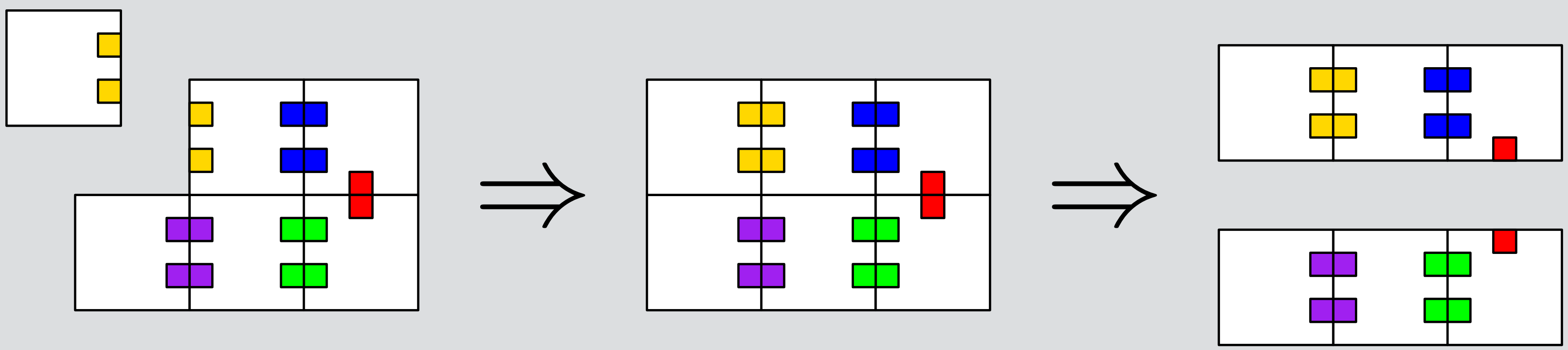
Size-Dependent Tile Self-Assembly:

Constant-Height Rectangles and Stability



Sándor Fekete, Robert Schweller, Andrew Winslow

Technische Universität Braunschweig

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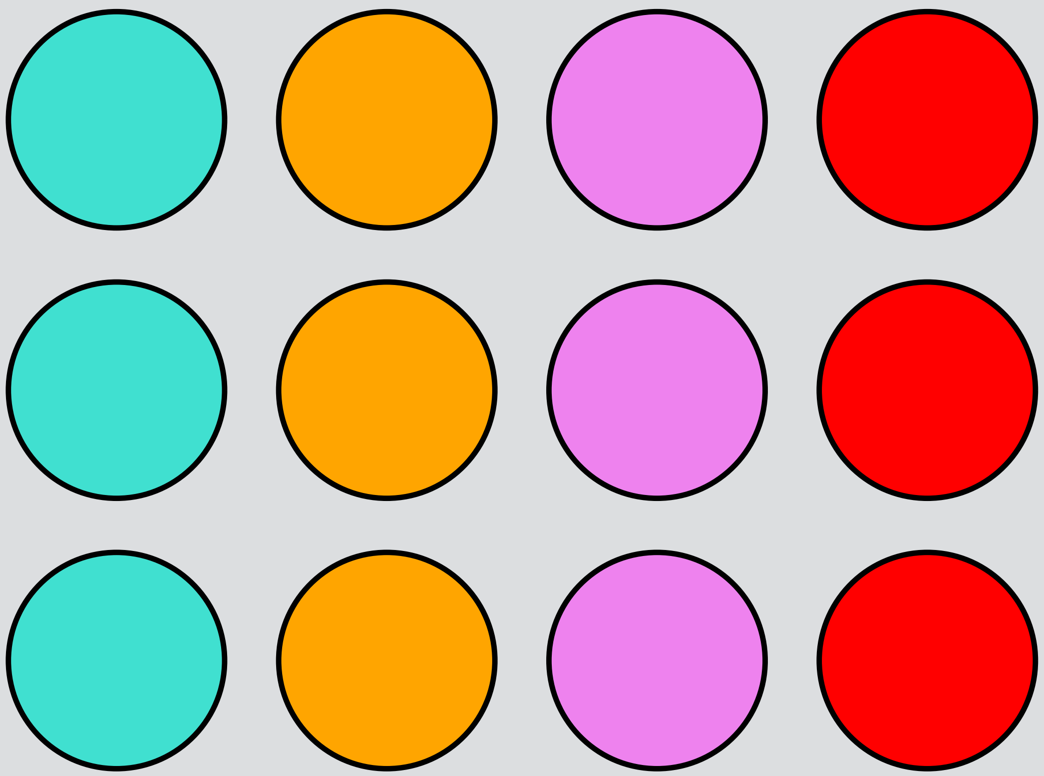
Self-Assembly

Self-Assembly

Simple particles coalescing into complex superstructures.



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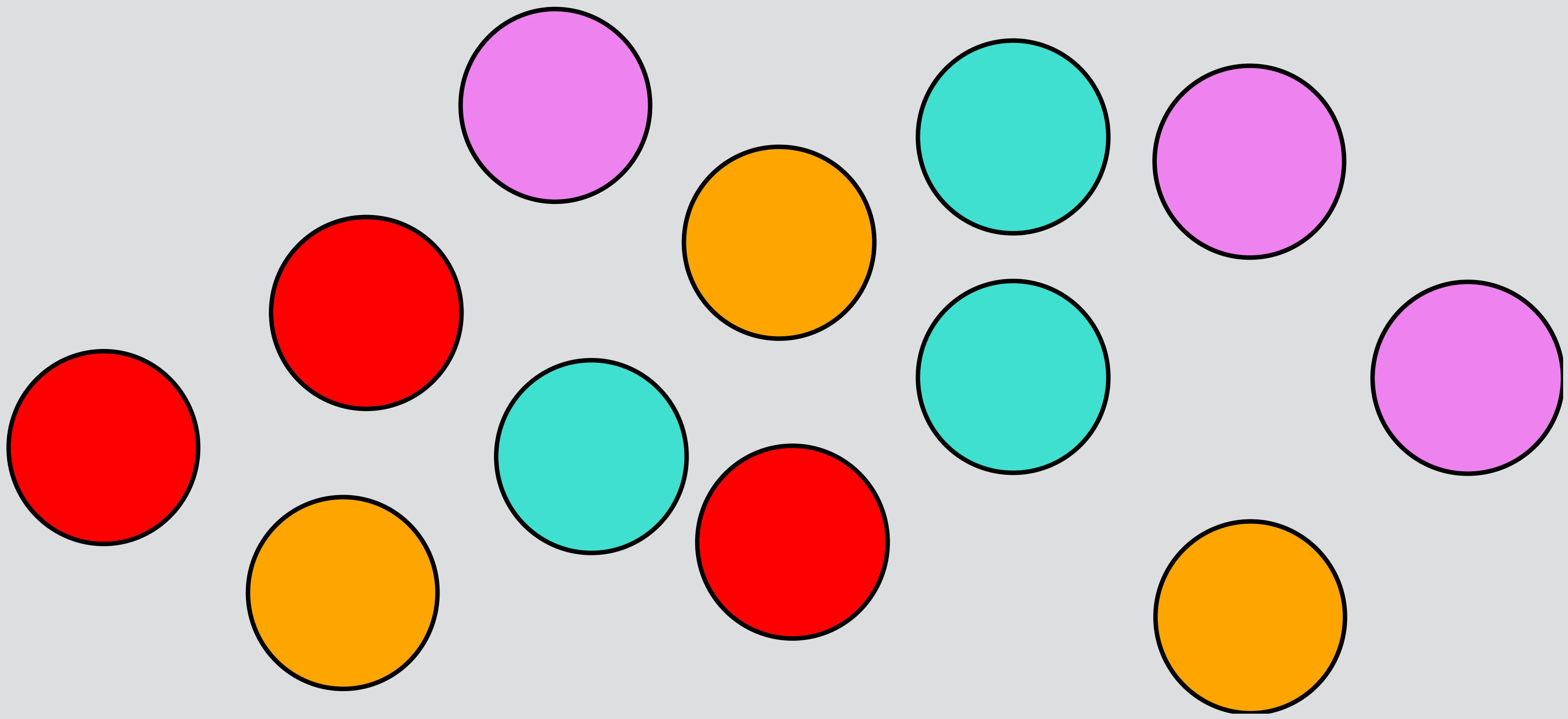

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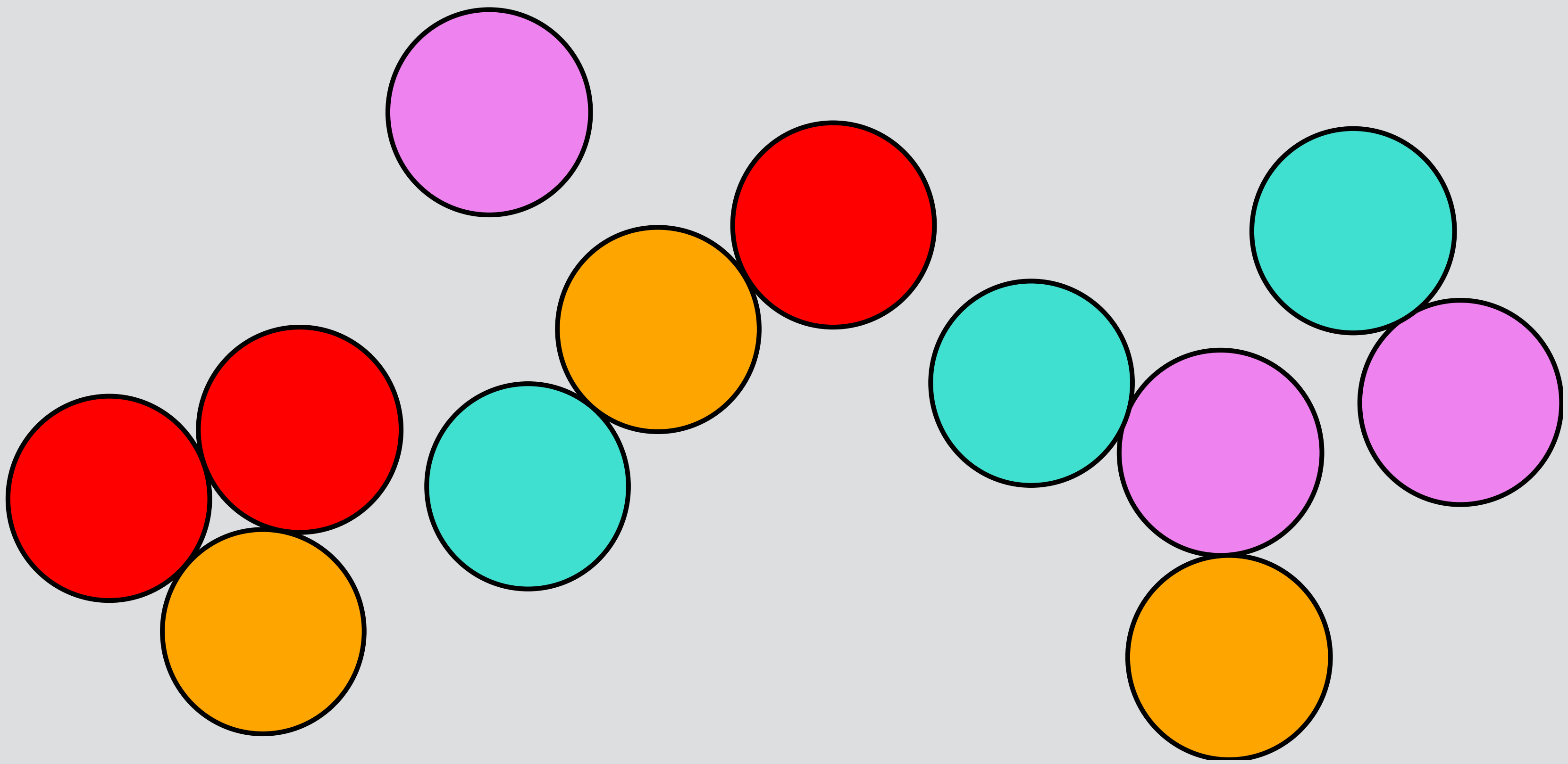

