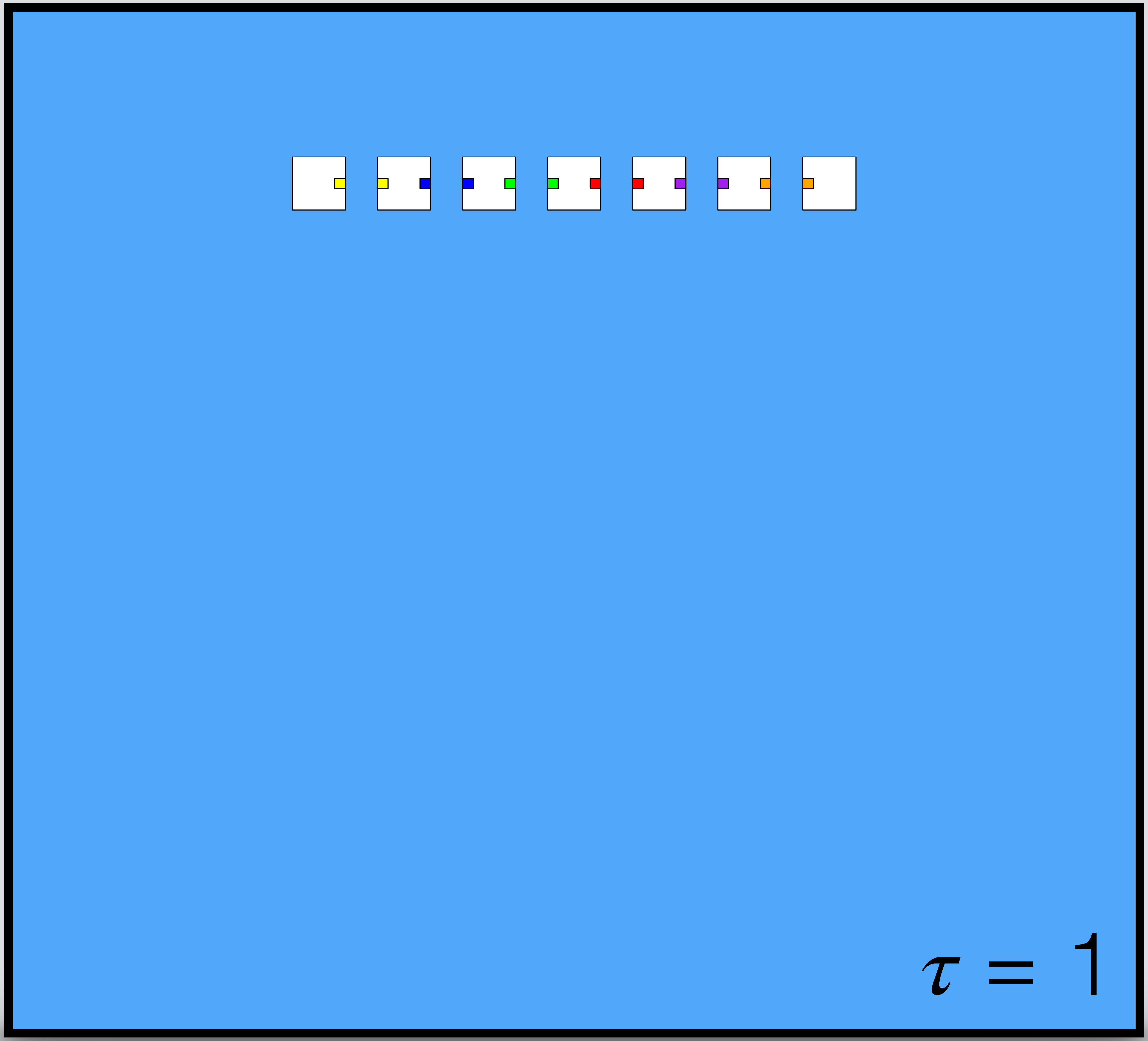

$\tau = 1$


$\tau = 1$

Terminal assembly has the same size as a non-terminal assembly.



$\tau = 1$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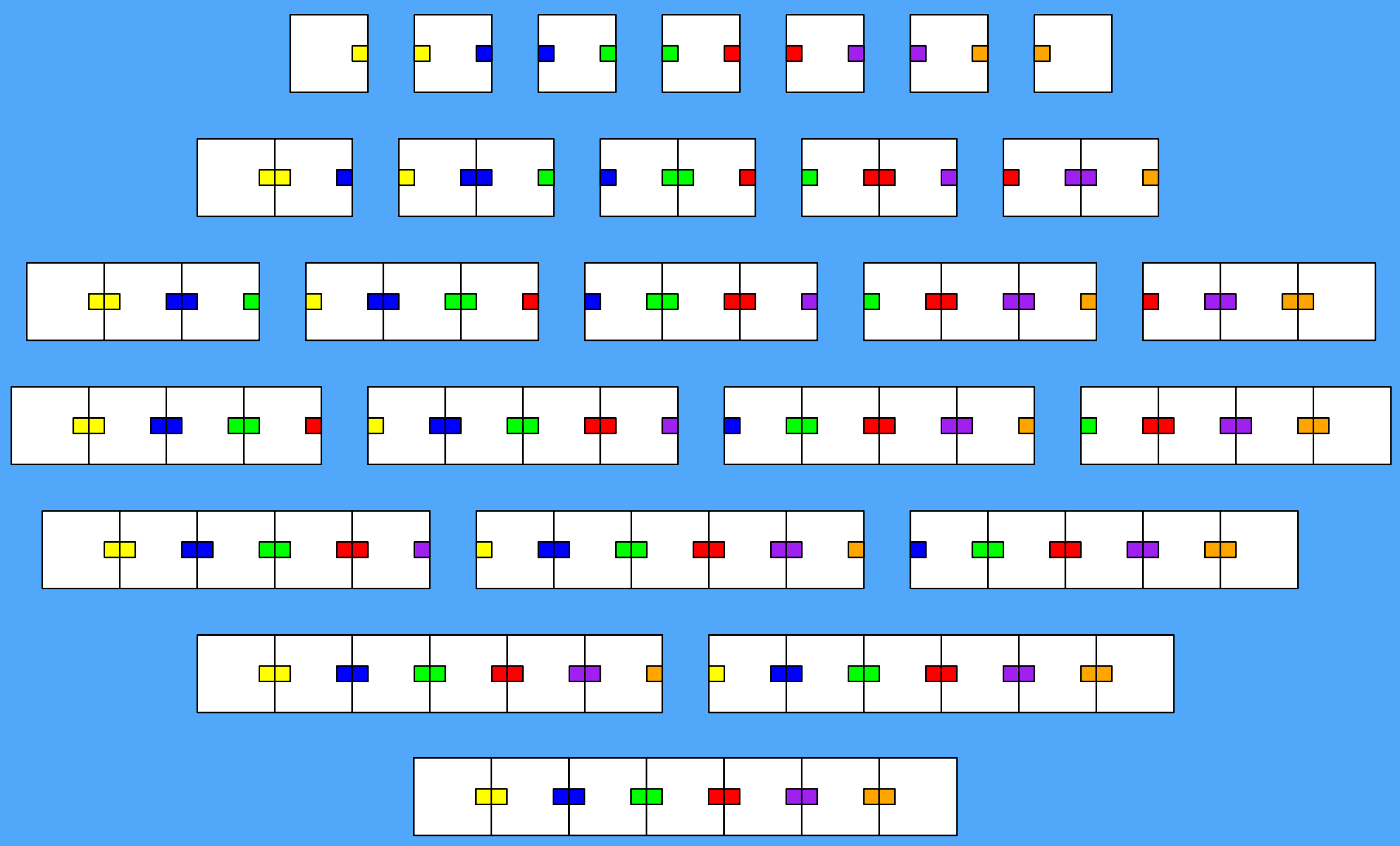
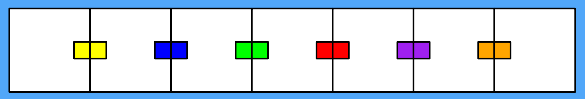


$\tau = 1$

Terminal assembly has almost the same size as a non-terminal assembly.



$\tau = 1$

Last year…

DNA19

19th International Conference on DNA Computing and Molecular Programming

September 22-27, 2013
Arizona State University, Tempe, AZ, USA

- Home
- Program
- Speakers
- Nanoday 2013

- Call for Papers
- Important Dates
- Registration
- Submissions
- Accepted Posters
- Prizes

- Program Committee
- Organizing Committee
- Steering Committee

- Location and Travel
- Accommodations
- Directions
- Meal Options
- Contact Us

ASU
Arizona State University

ISNSCE
International Society for Nanoscale Science, Computation and Engineering

The 19th International Conference on DNA Computing and Molecular Programming will be held September 22-27, 2013 at Arizona State University in Tempe, Arizona. On Friday September 27, there will be a Nanoday focused on nanotechnology research related to the field's interest.

Research in DNA computing and molecular programming draws together many disciplines (including mathematics, computer science, physics, chemistry, material science and biology) to address the analysis, design, and synthesis of information-based molecular systems. This annual meeting is the premier forum where scientists with diverse backgrounds come together with the common purpose of applying principles and tools of computer science, physics, chemistry and mathematics to advance molecular-scale computation and nanoengineering. Continuing this tradition, the 19th International Conference on DNA Computing and Molecular Programming (DNA19), organized under the auspices of the International Society for Nanoscale Science, Computation and Engineering (ISNSCE), will focus on important recent experimental and theoretical results.

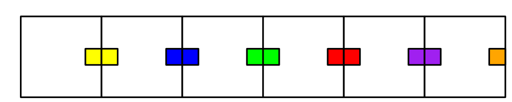
We are looking forward to seeing you in the desert!

Hao Yan
Chair of organizing committee

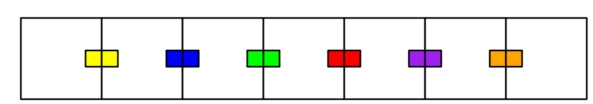
Erik Winfree

The filtering is not practical.

Removing almost-terminal assemblies is difficult.



vs.

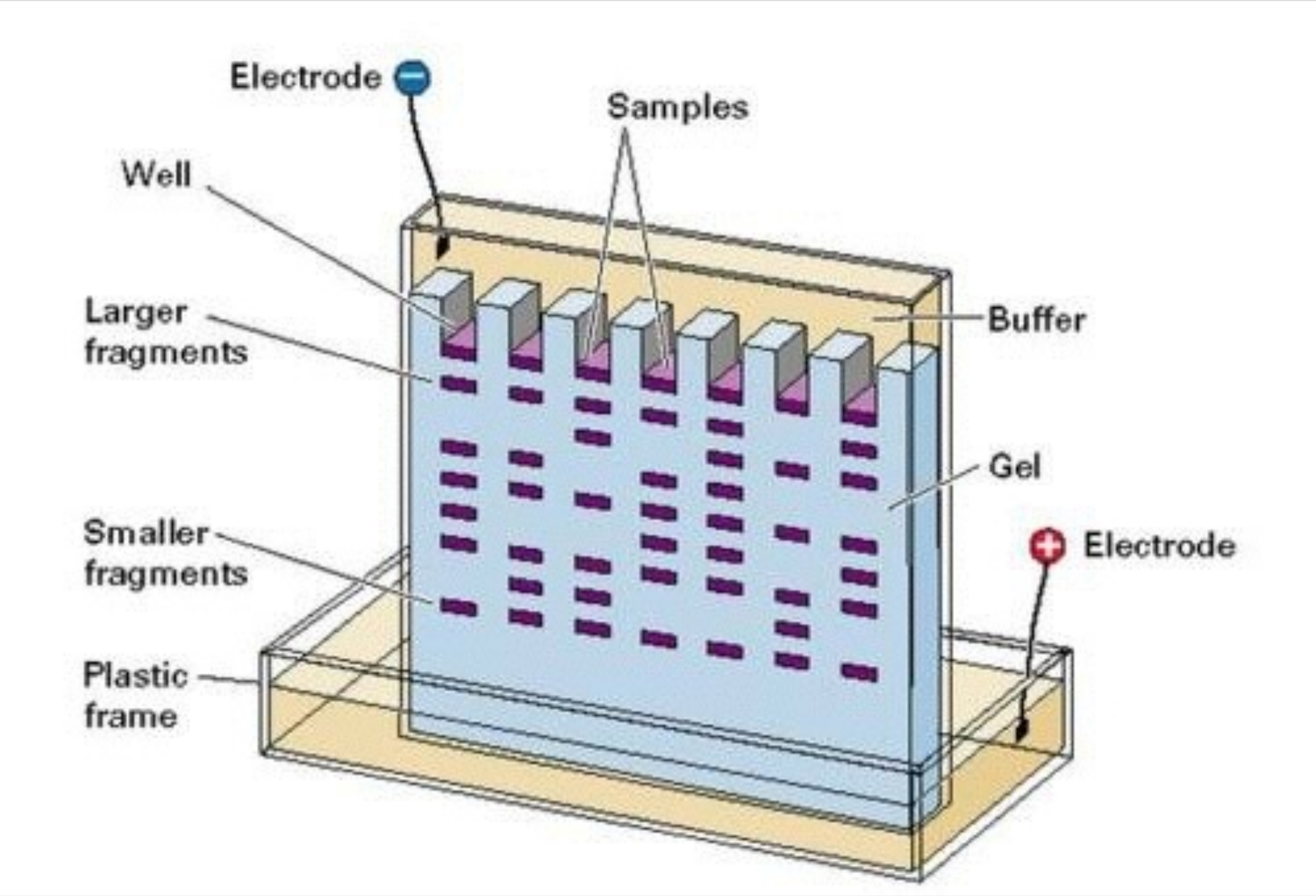


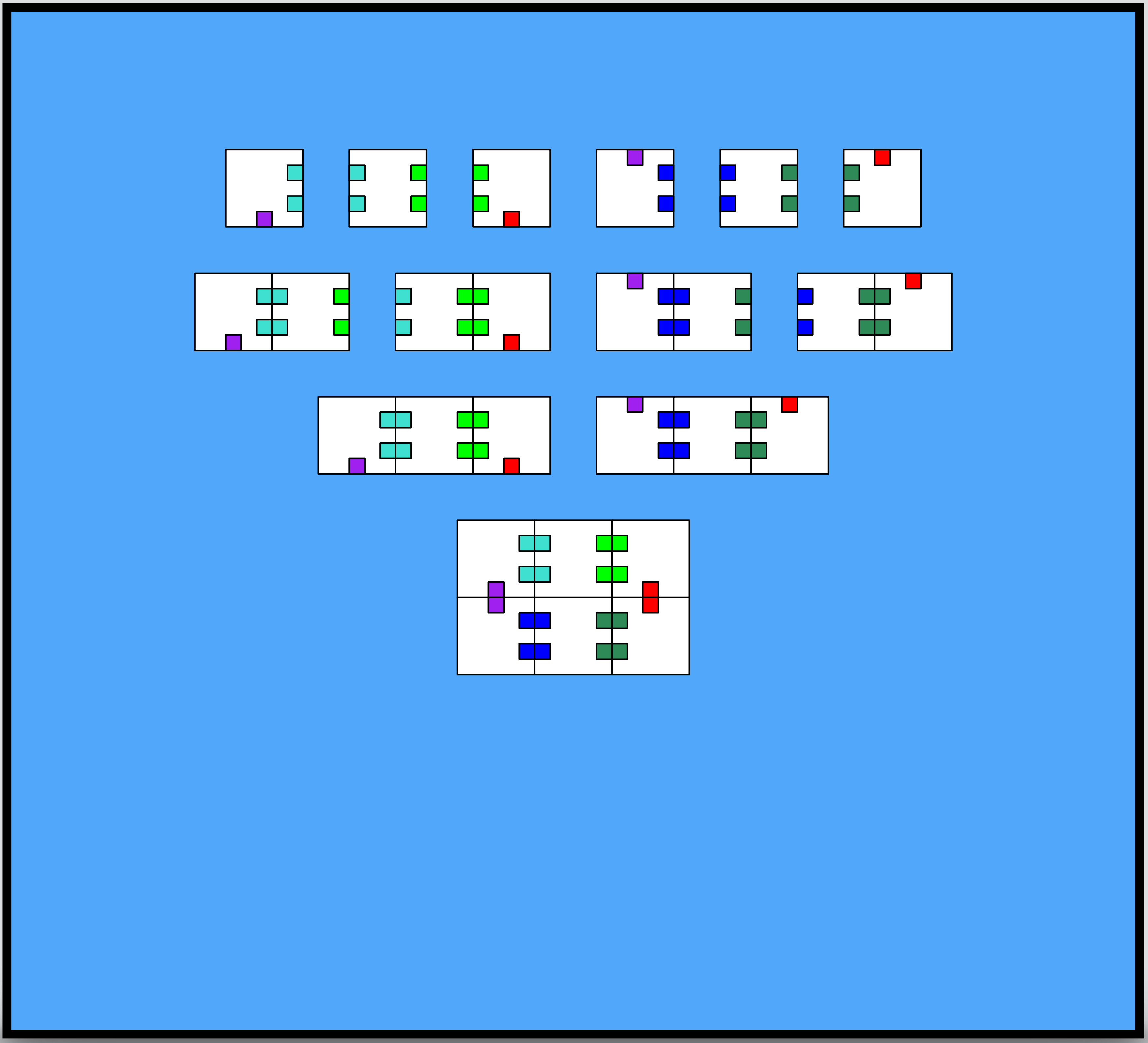
Our work

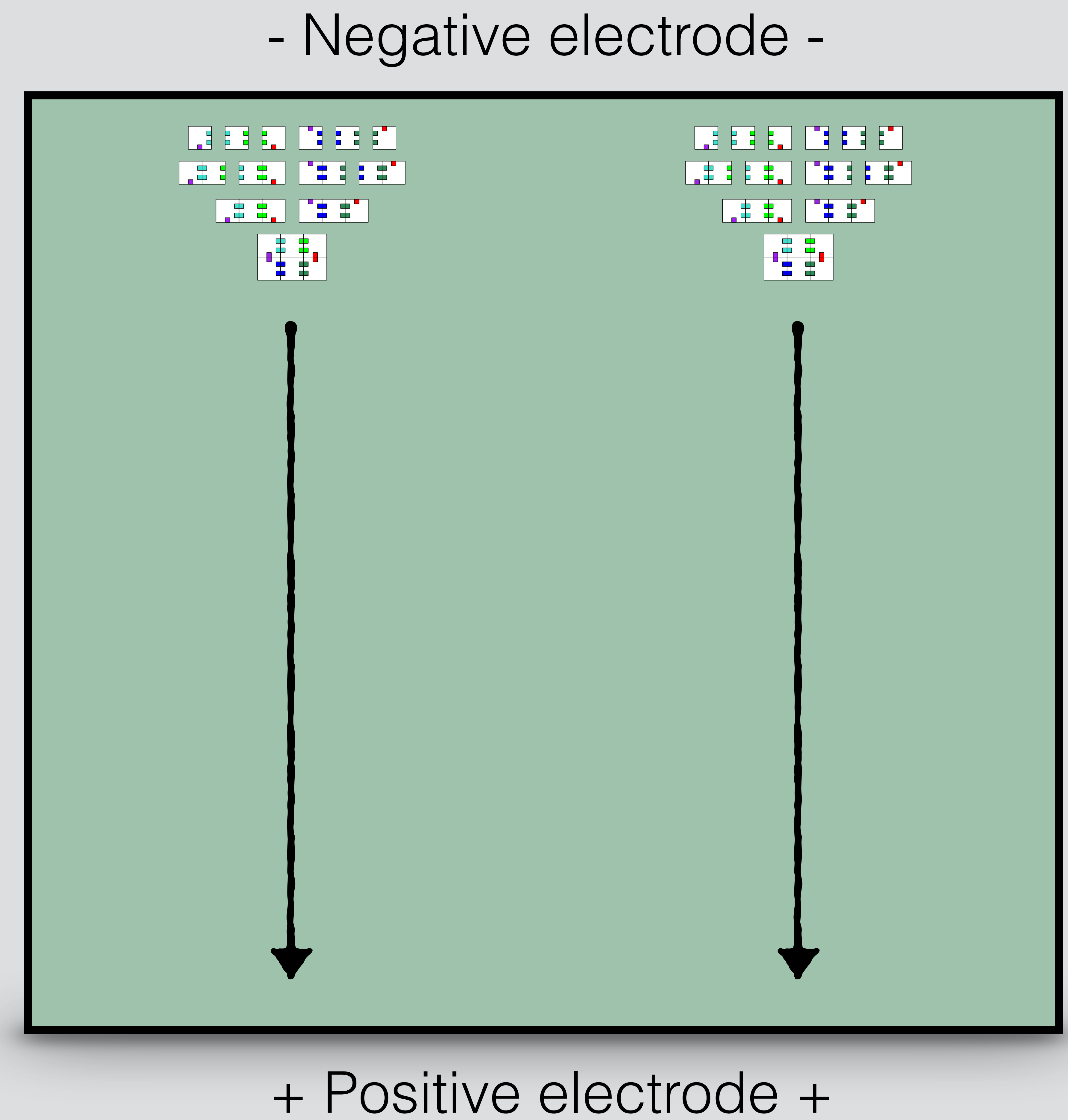
- Define a notion of how amenable a system is to size-based filtering.
    - *Size-separability*
- Give an algorithm for making a restricted class of systems size-separable.