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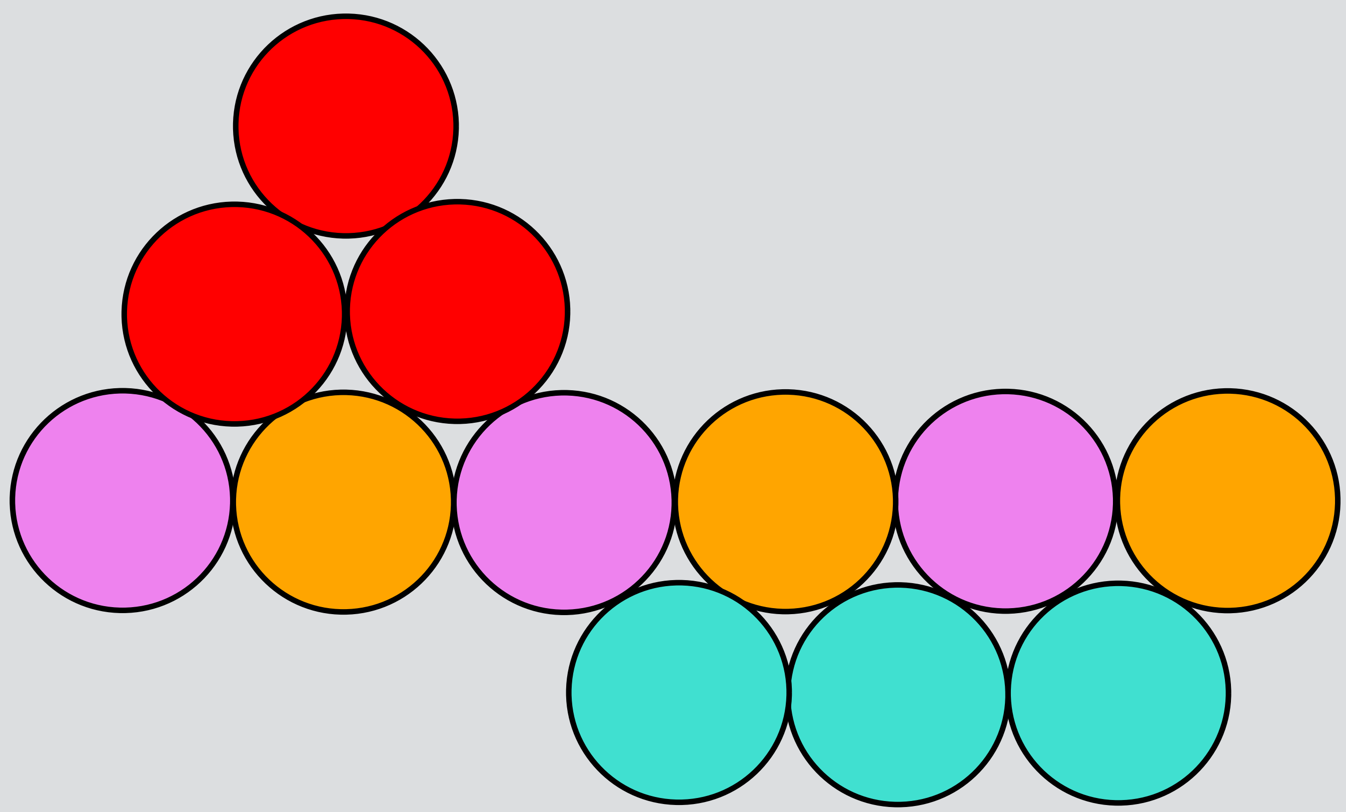
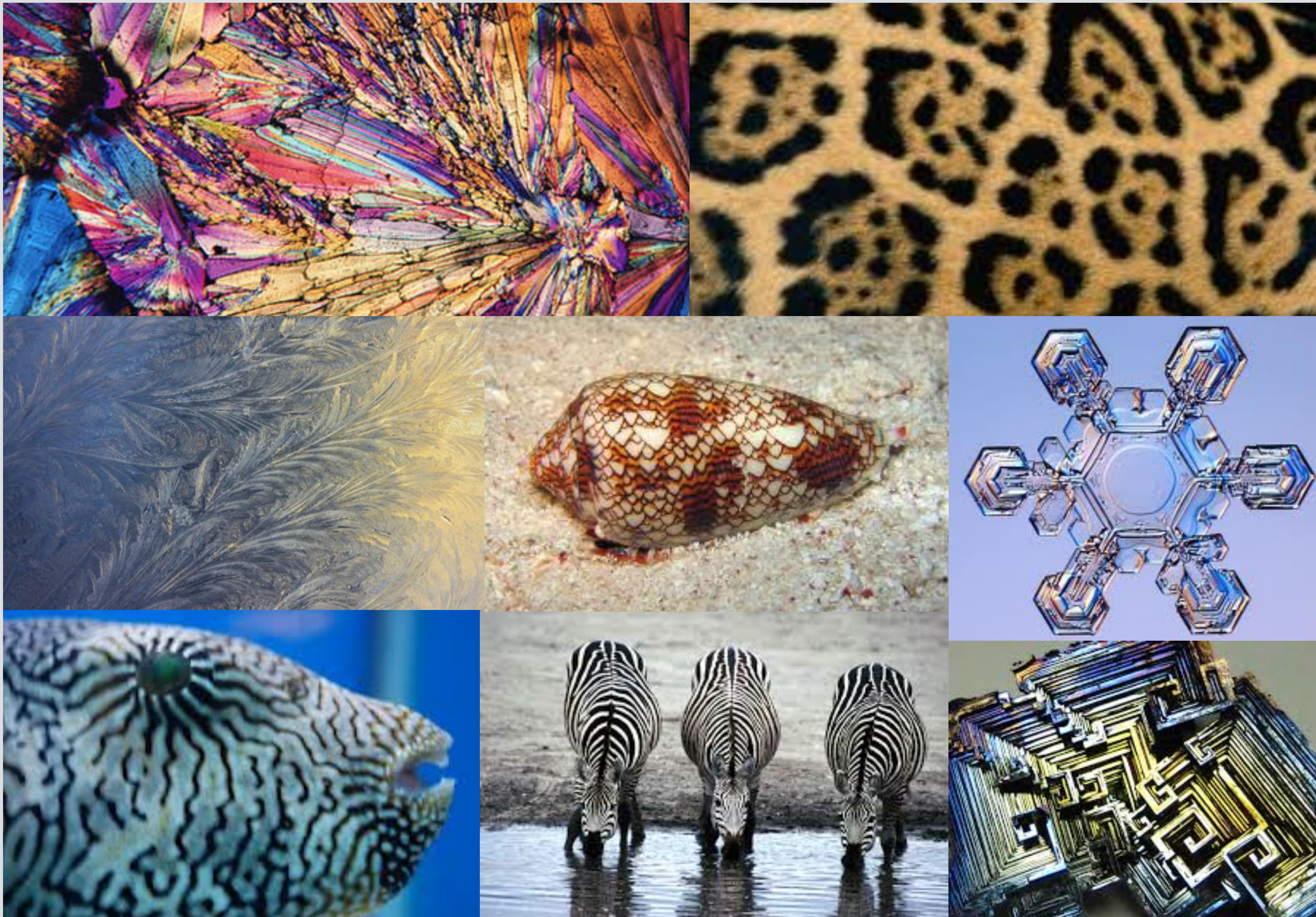
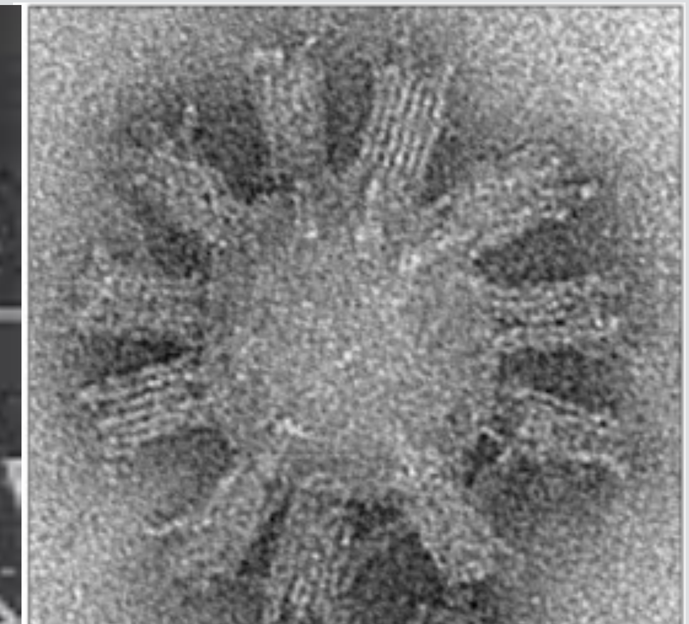
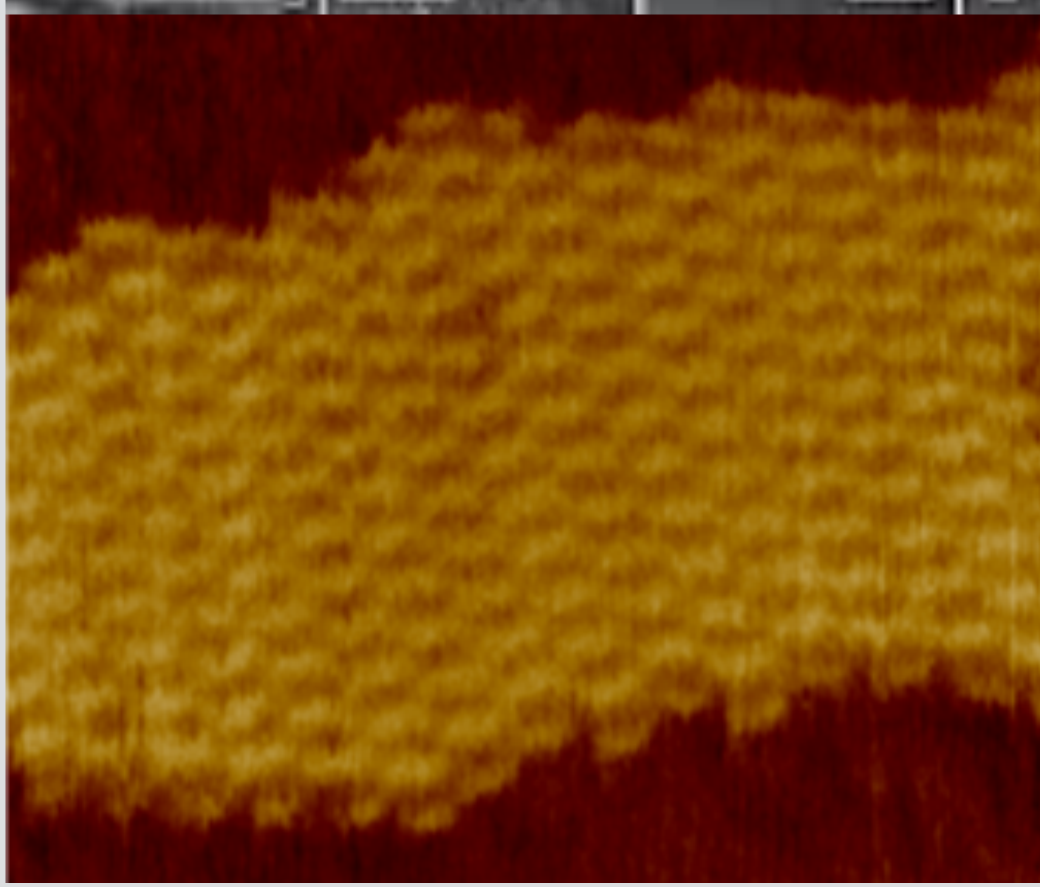


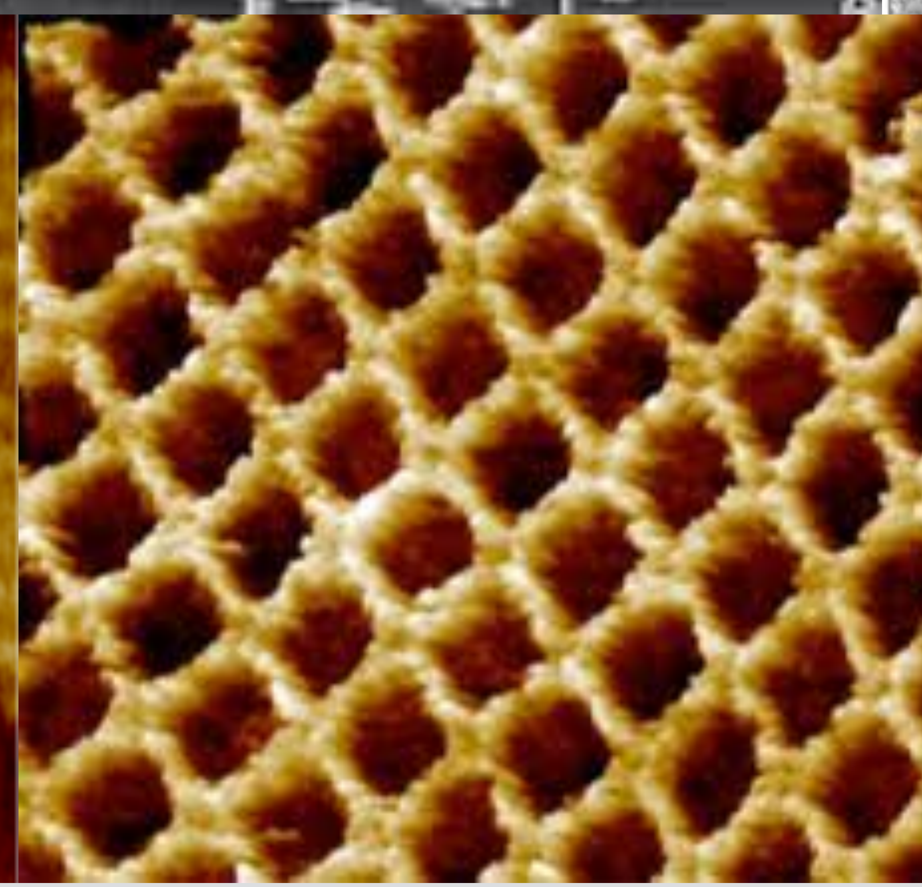
Natural Self-Assembly



Synthetic Self-Assembly with DNA







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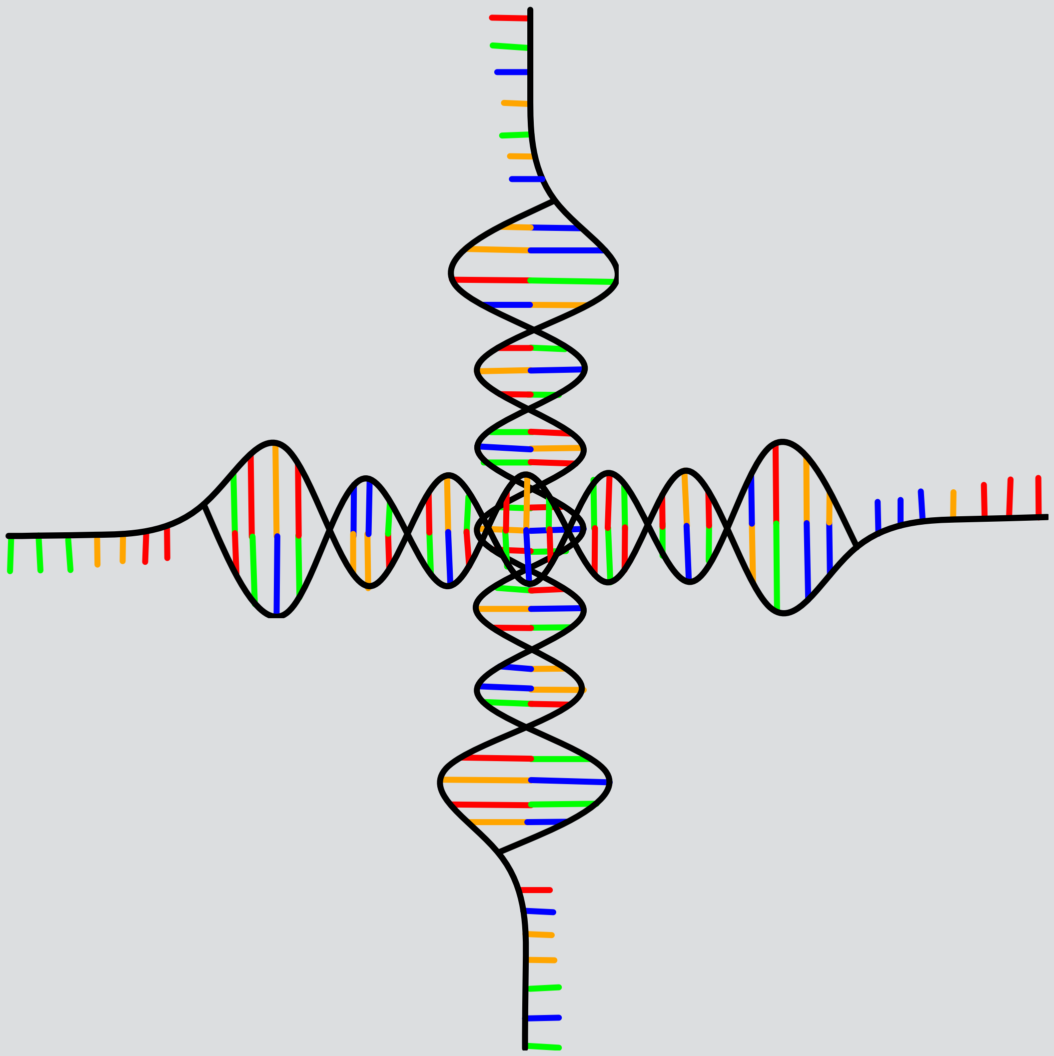