Gel electrophoresis

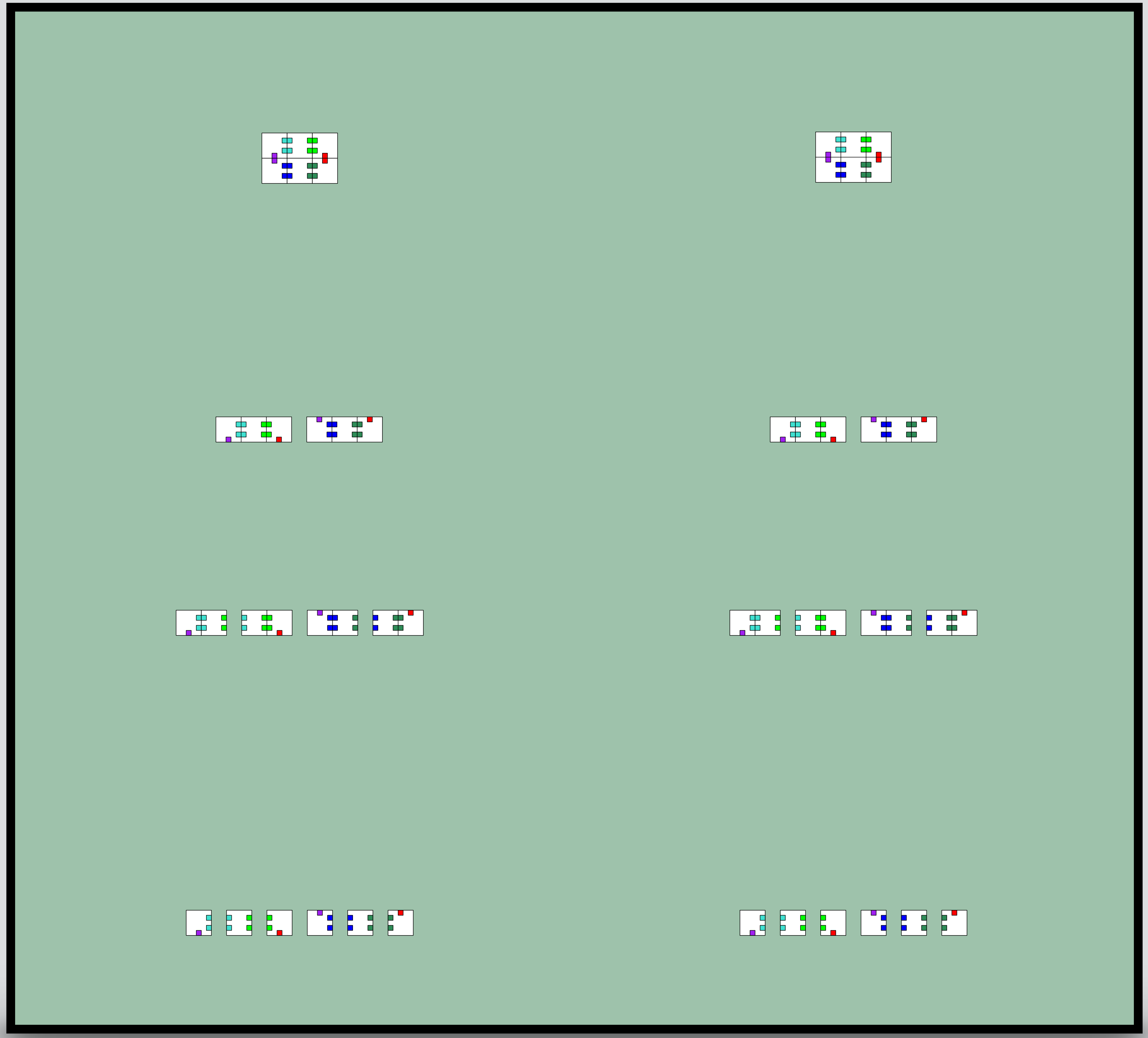
Gel electrophoresis



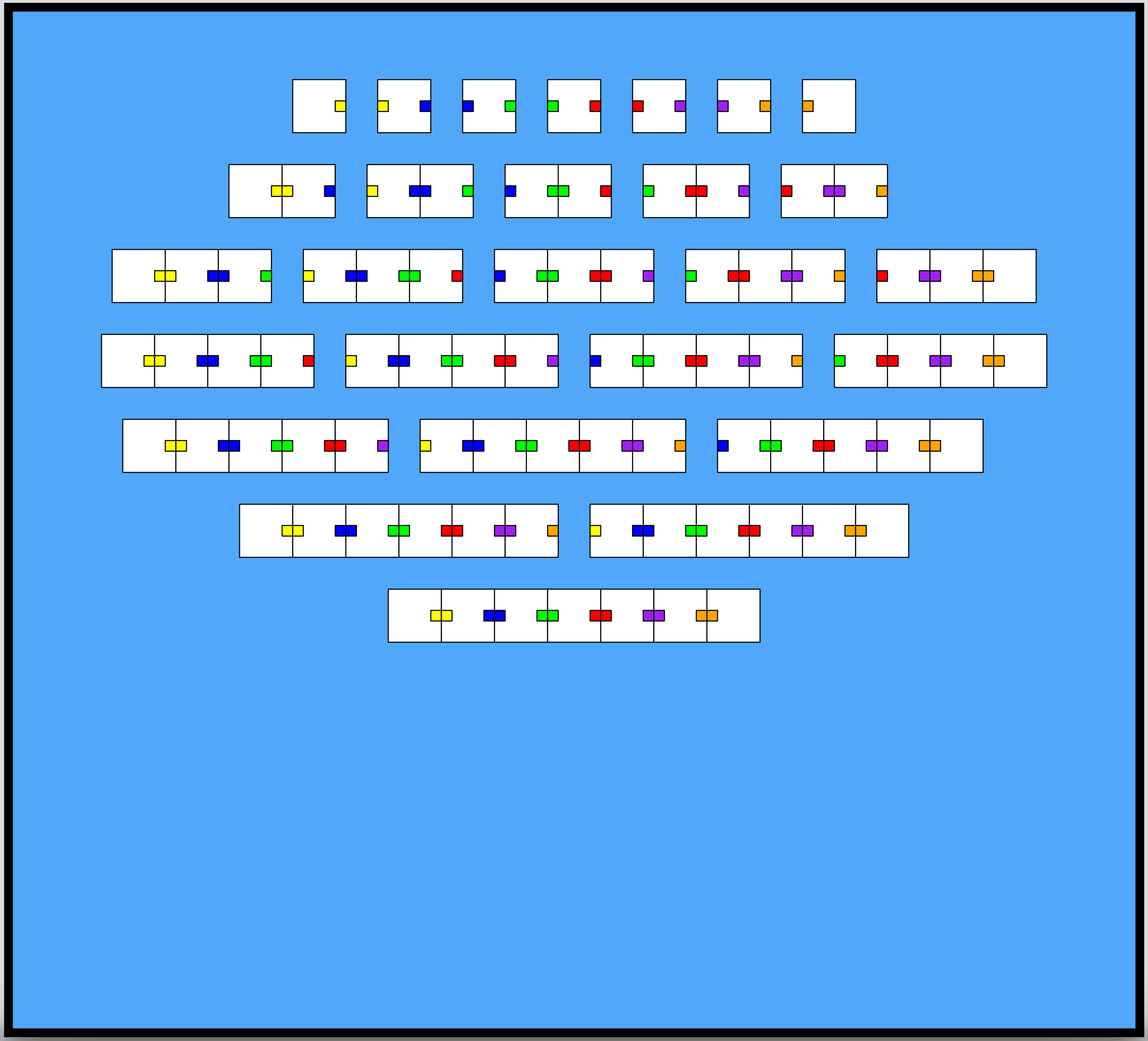



- Negative electrode -
+ Positive electrode +

- Negative electrode -



+ Positive electrode +



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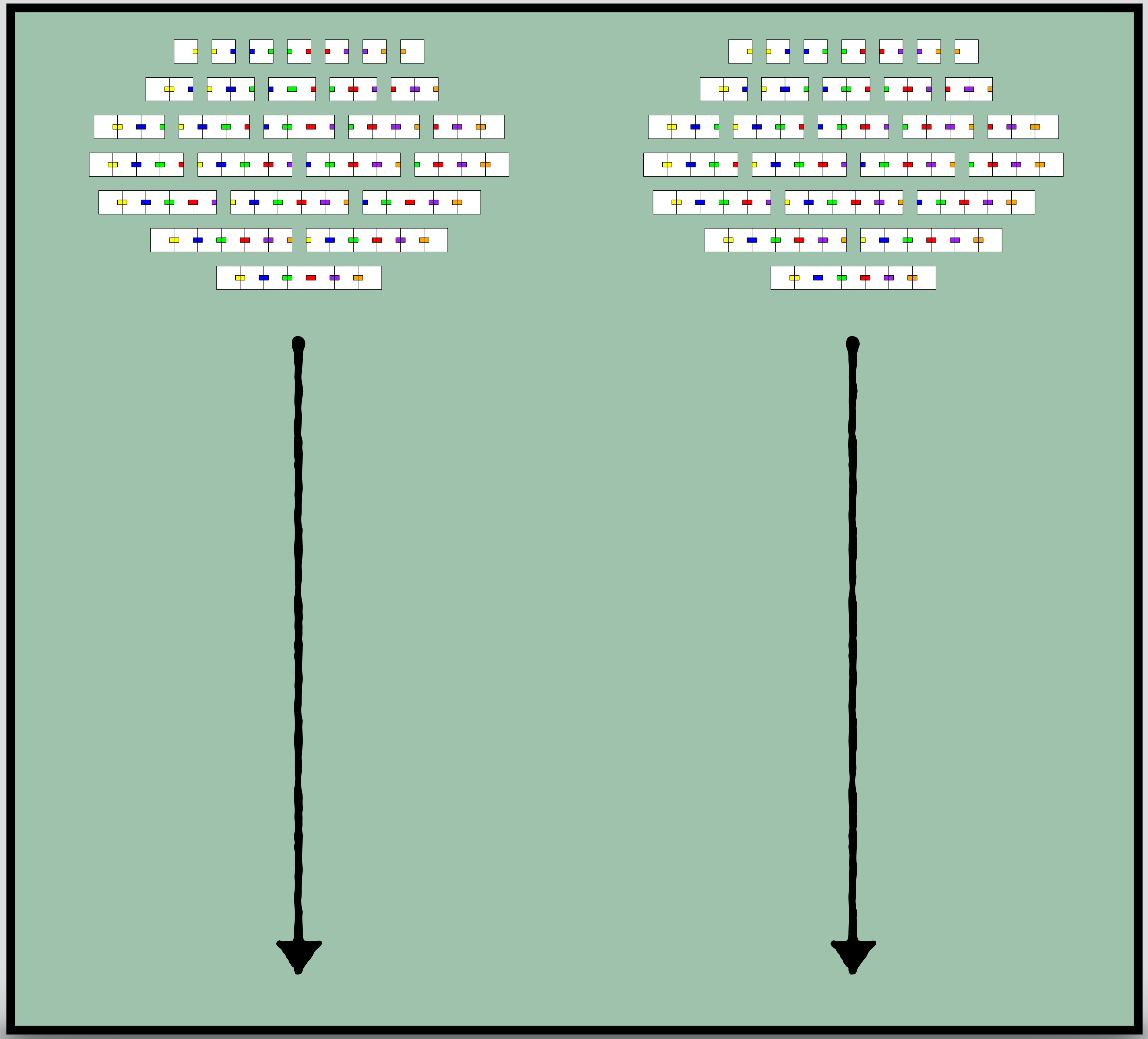
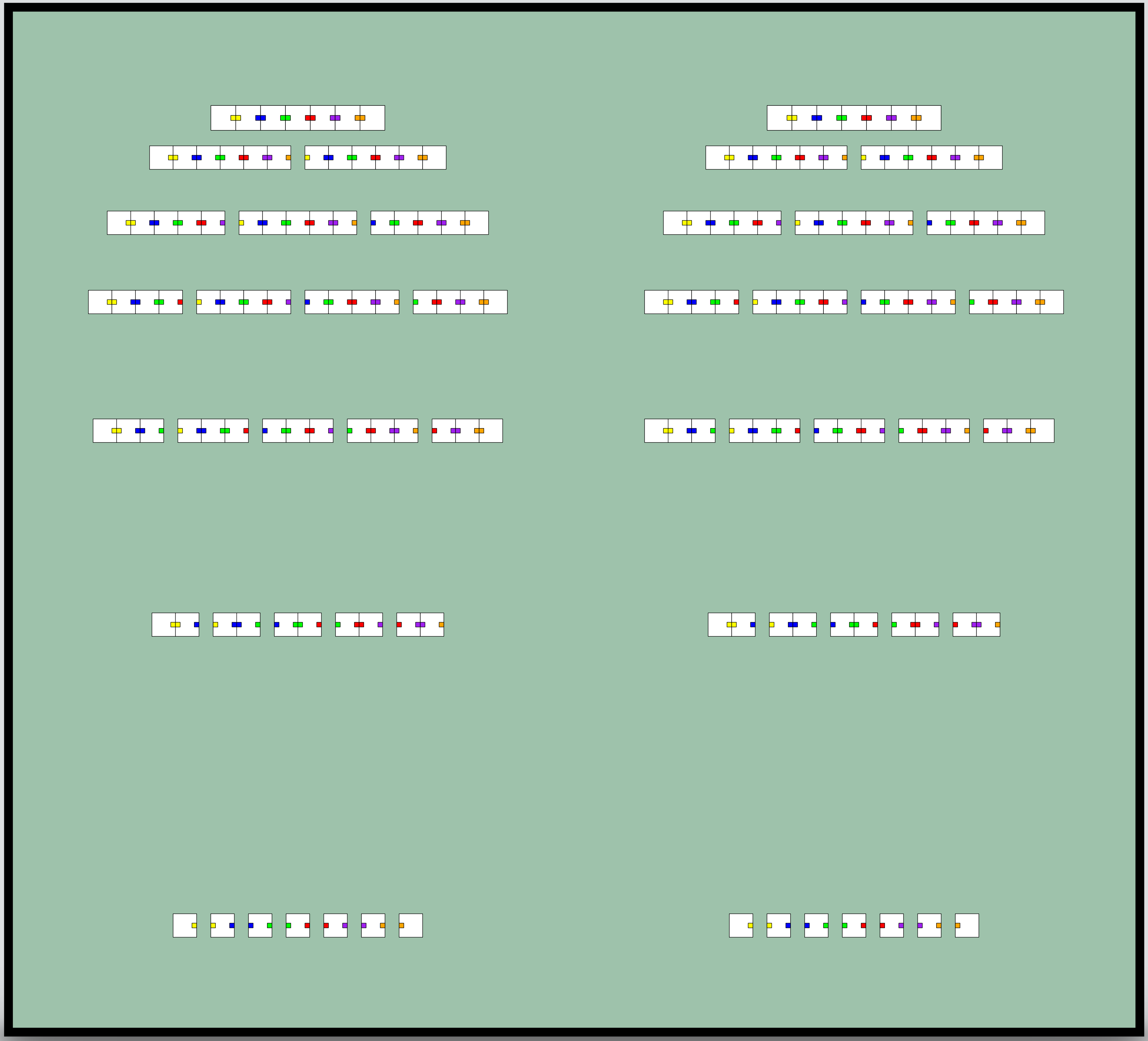
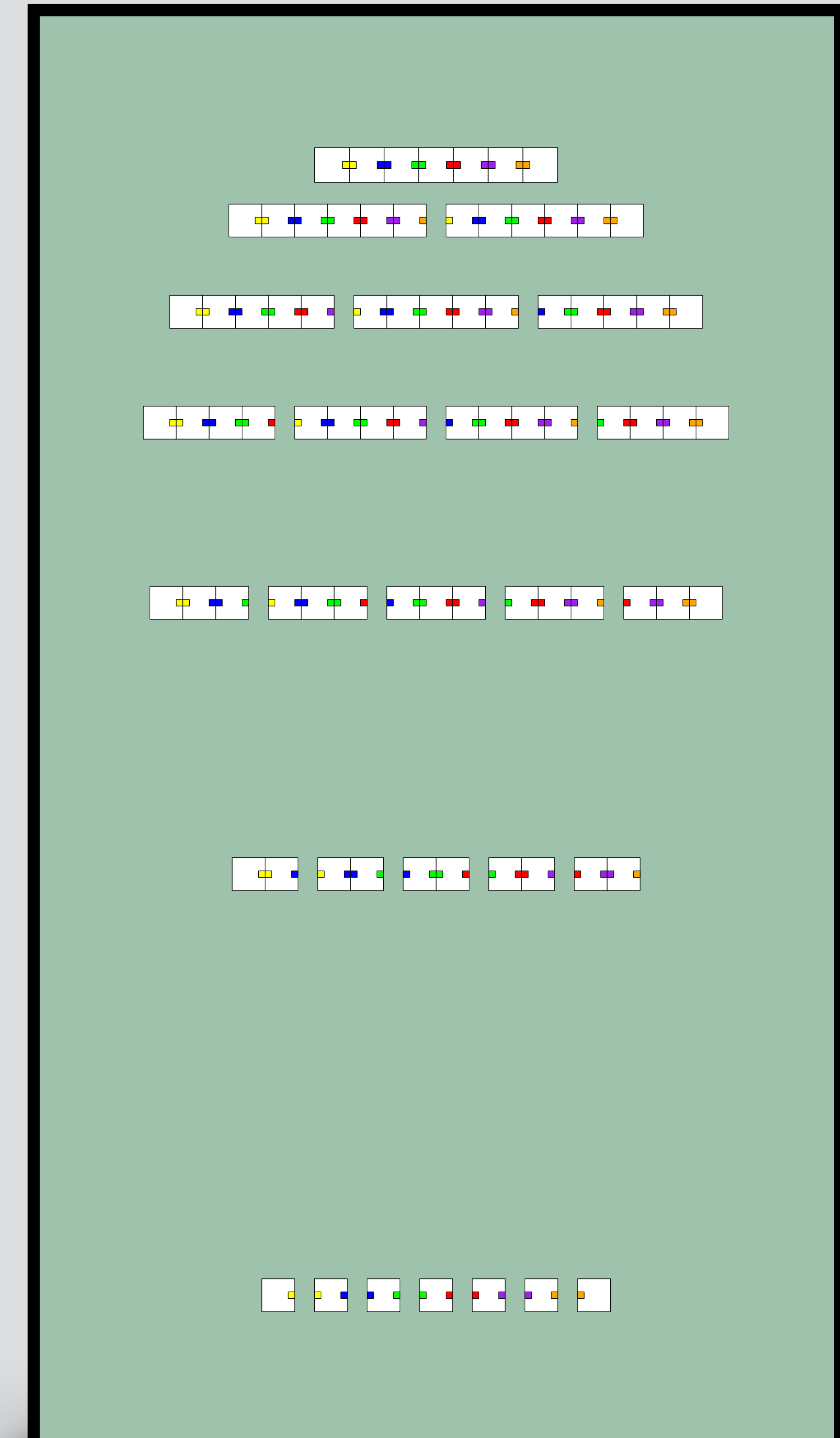
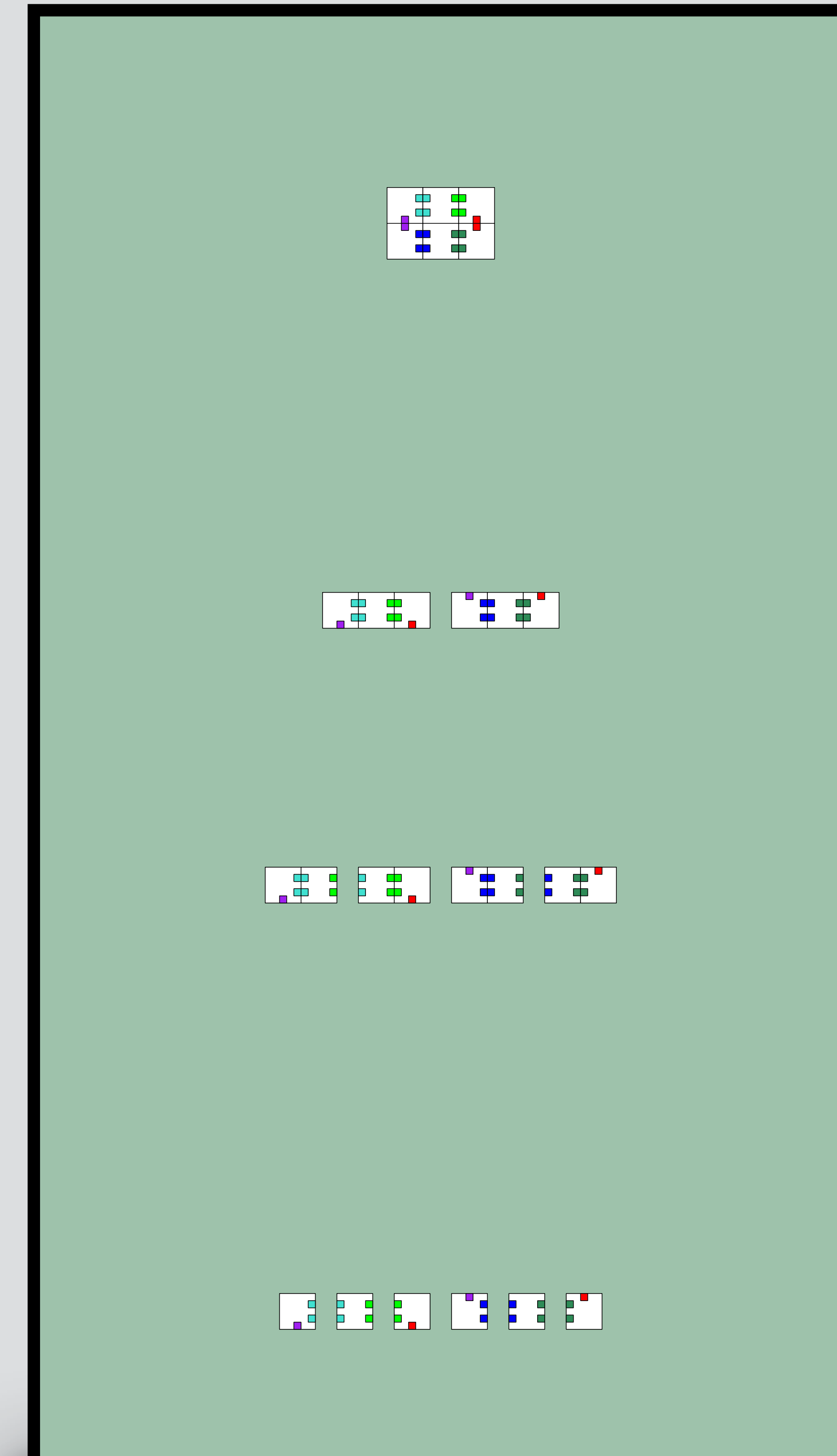