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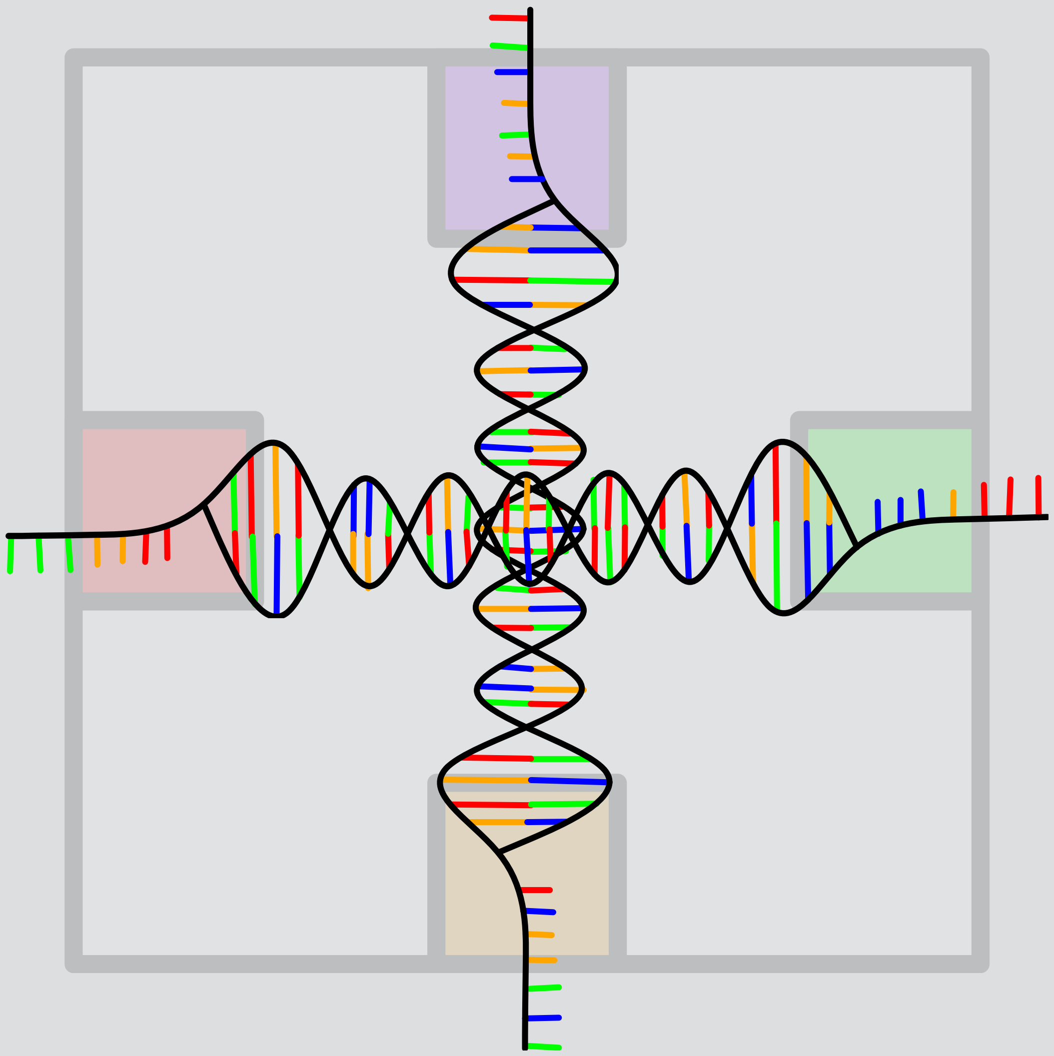

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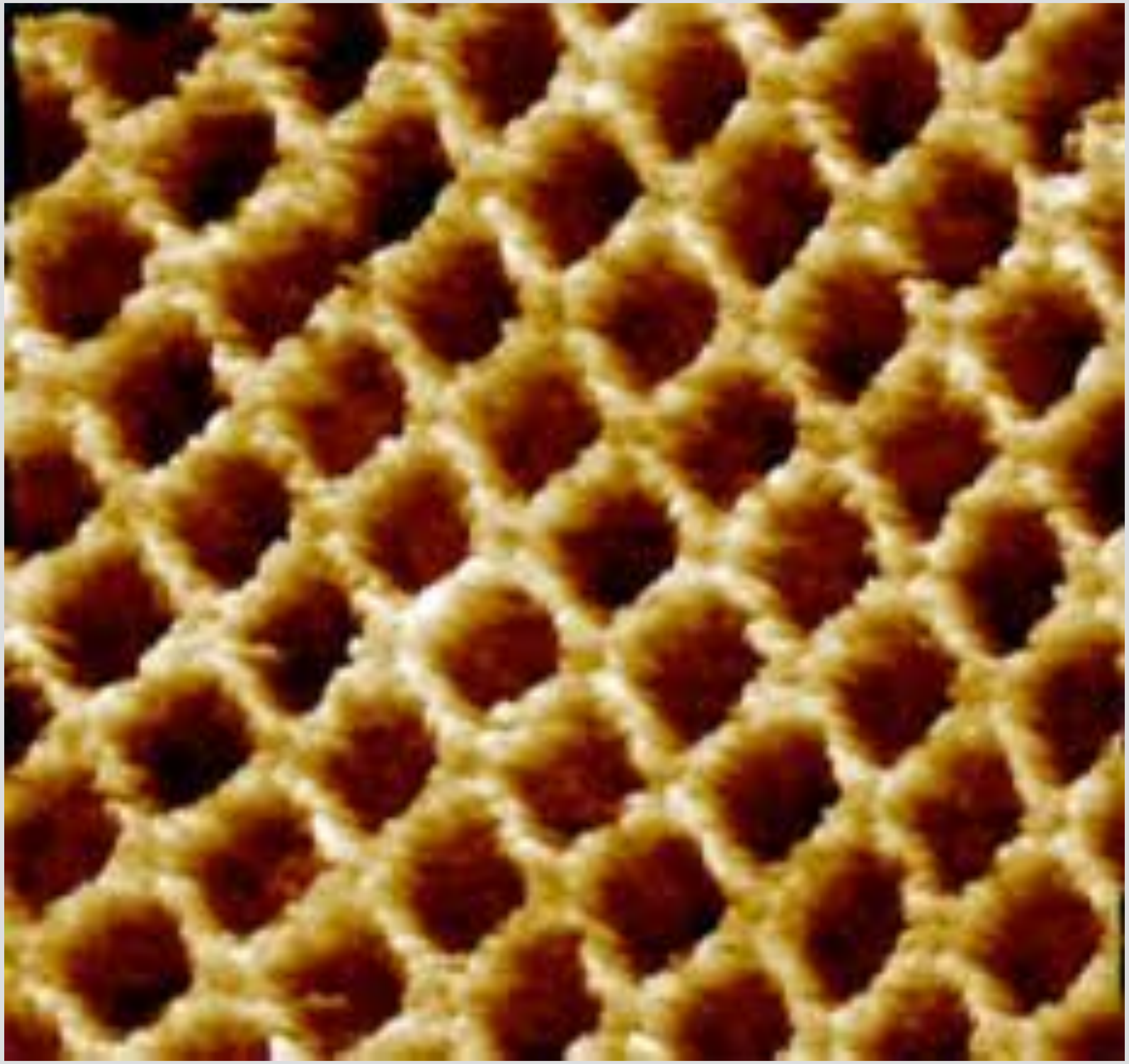

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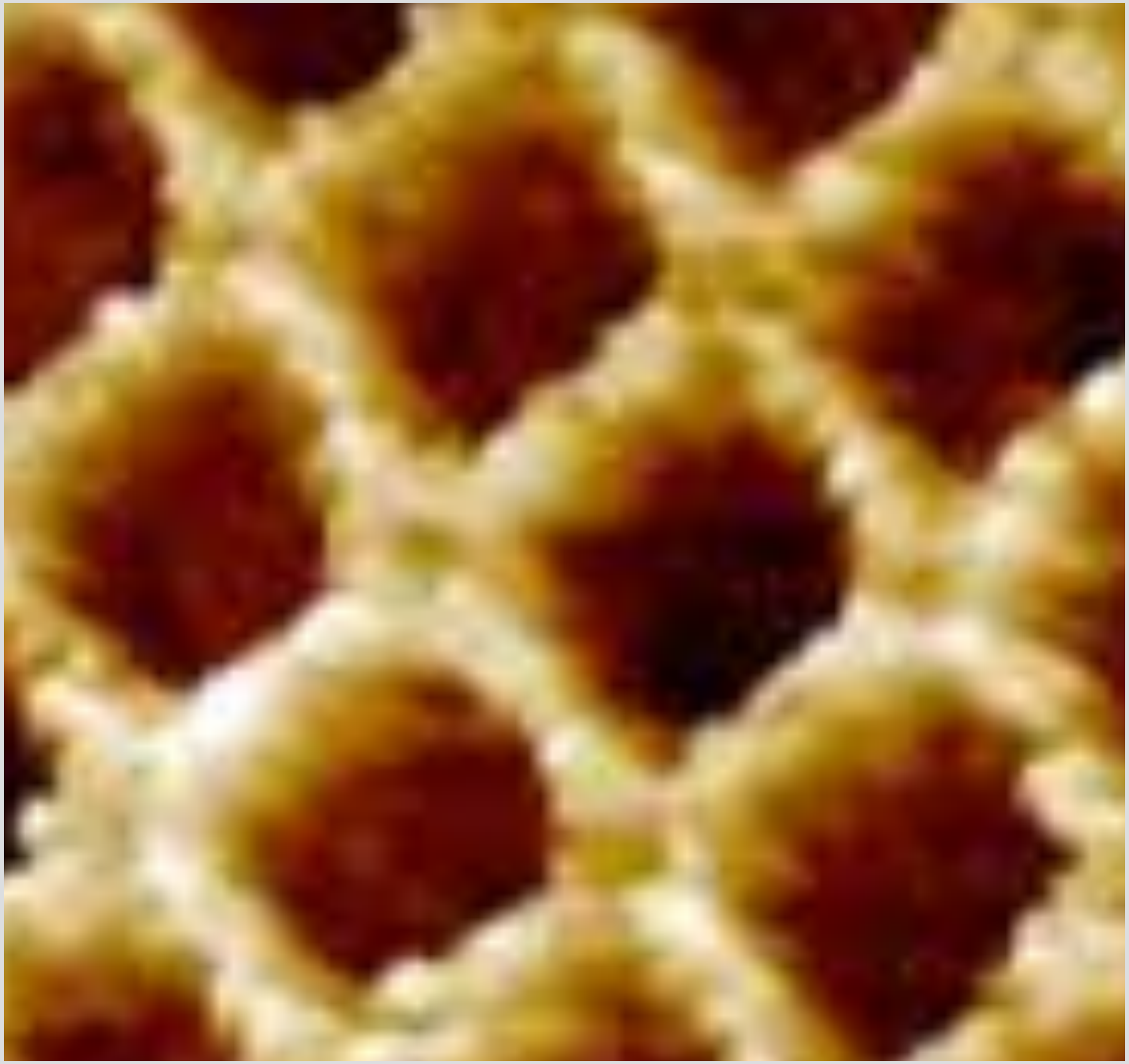