+ Positive electrode +



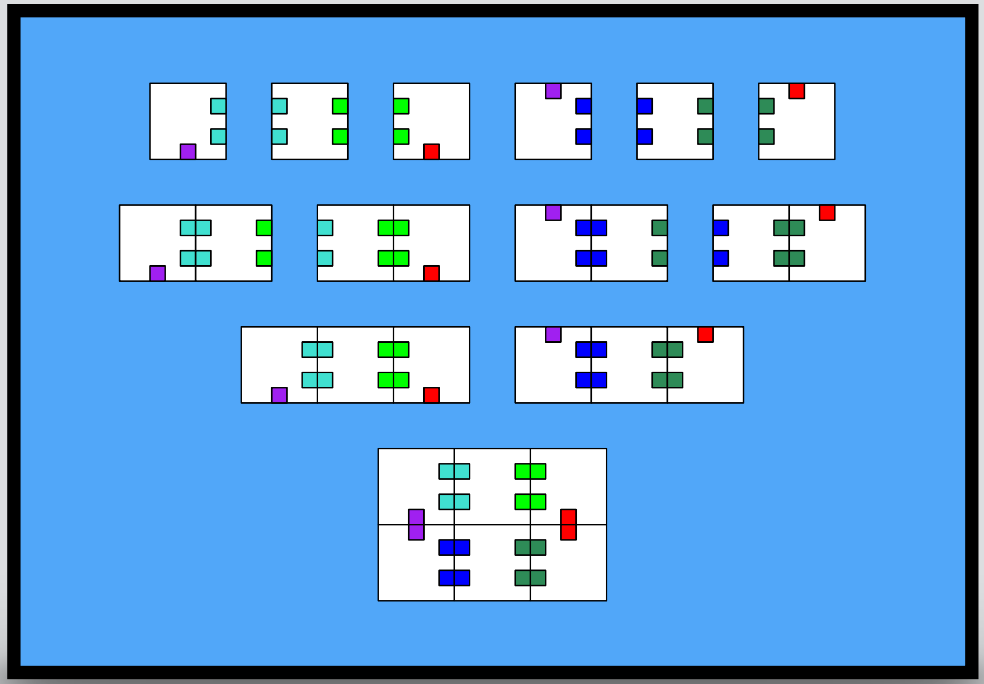




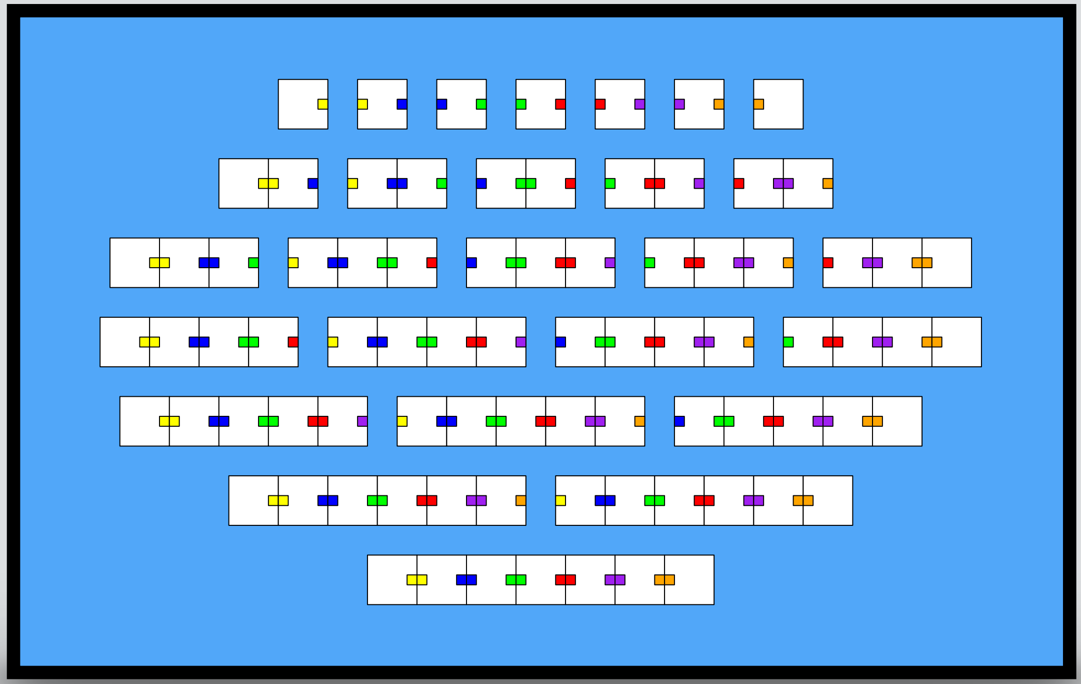
Size-separability

- A system is *factor-c size-separable* if the size ratio of the smallest terminal over largest non-terminal assembly is at least c.

- Every system is factor-c for some c in (0, 2].



Size-separability = 2



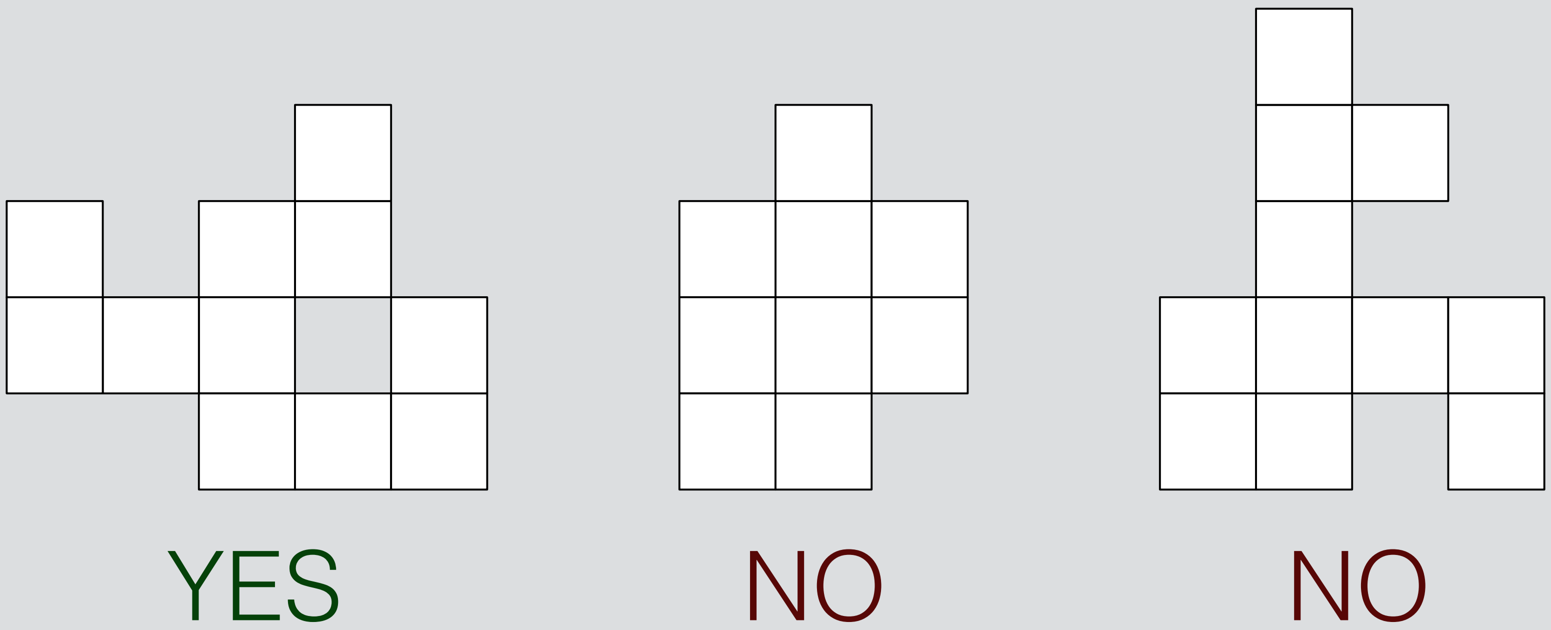
Size-separability = 7/6

Basic size-separability results

- A **temperature-1 system** with terminal assembly T is at most factor-|T|/(|T|-1) size-separable.

Basic size-separability results

- A **temperature-1 system** with terminal assembly T is at most factor-|T|/(|T|-1) size-separable.
- A system with a **tree-shaped terminal assembly** T is at most factor-|T|/(|T|-1) size-separable.



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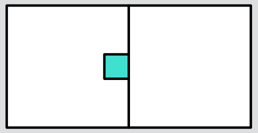
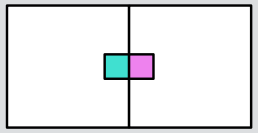
- A system with a **tree-shaped terminal assembly** T is at most factor-|T|/(|T|-1) size-separable.

- A system with a **unique terminal assembly** T is at least factor-|T|/(|T|-1) size-separable.

Quick definitions

- A system is *mismatch-free* if no producible assembly has a mismatch:  

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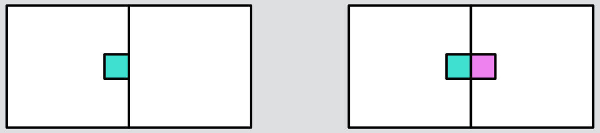
- A system is *mismatch-free* if no producible assembly has a mismatch: 

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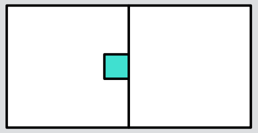
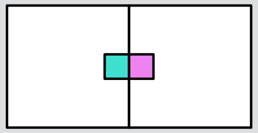
- A system is *mismatch-free* if no producible assembly has a mismatch:  

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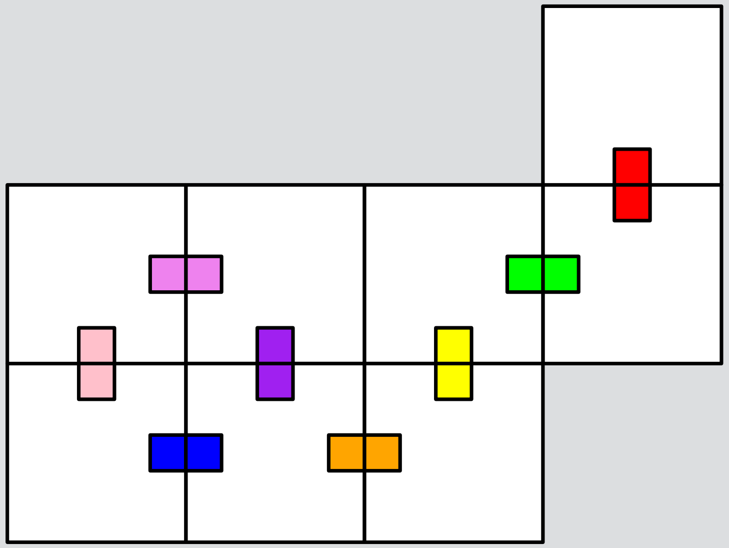