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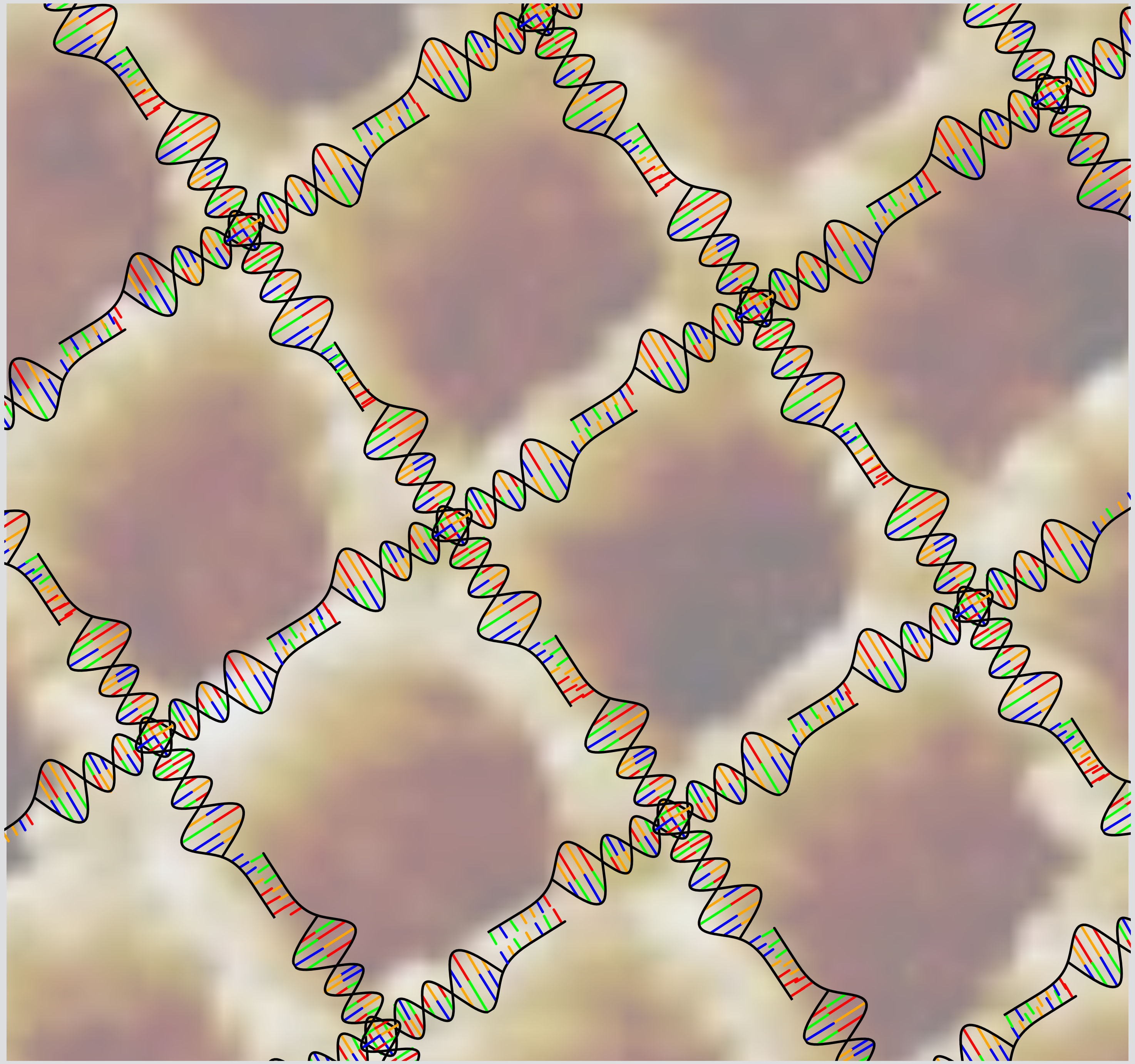
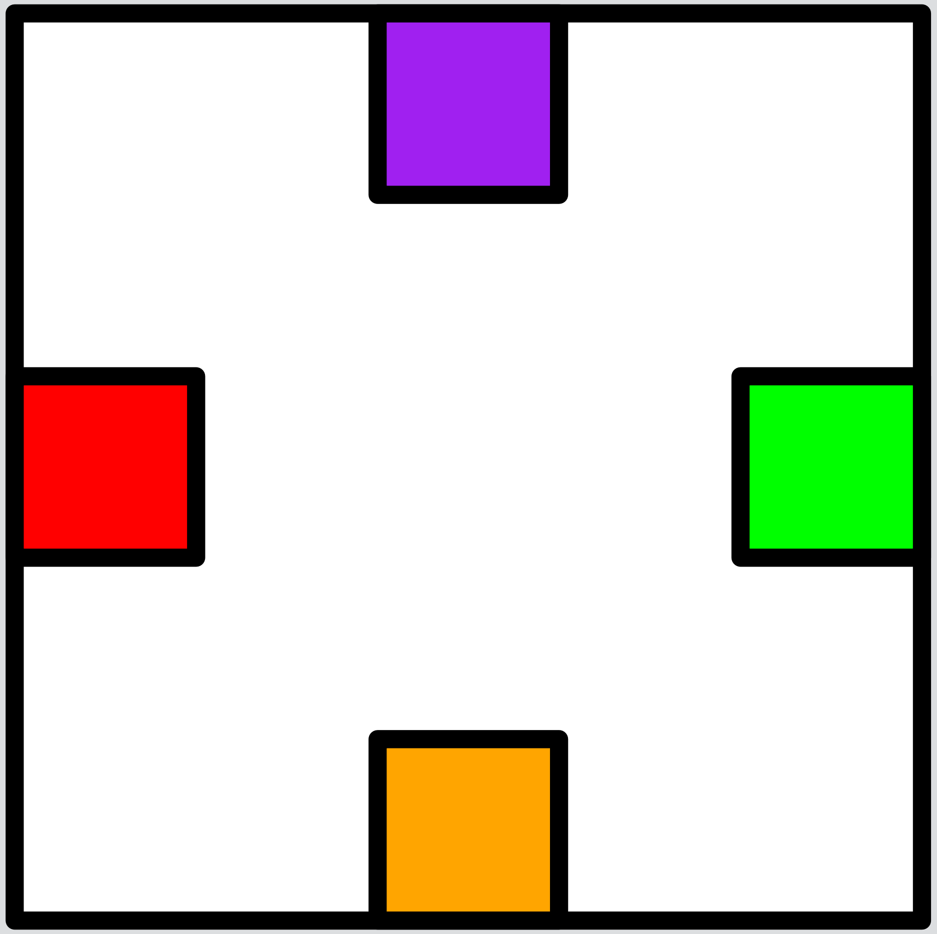


Tile Self-Assembly



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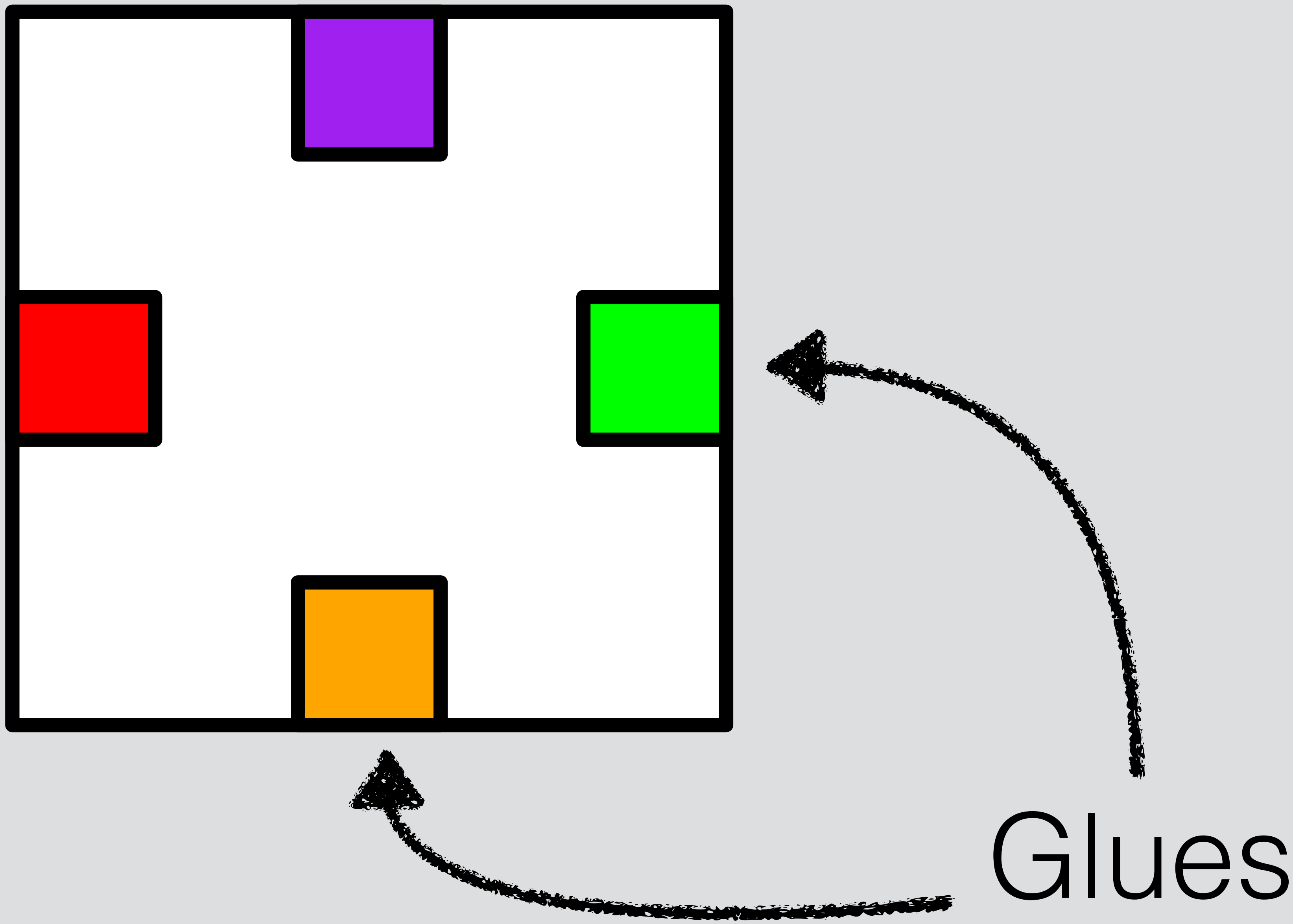

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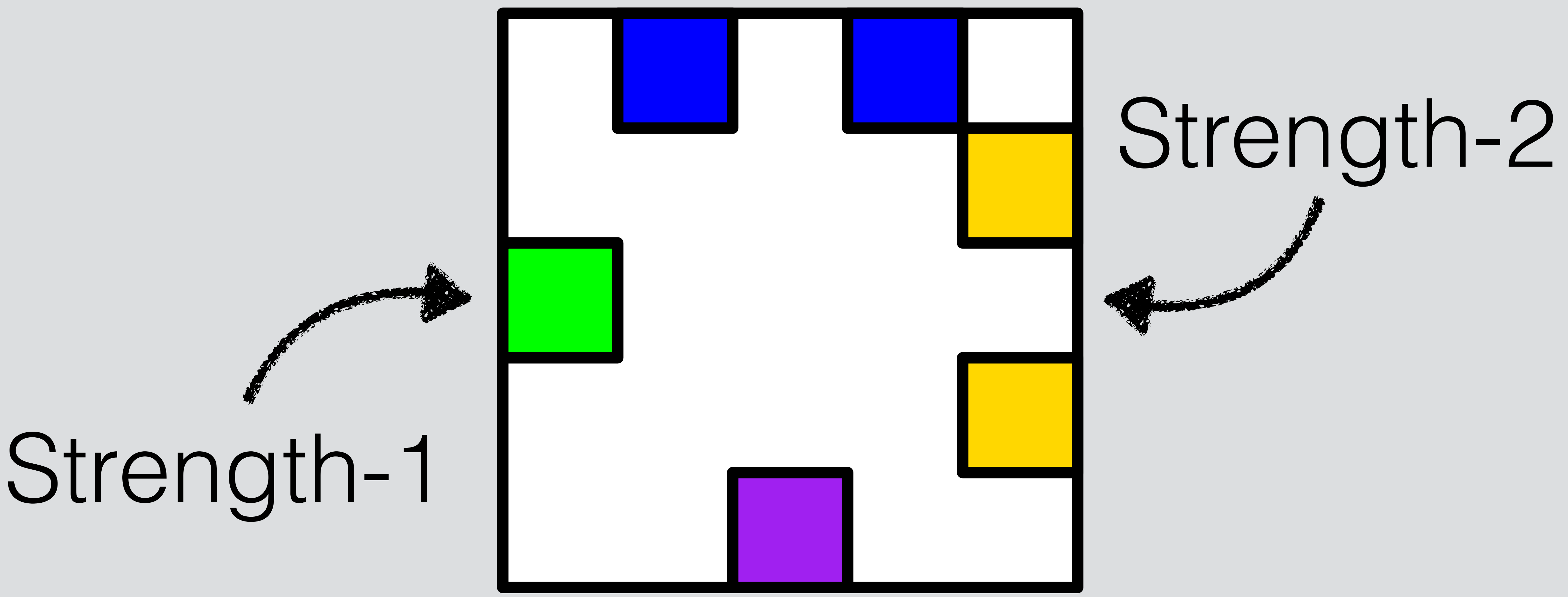
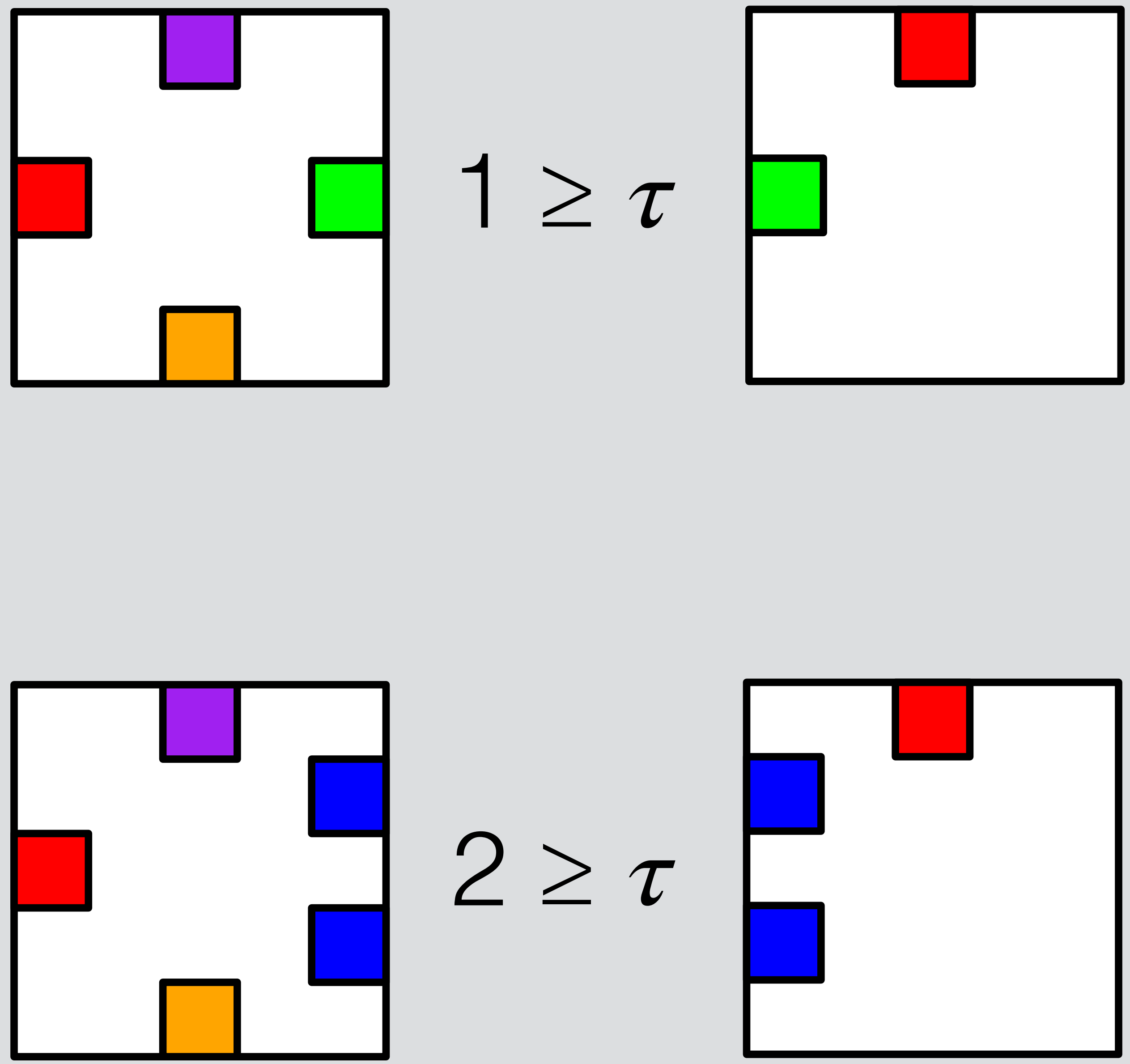


At temperature $\tau = 1$



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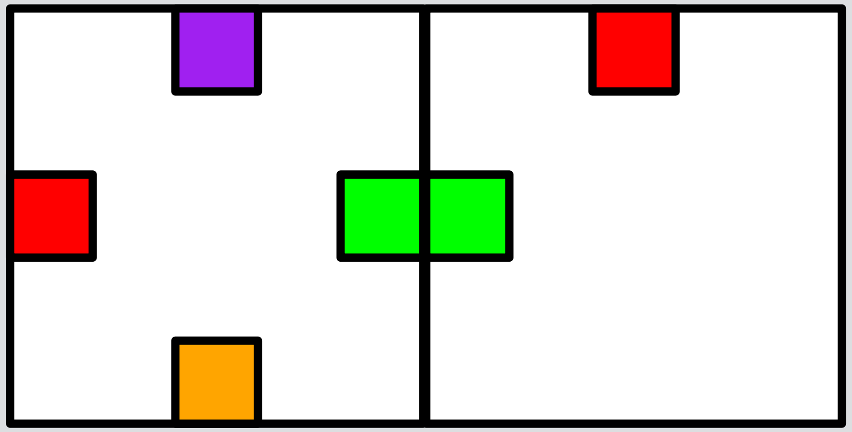
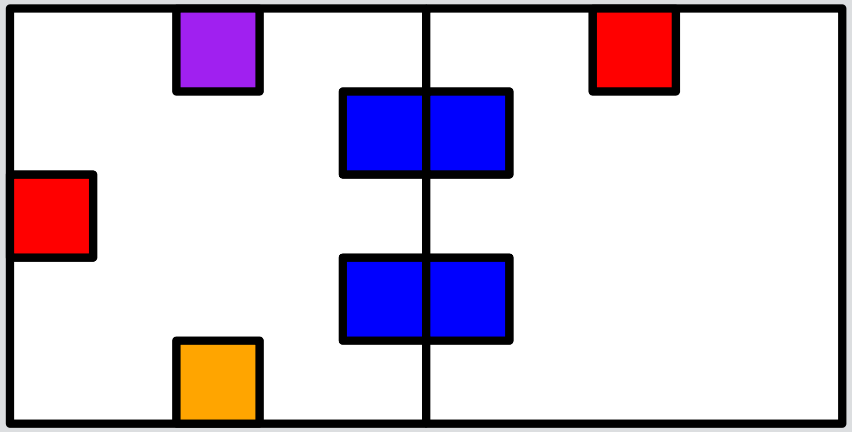
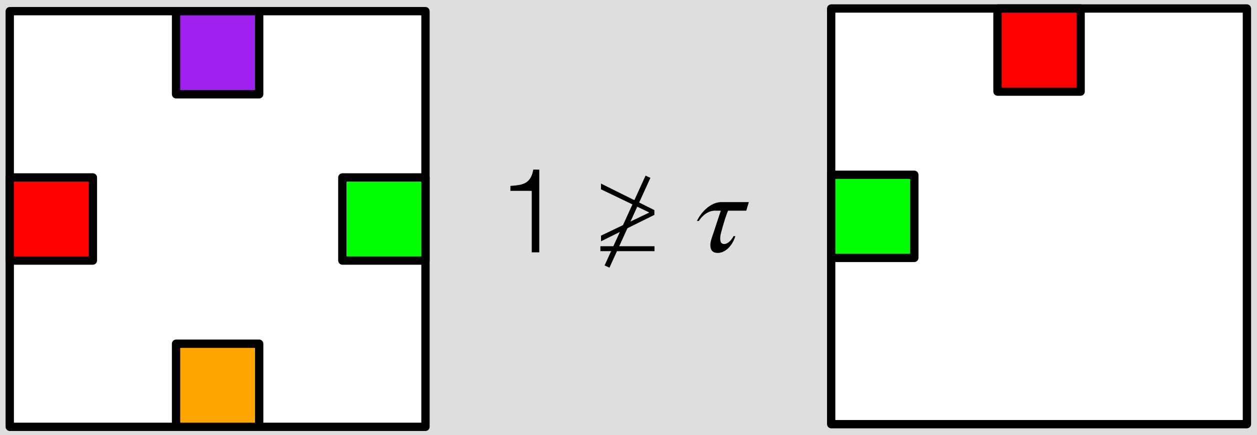
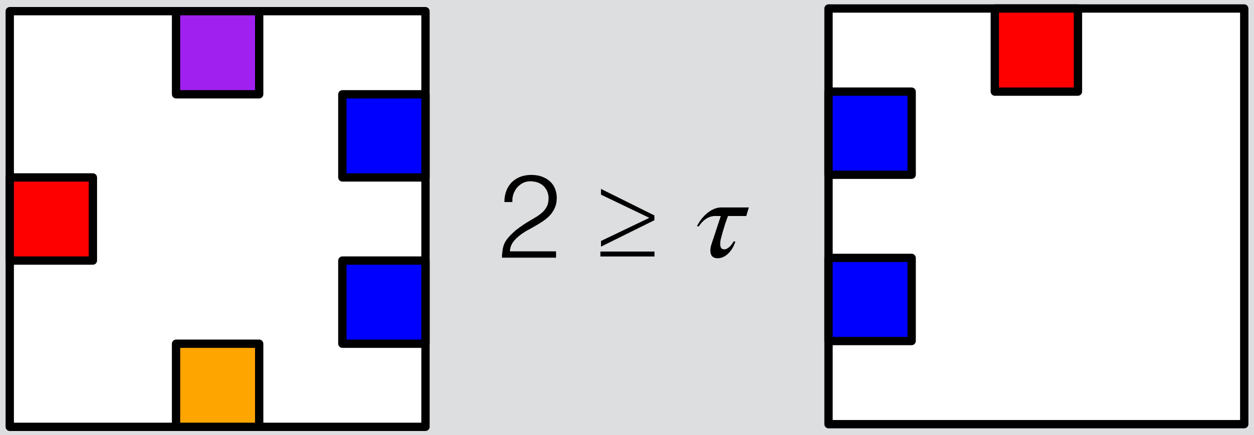