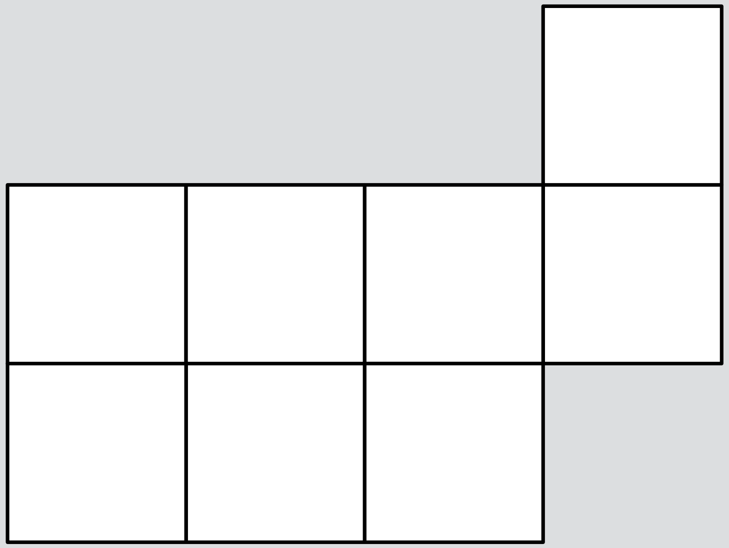

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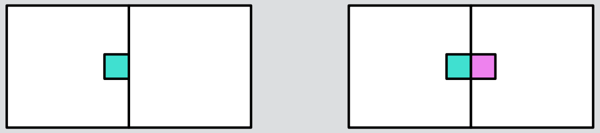
- A system is *mismatch-free* if no producible assembly has a mismatch: 
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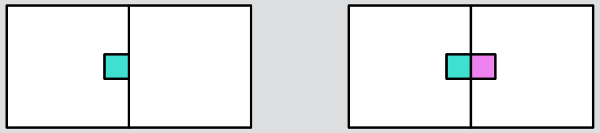
- A system is *mismatch-free* if no producible assembly has a mismatch: 
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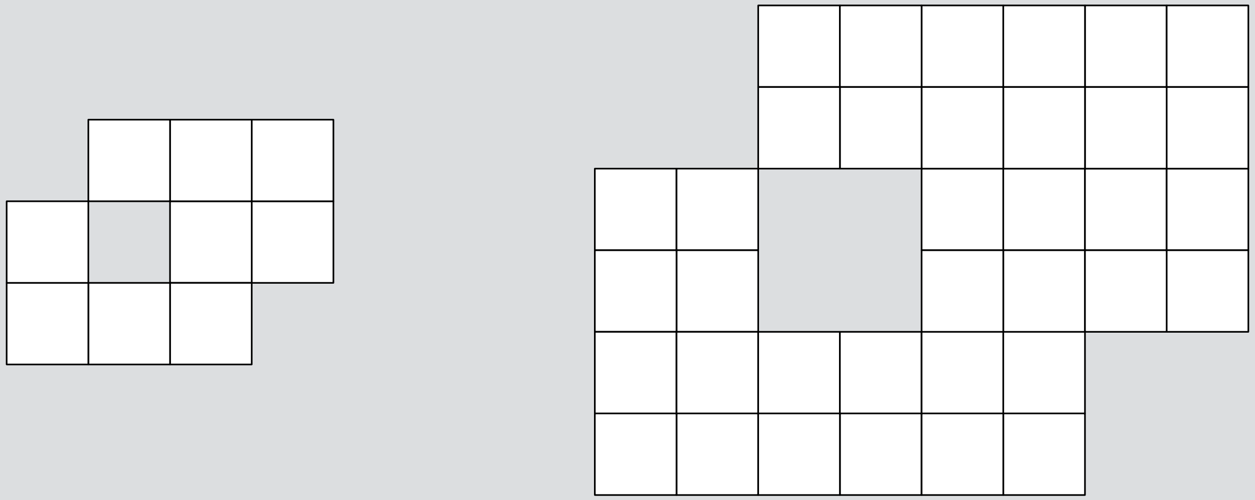

Previous results

- [Soloveichik, Winfree 2007]: any shape can be assembled at some scale using O(K/log(K)) tiles at $\tau = 2$.
    - K = Kolmogorov complexity.
    - Optimal for tile types, bad scale factor.
    - Can be adapted to be factor-2 size-separable.

Previous results

- [Chen, Doty 2012]: N x N square can be assembled using O(log(N)/loglog(N)) tiles at τ = 2 by a factor-2 size-separable system.
    - Optimal for tile types.
    - Just for squares.

Our result

Theorem: Let there be a $\tau = 1$ mismatch-free system of t tiles with unique terminal assembly A.

Then there exists a $\tau = 2$ factor-2 size-separable system of at most 8t tiles with unique terminal assembly A' with shape of A scaled by a factor of 2.

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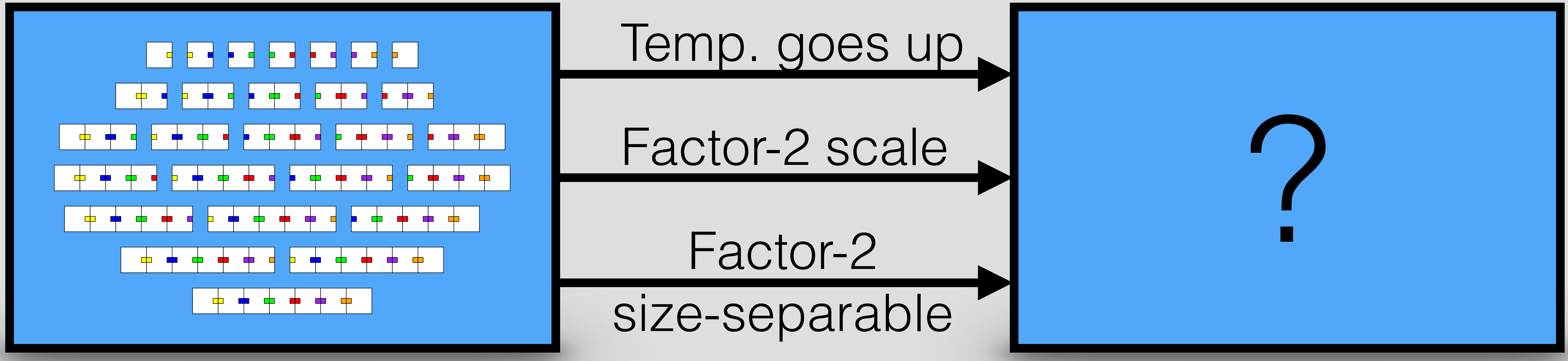

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Generic way to make an existing system size-separable
(with optimal scale factor and tile types: 2)

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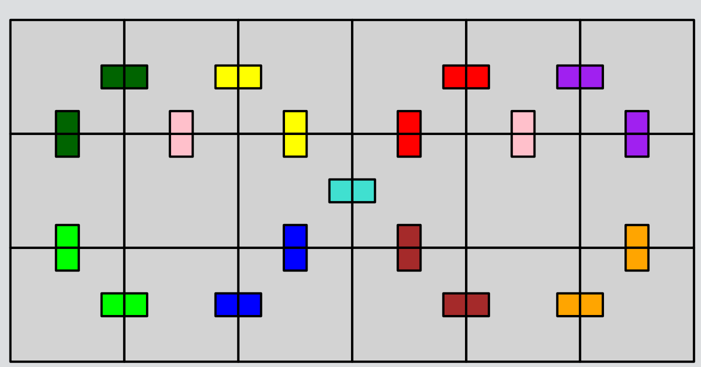
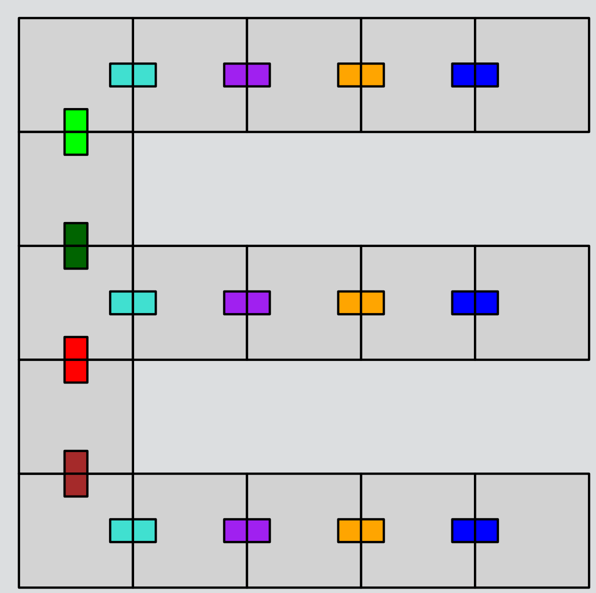
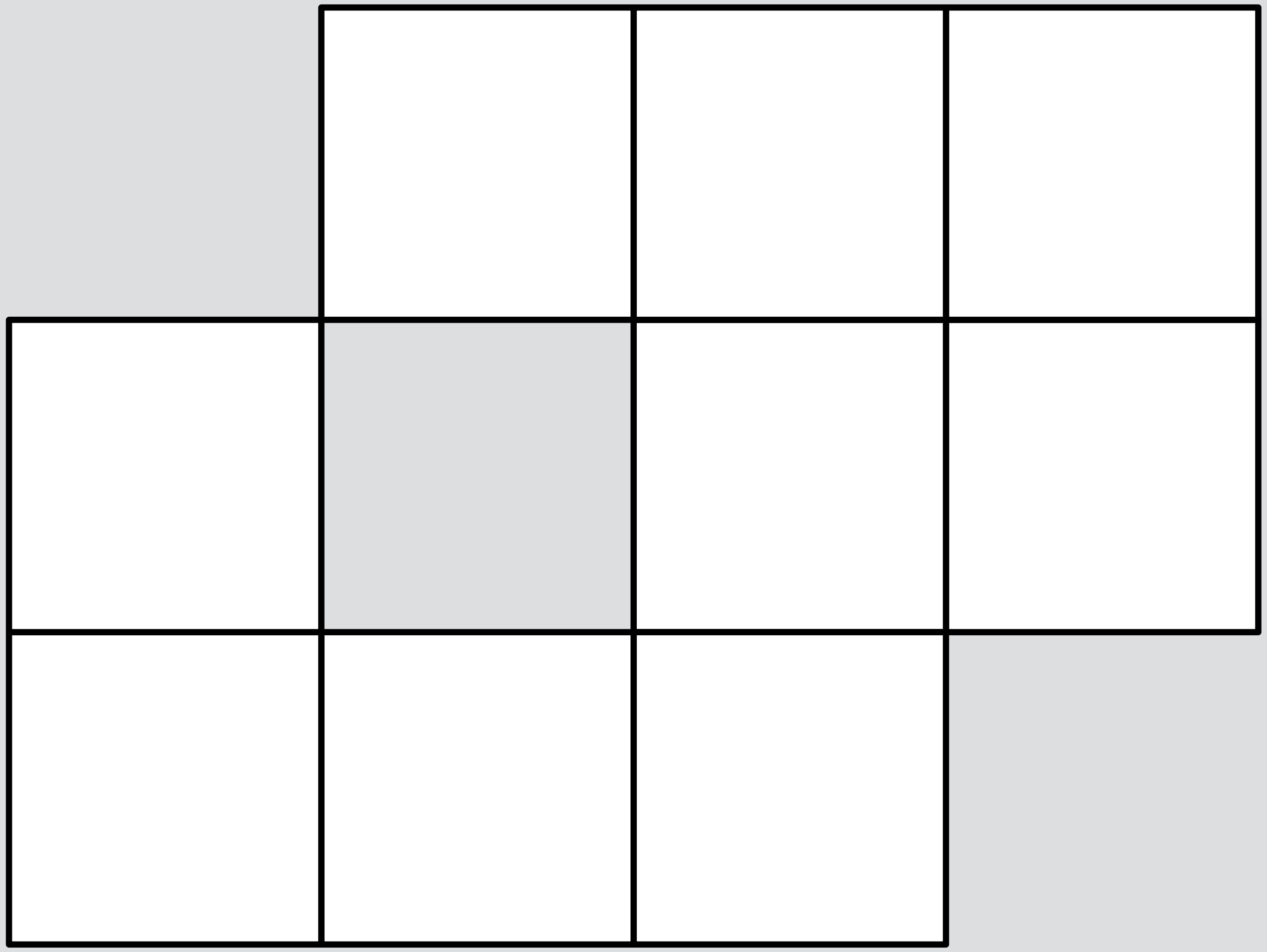
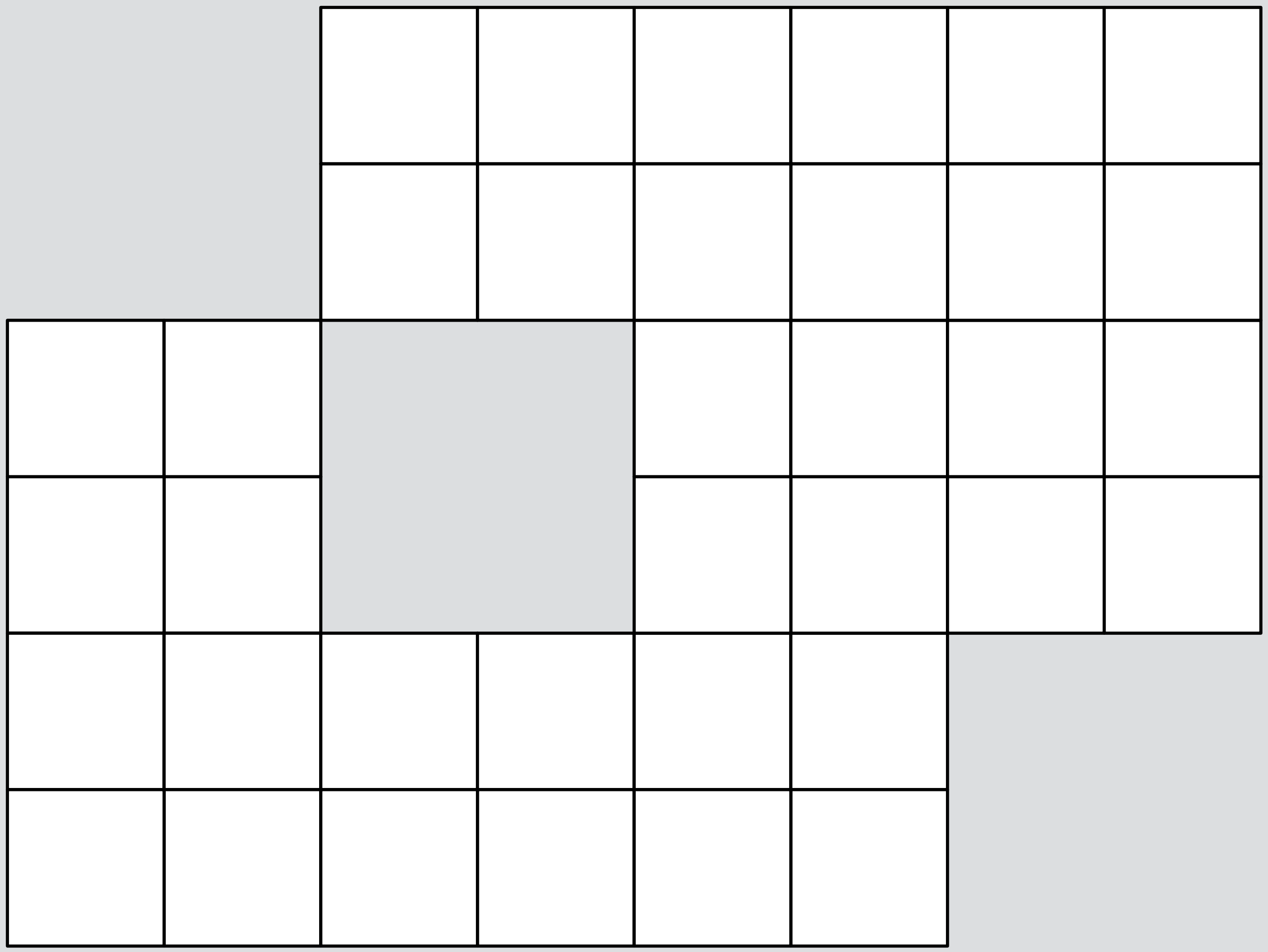