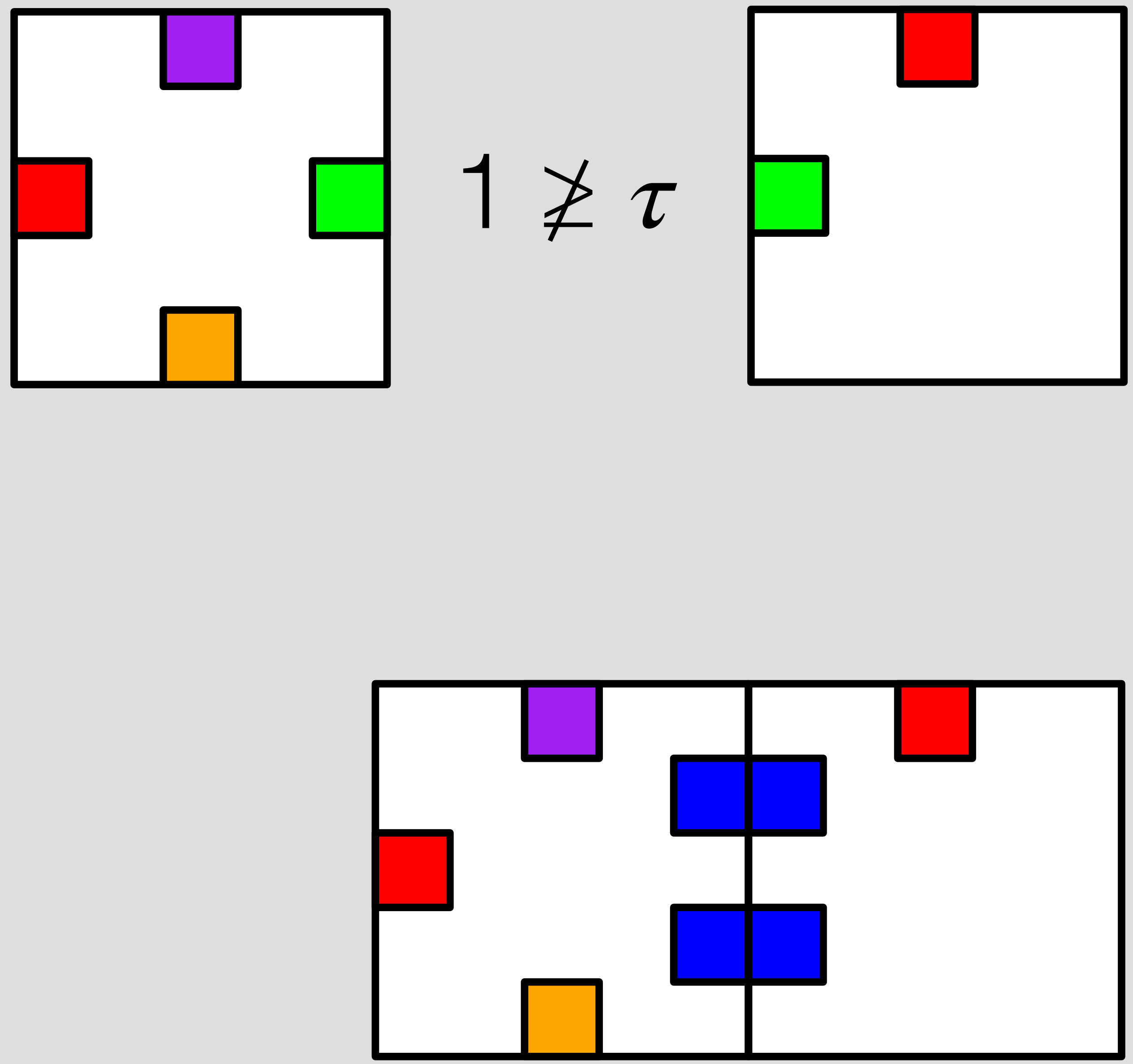


At temperature $\tau = 2$

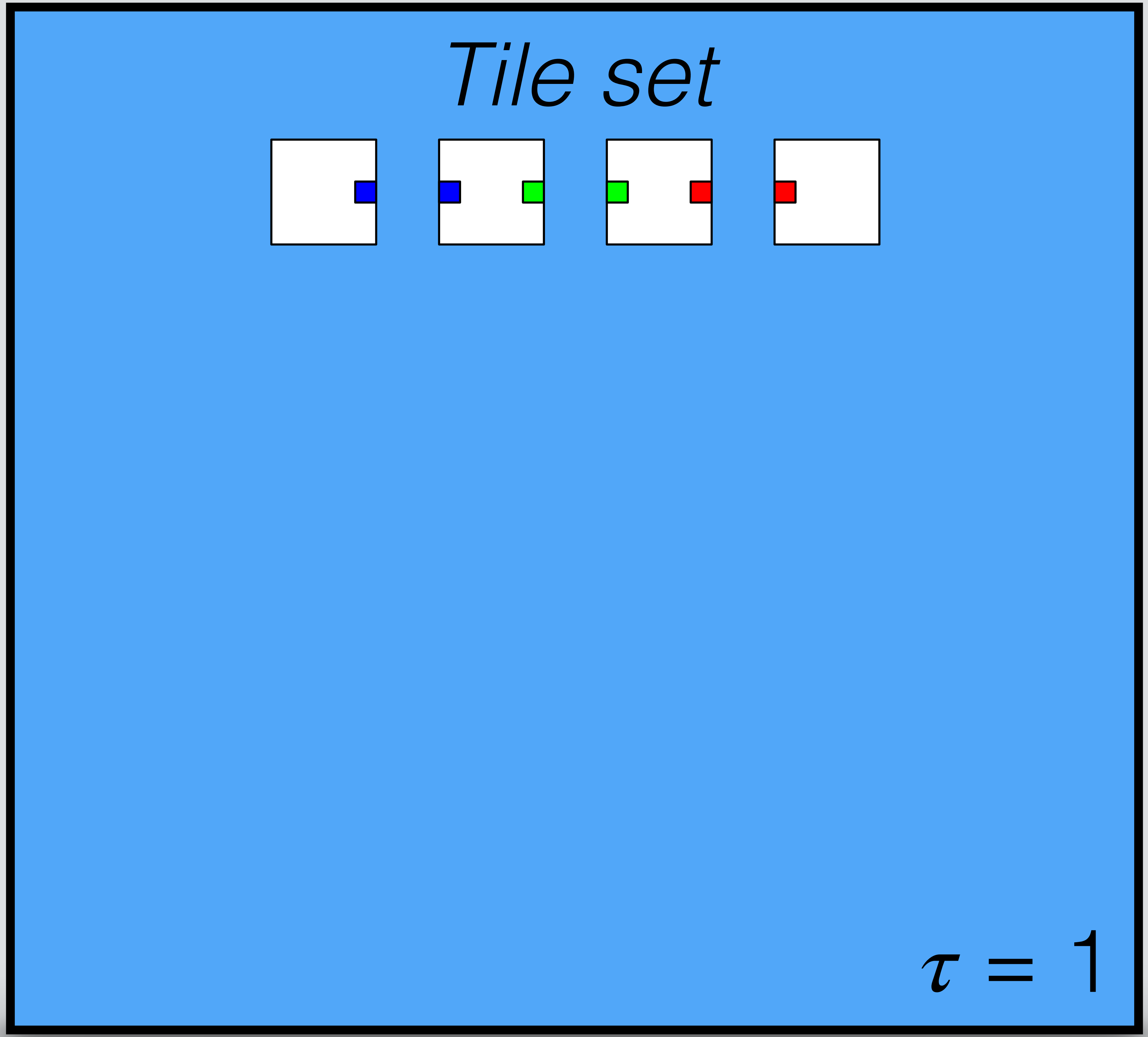




At temperature $\tau = 2$



Two-handed assembly



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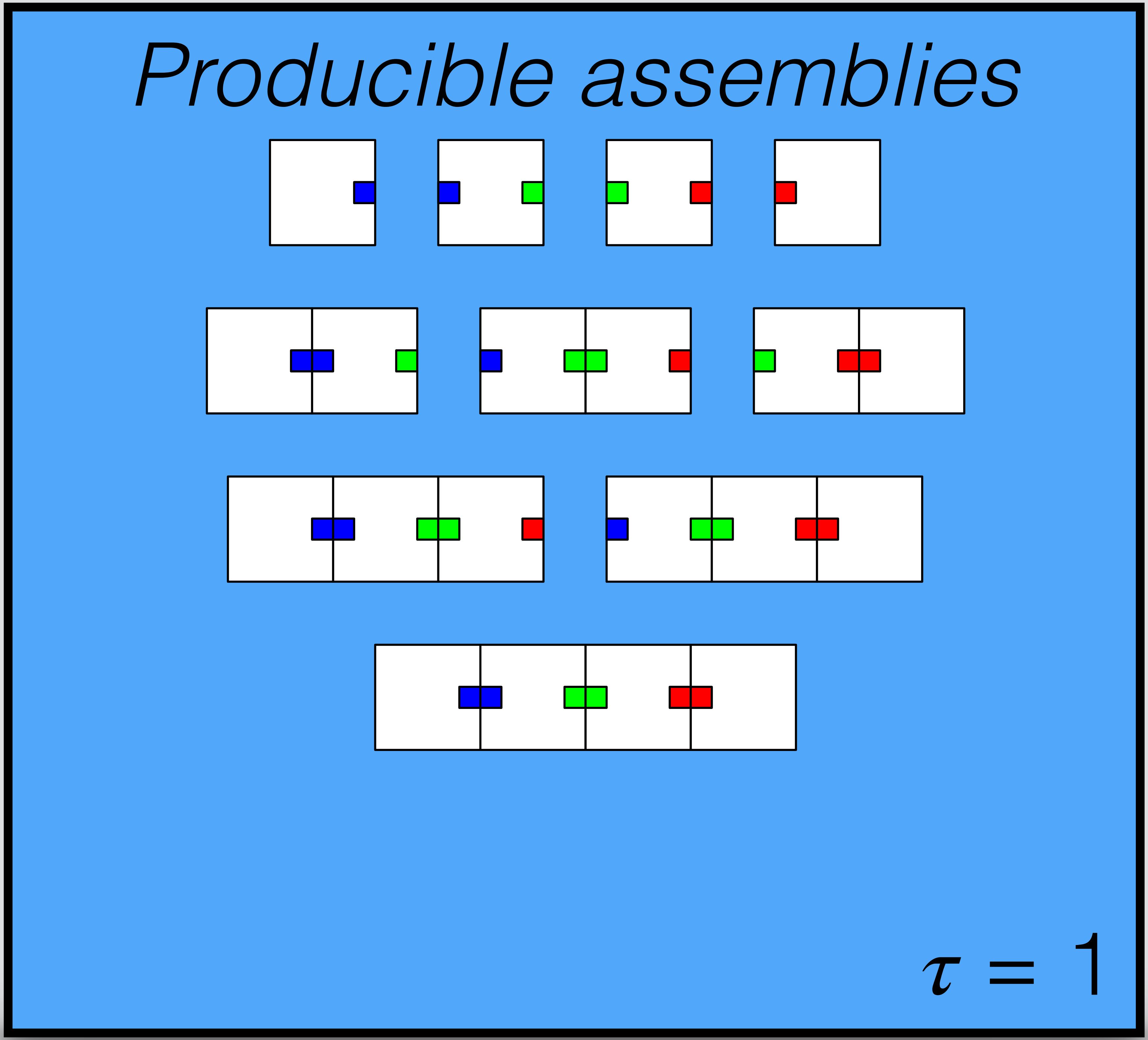

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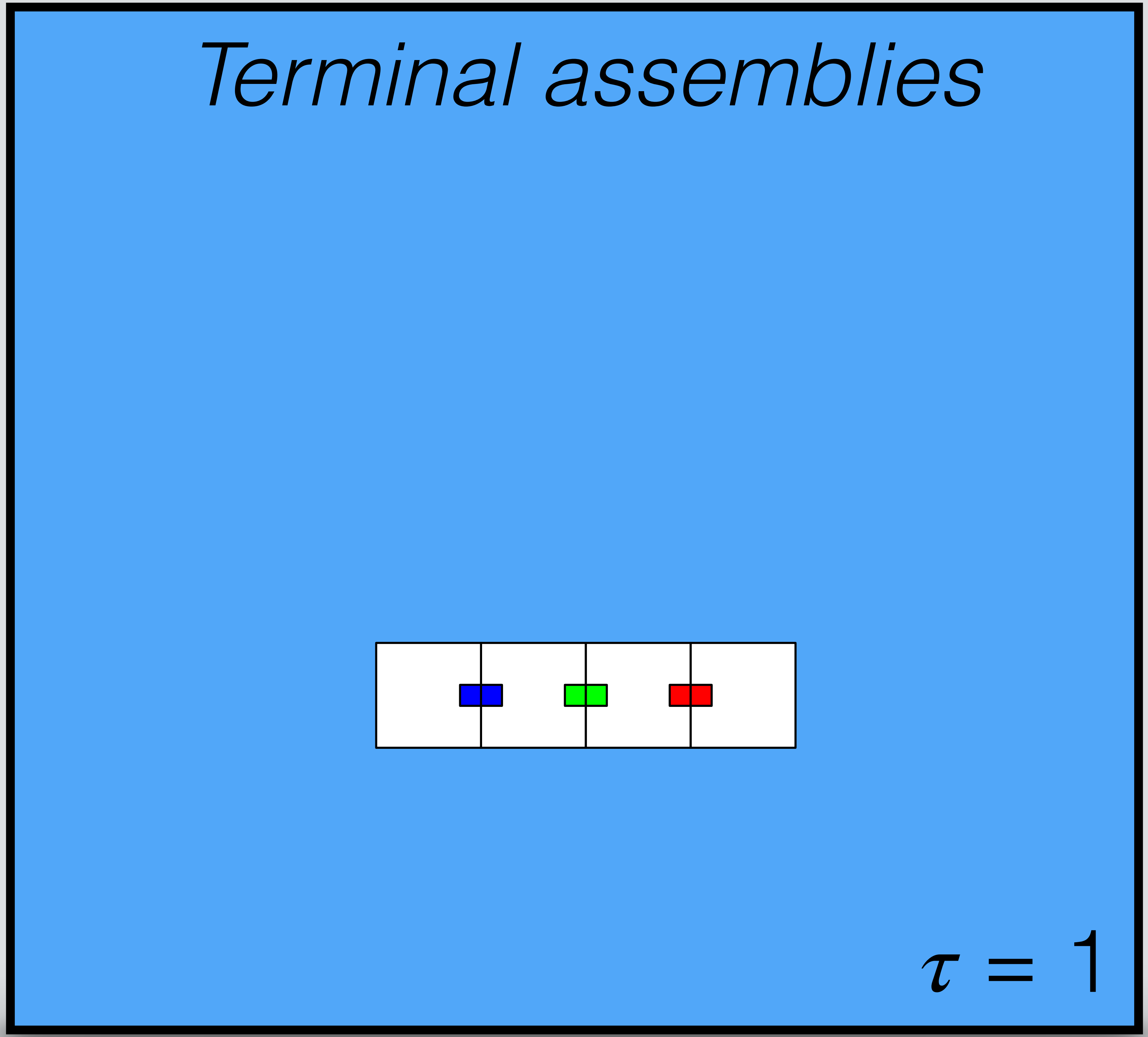
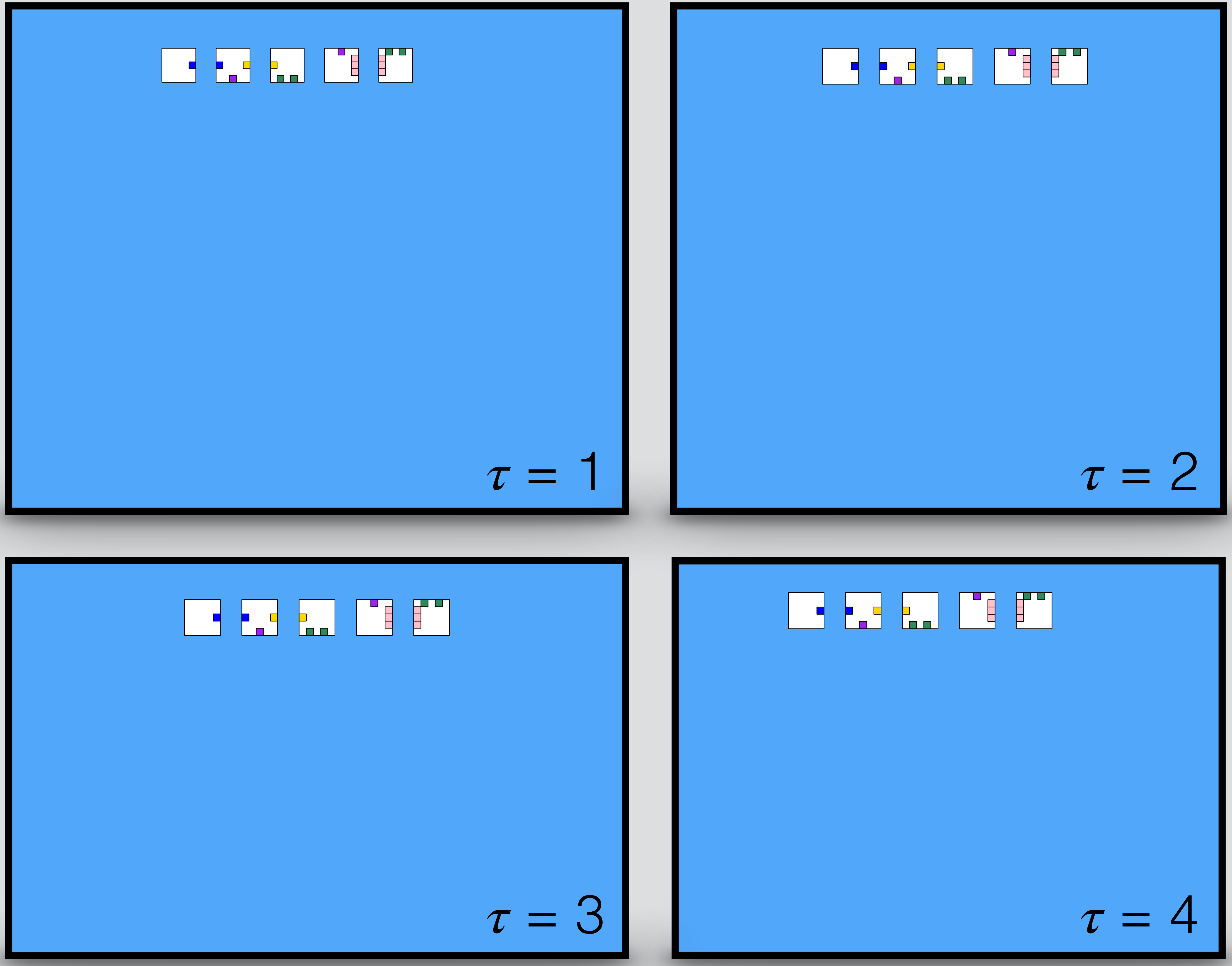
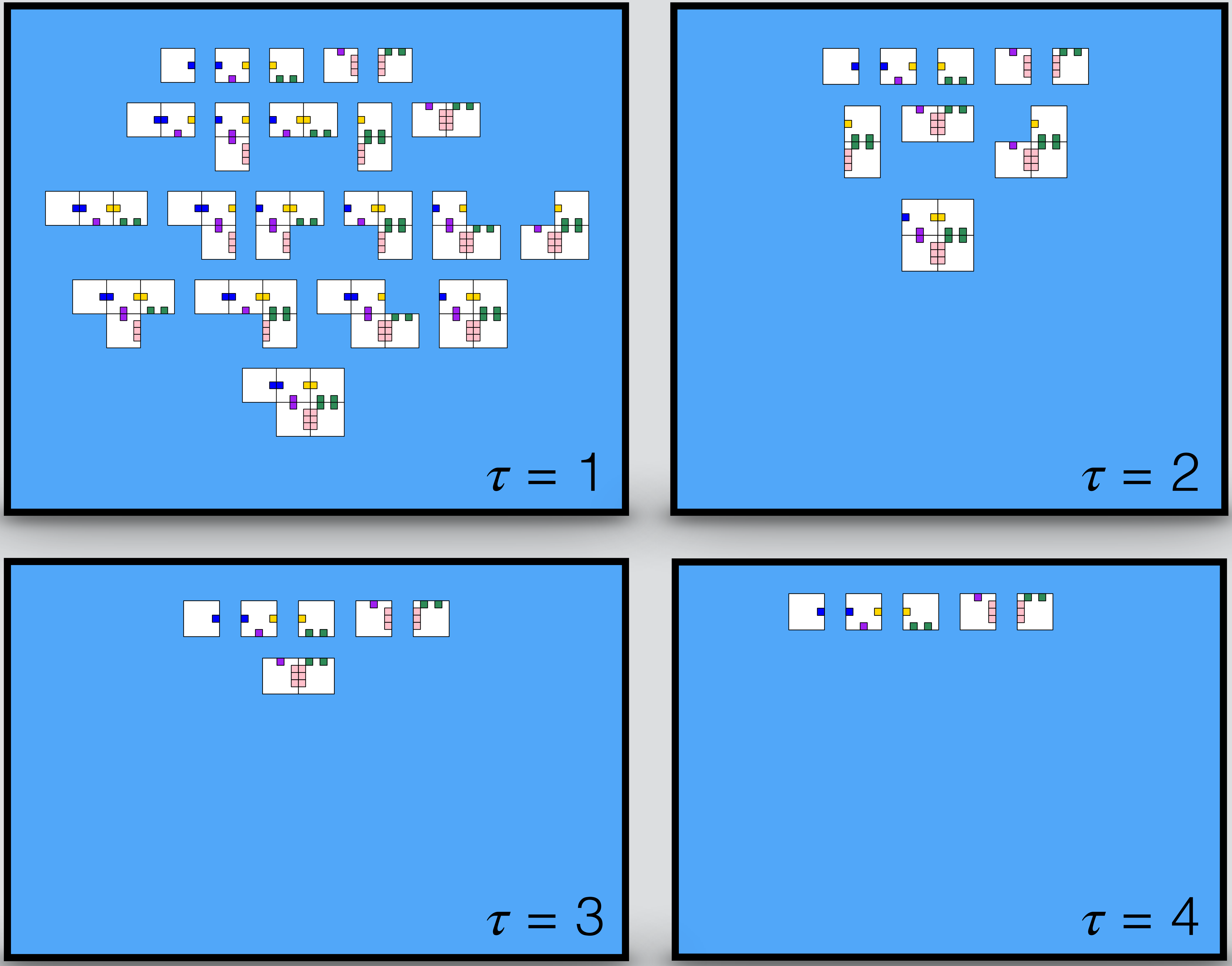