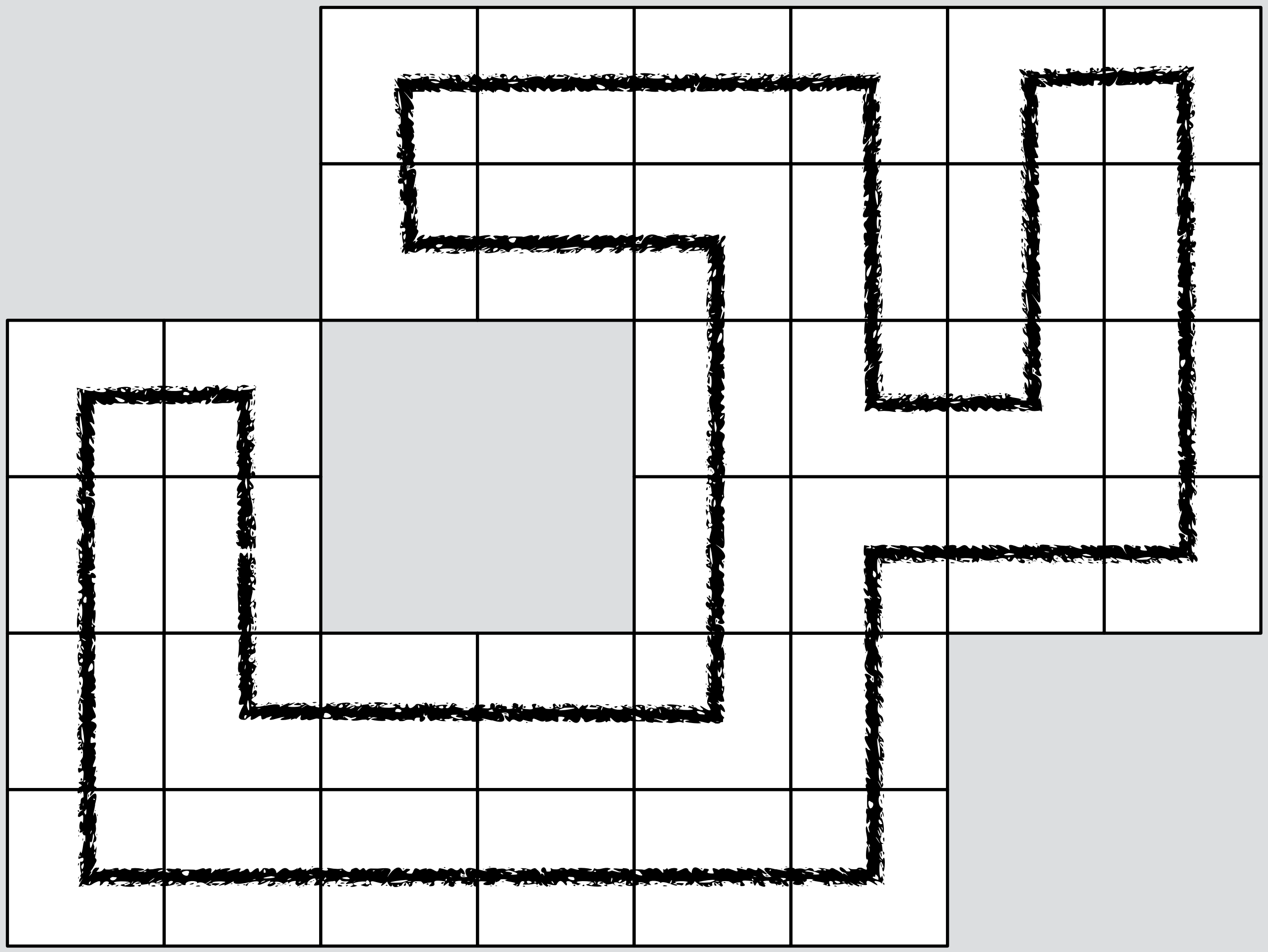
Algorithm idea



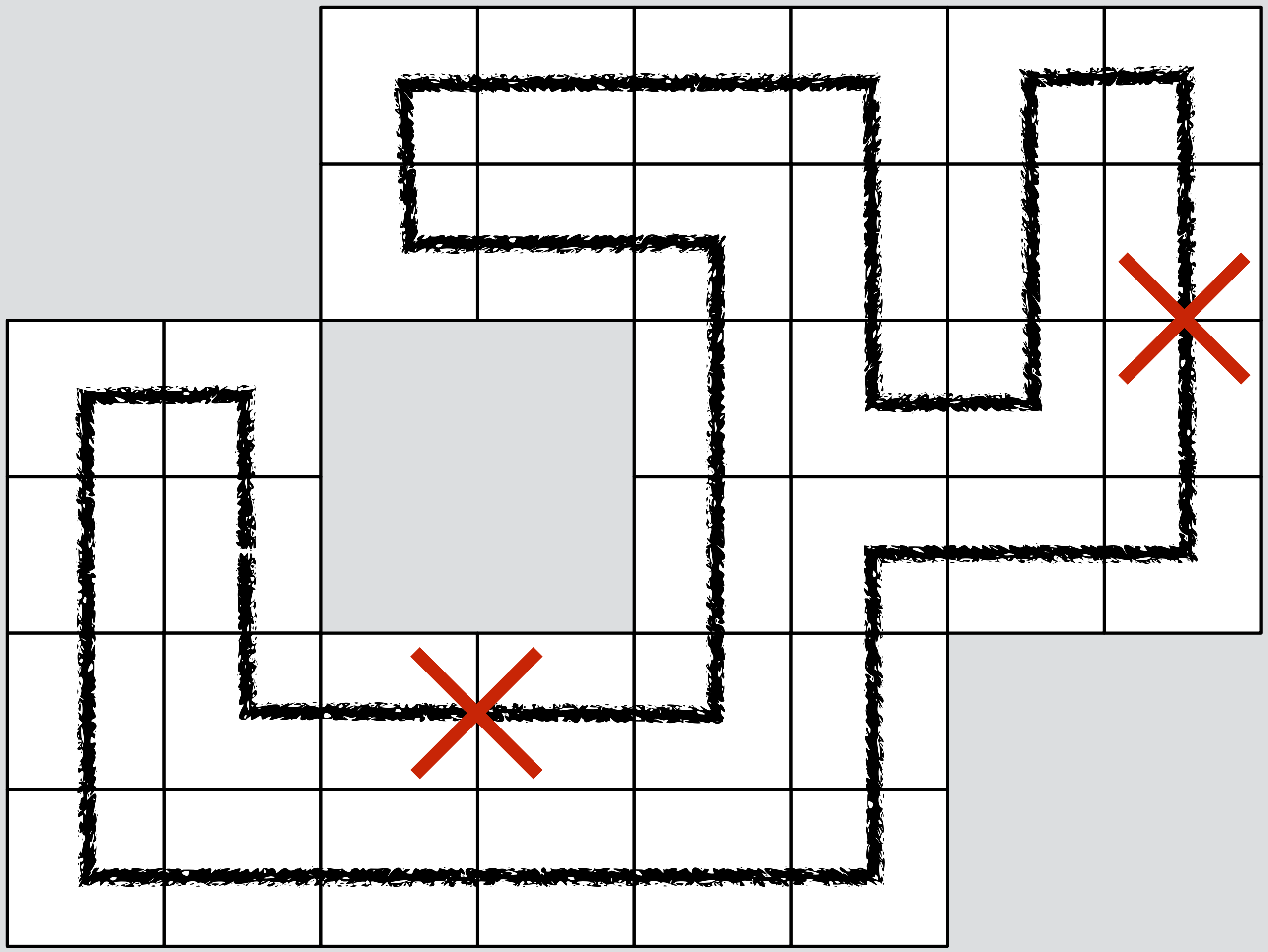
Algorithm idea



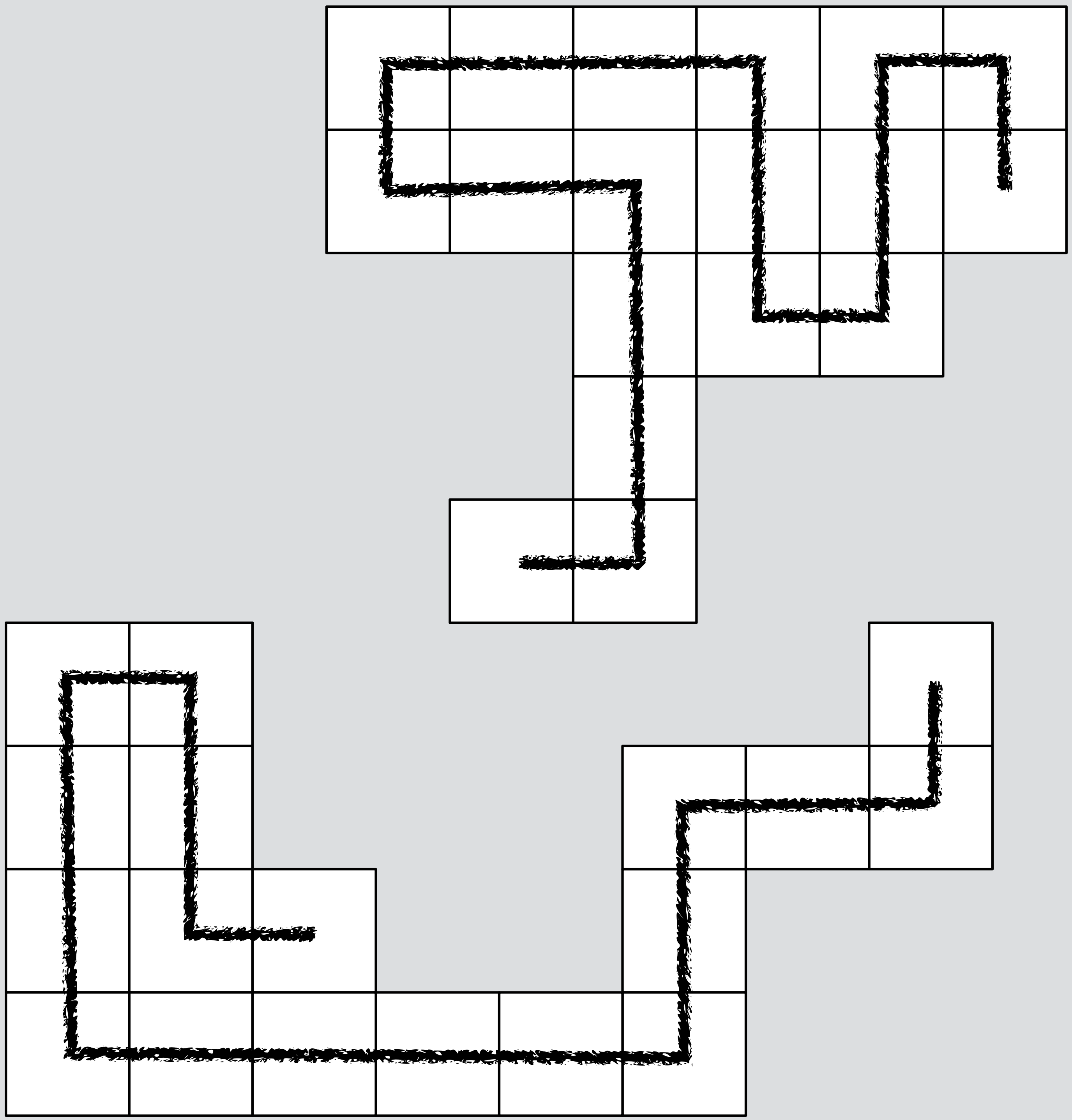
Algorithm idea



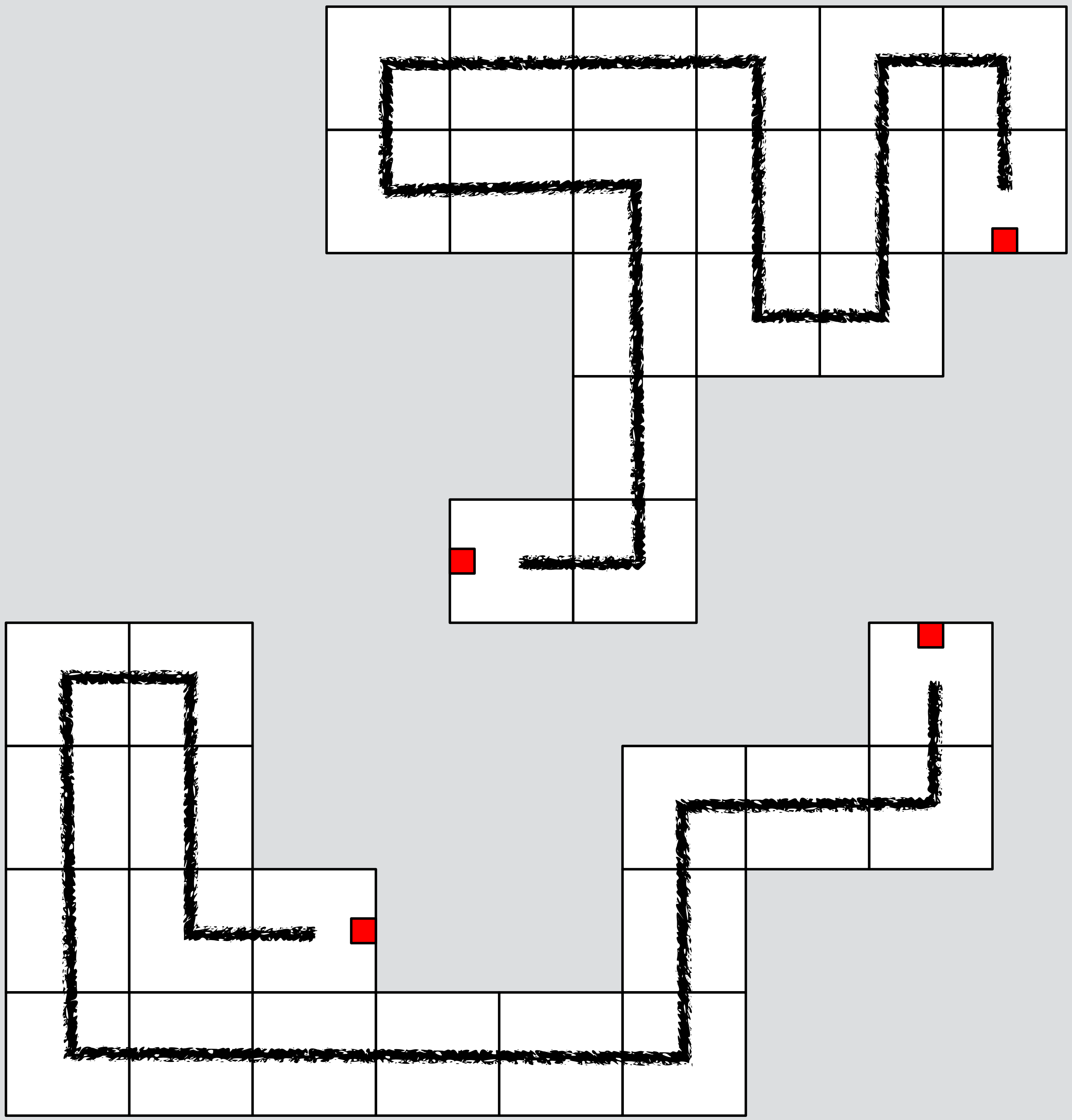
Algorithm idea



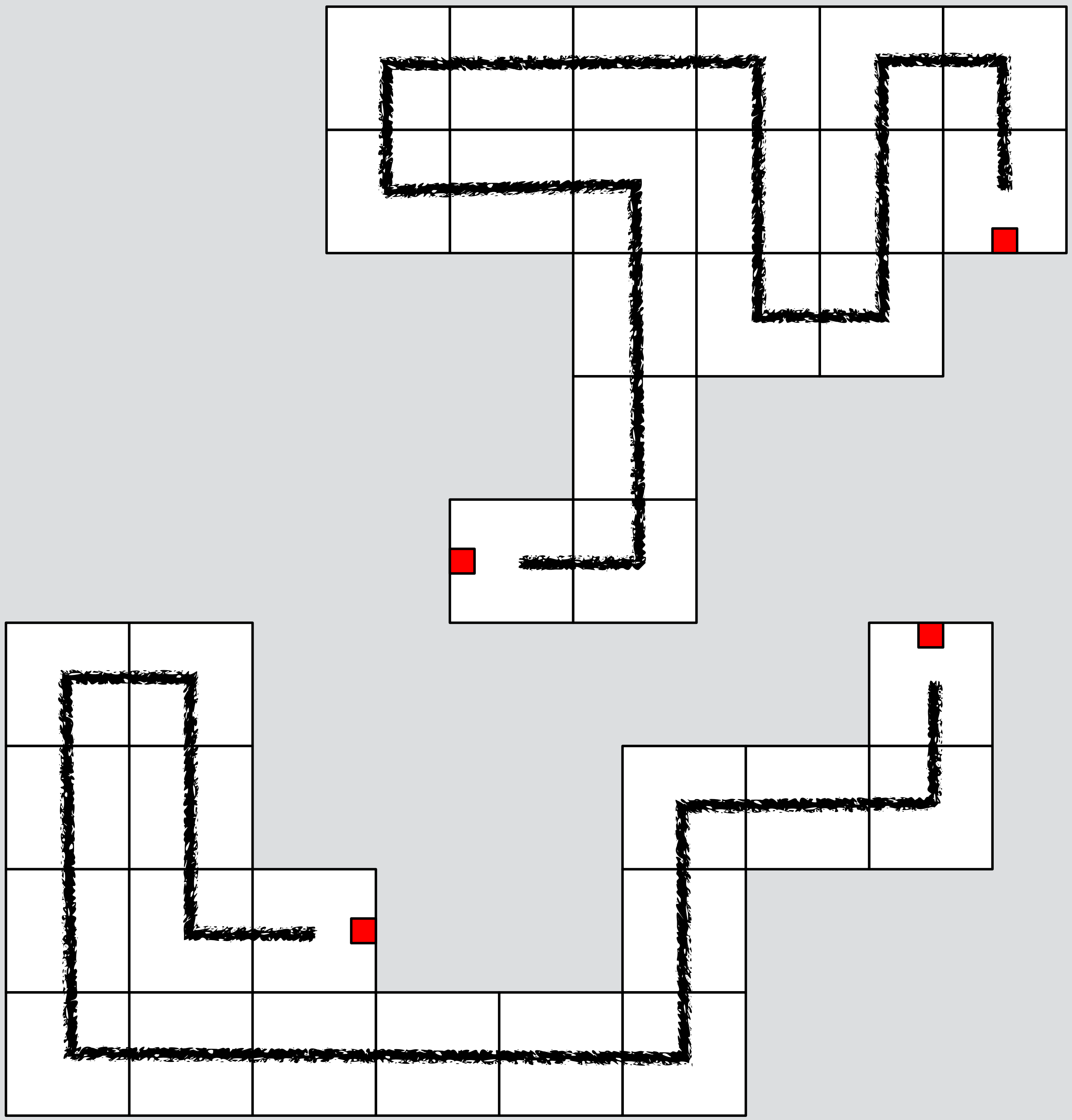
Algorithm idea



Algorithm idea



Algorithm idea



$\tau = 2$

Conclusions

- The two-handed tile assembly assumes any non-terminal assembly is removed.
- The practicality of this is captured by the *size-separability* of the system.
- Any $\tau = 1$ mismatch-free system with a unique terminal assembly can converted to a factor-2 size-separable system with optimal increase in scale and temperature.

Open problems

- Algorithms for making larger classes of tile systems optimally size-separable:
  - Systems with multiple terminal assemblies.
  - $\tau > 1$ mismatch-free systems.
  - $\tau = 1$ systems with mismatches.
- Examples of systems that cannot be made size-separable without large scale, tile set blowup.