$\tau = 1$
$\tau = 2$
$\tau = 3$
$\tau = 4$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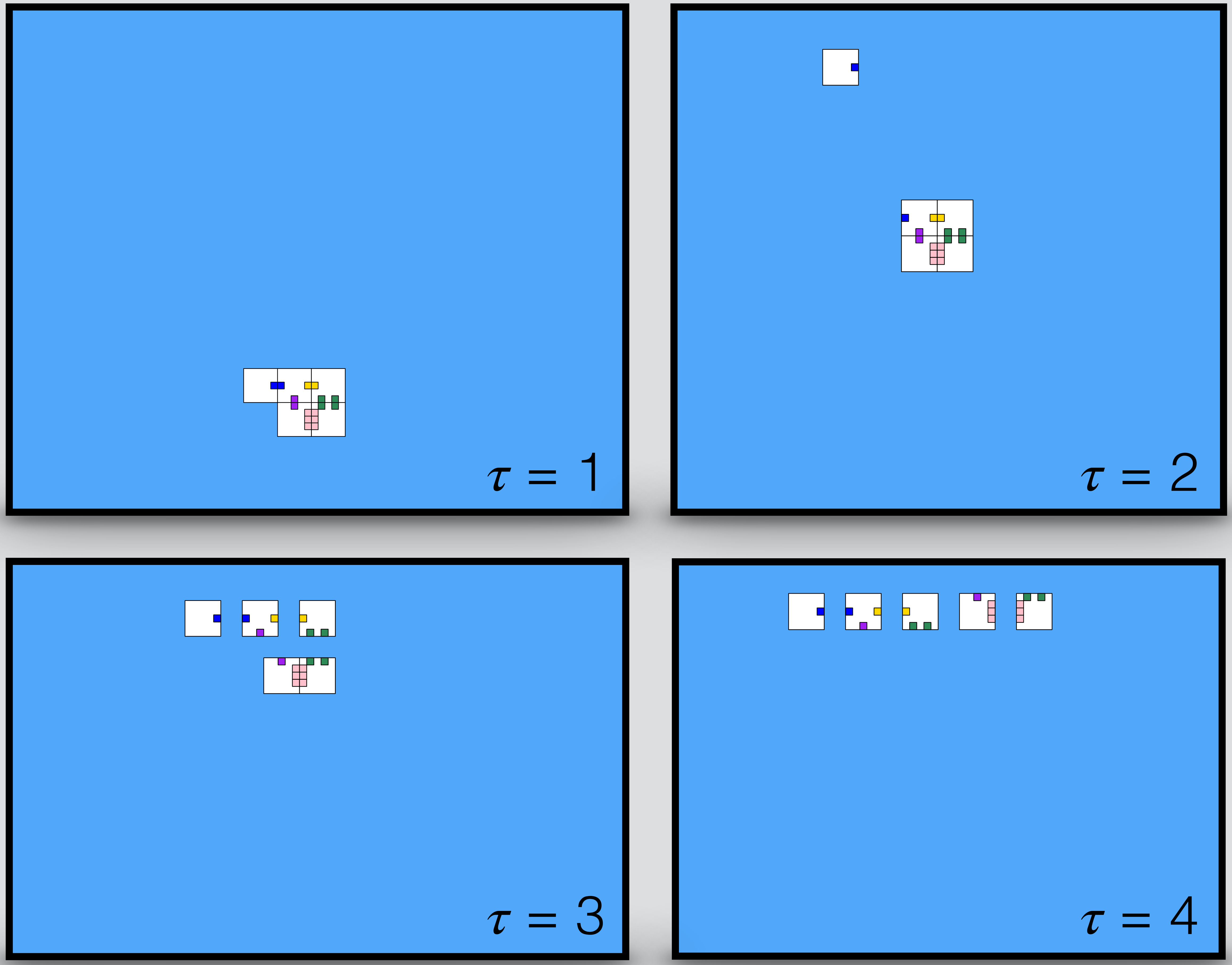
τ = 1
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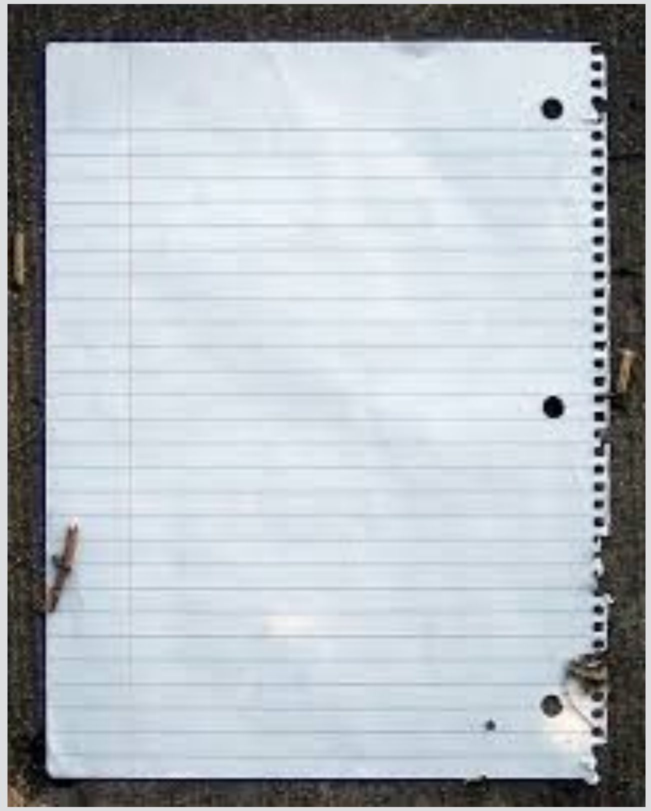
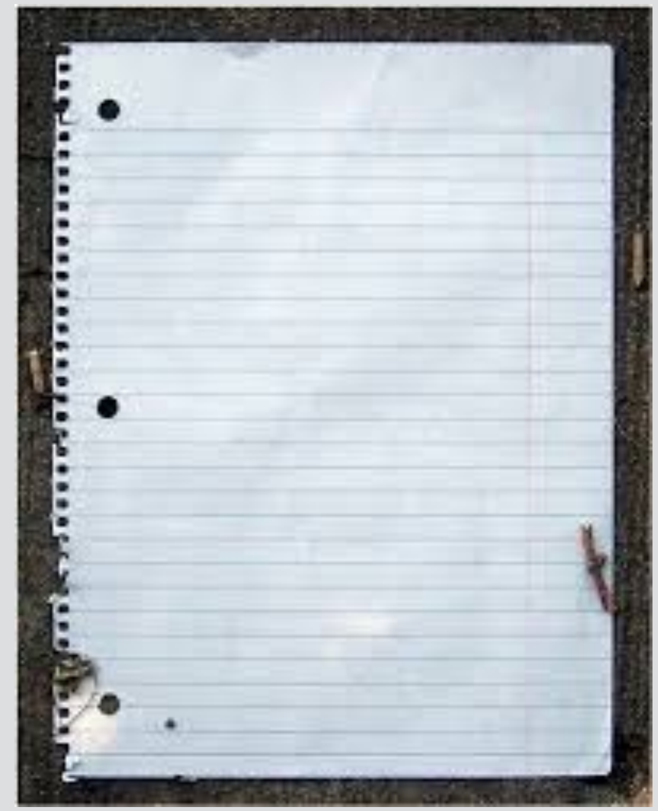
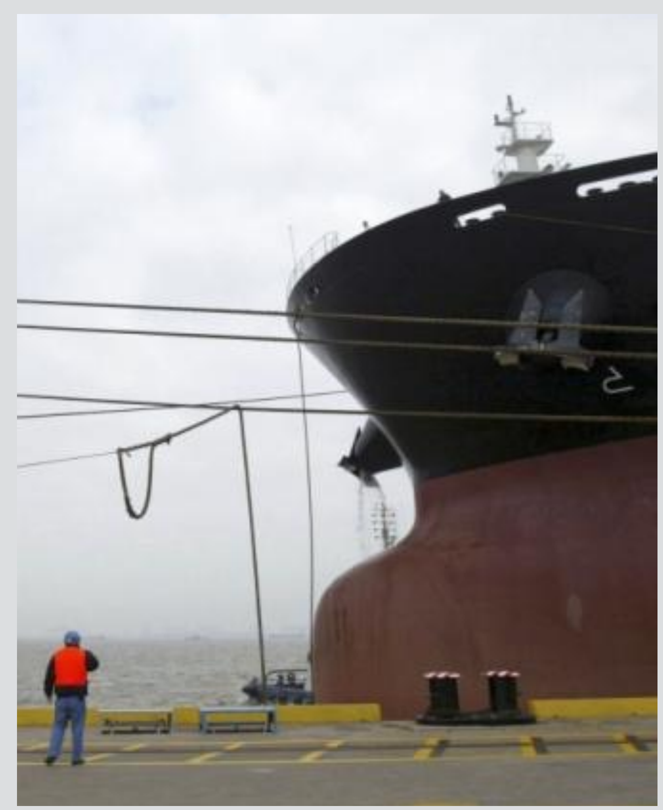
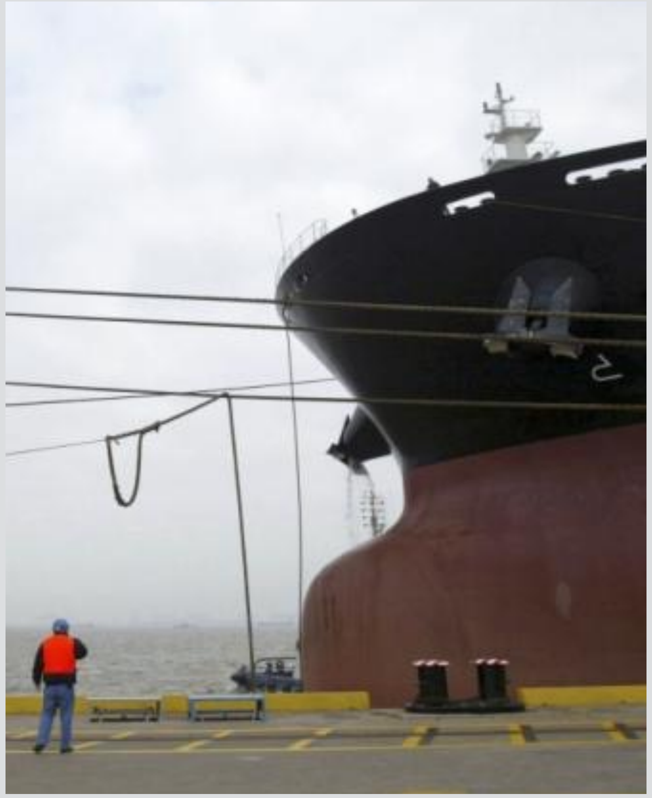






Swingline




Swingline


Swingline

Bigger assemblies require more bond strength.

Size-dependent assembly

- Replace temperature τ with increasing temperature function $\tau : \mathbb{N} \rightarrow \mathbb{N}$.
- Assemblies α, β can bond if total bond strength is $\geq \tau(\min(|\alpha|, |\beta|))$.

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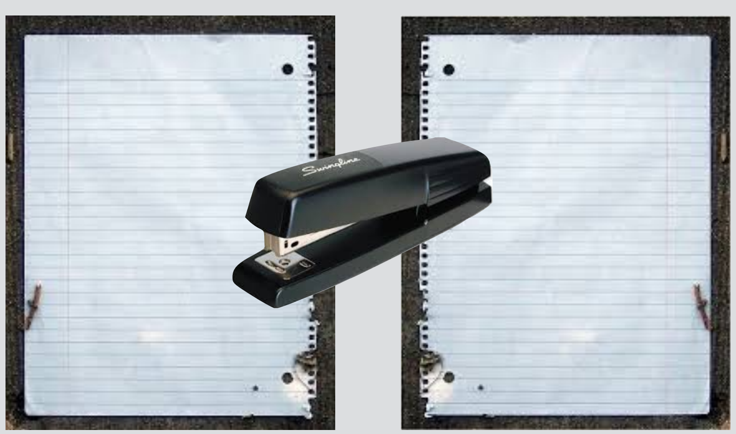
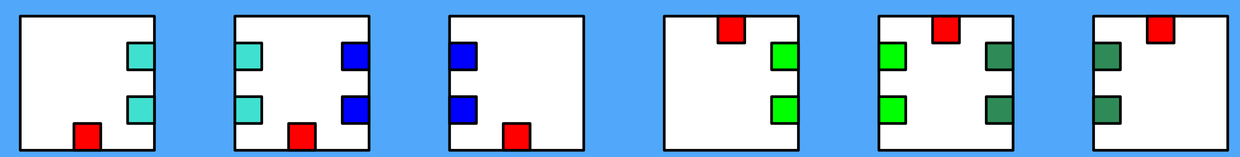
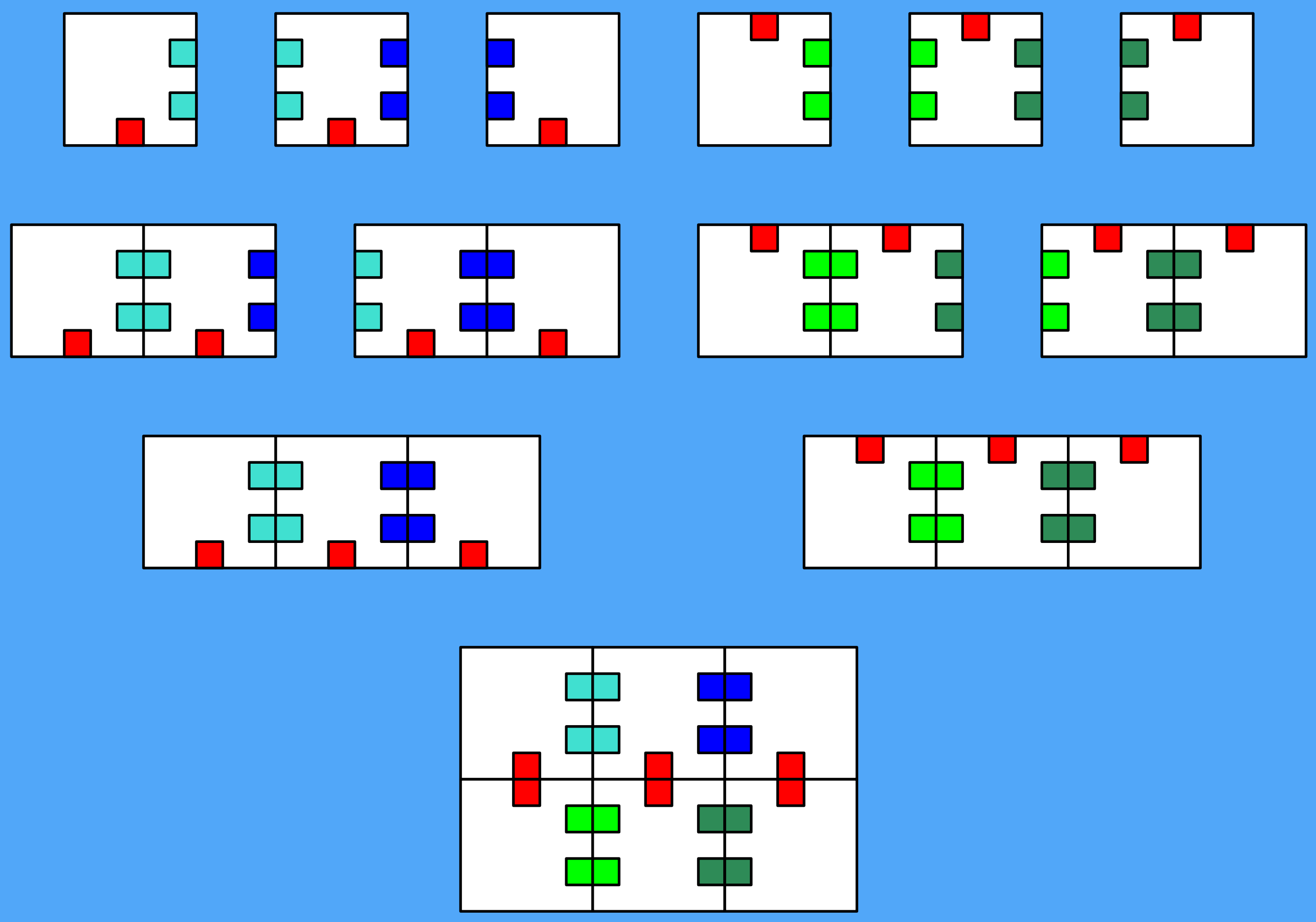


Size-Dependent Assembly



$$\tau(n) = \begin{cases} 2 : n \leq 1 \\ 3 : \text{otherwise} \end{cases}$$

Size-Dependent Assembly



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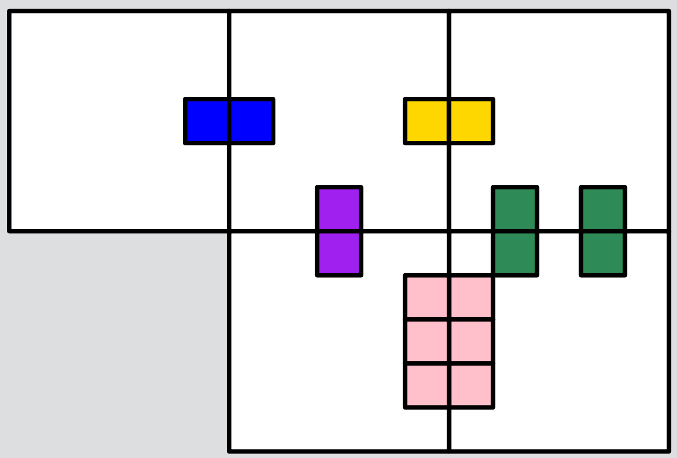
Stability and Cuts

An assembly is stable at temperature τ

if all cuts have strength $\geq \tau$.



Cuts of strength:

Stability and Cuts

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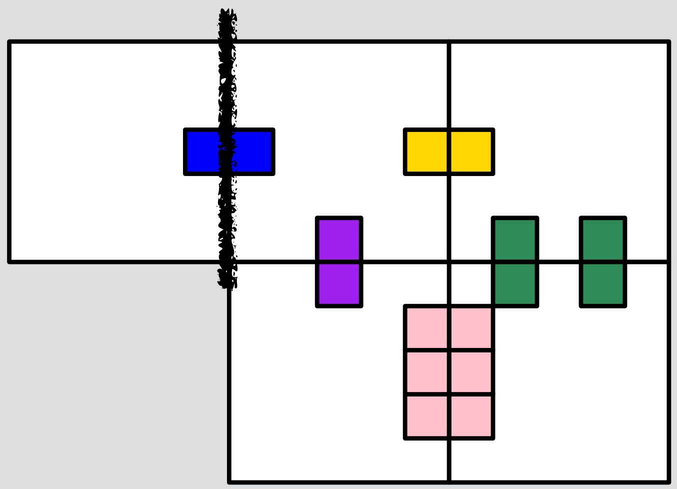

Cuts of strength:

1

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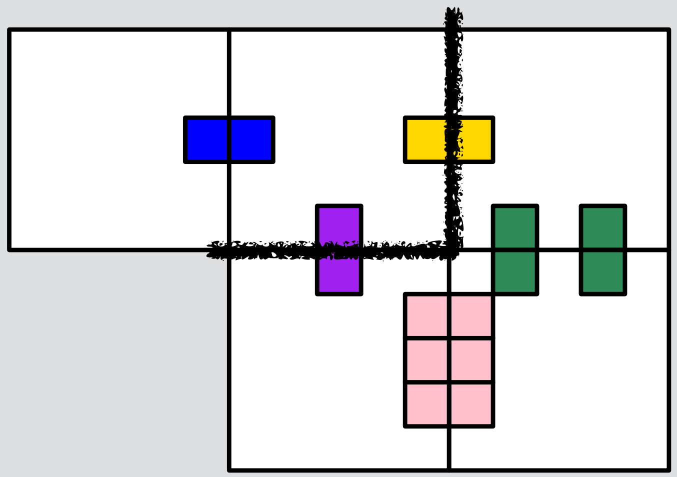

Cuts of strength:
1, 2

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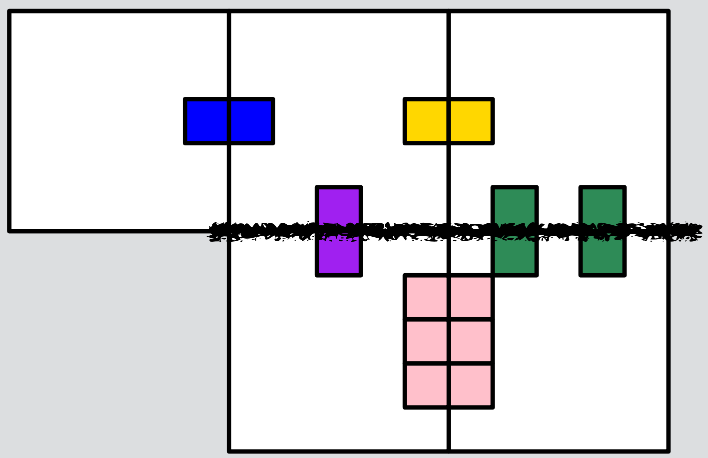


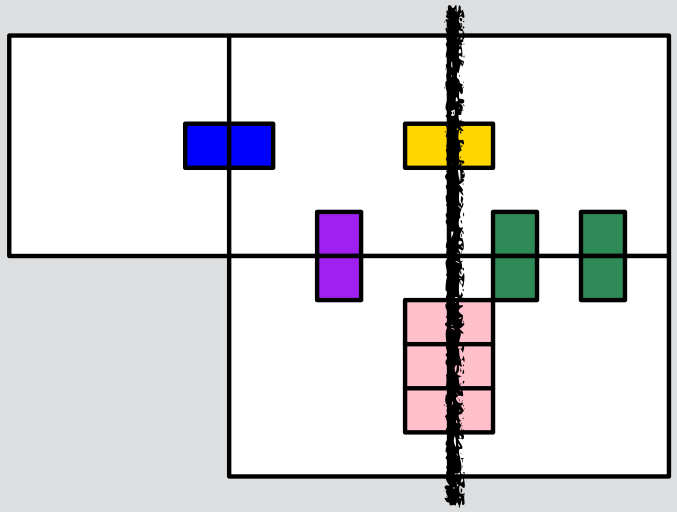
Cuts of strength:

1, 2, 3

Stability and Cuts

An assembly is <u>stable</u> at temperature τ

if all <u>cuts</u> have strength $\geq \tau$.



Cuts of strength:

1, 2, 3, 4

Stability and Cuts

An assembly is <u>stable</u> at temperature τ

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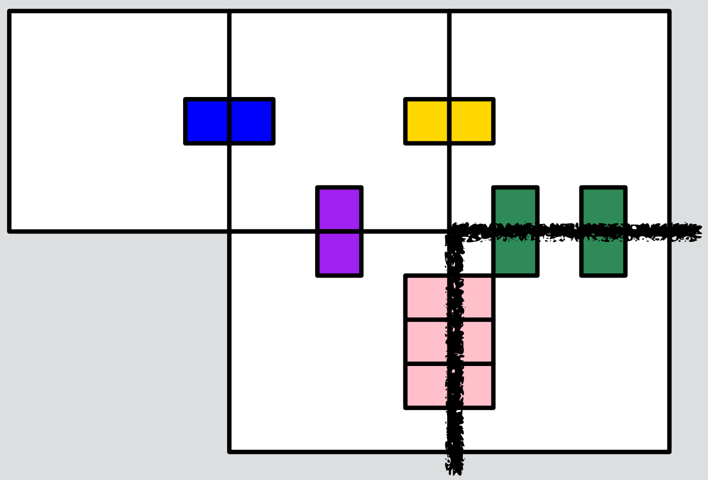

Cuts of strength:

1, 2, 3, 4, 5

Stability and Cuts

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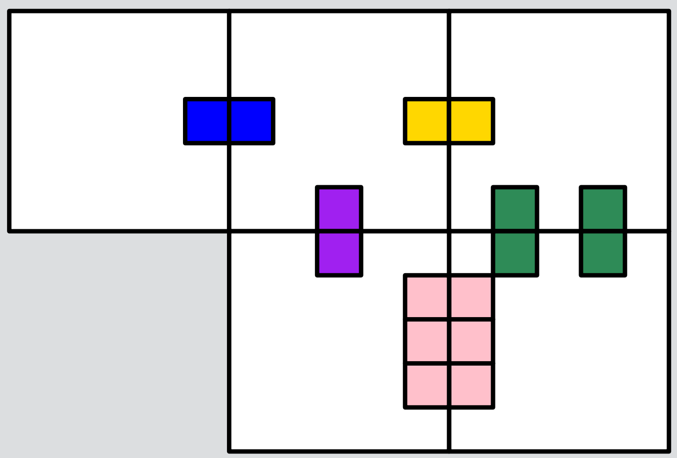

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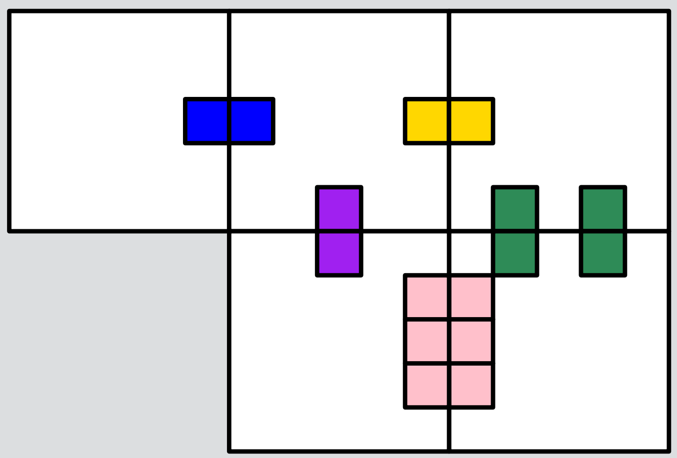


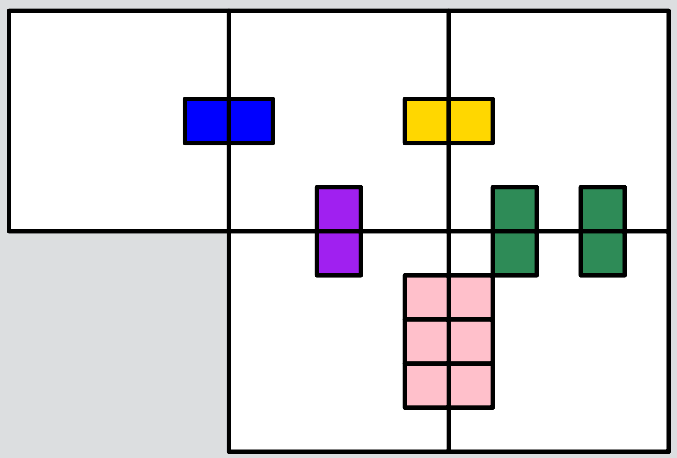
Cuts of strength:

1, 2, 3, 4, 5

Stable at $\tau \leq 1$

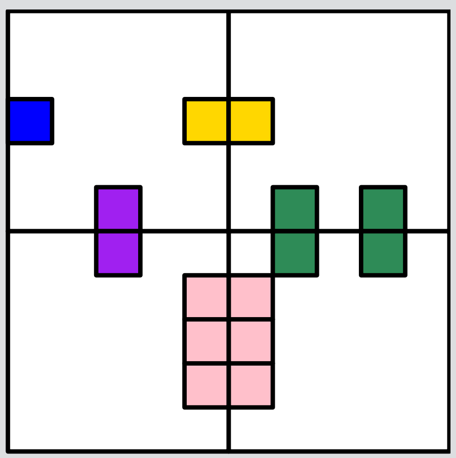
Stability and Cuts

An assembly is stable at temperature τ if all cuts have strength $\geq \tau$.



Cuts of strength:
1, 2, 3, 4, 5

Stable at $\tau \leq 1$



Cuts of strength:
2, 3, 4, 5

Stable at $\tau \leq 2$

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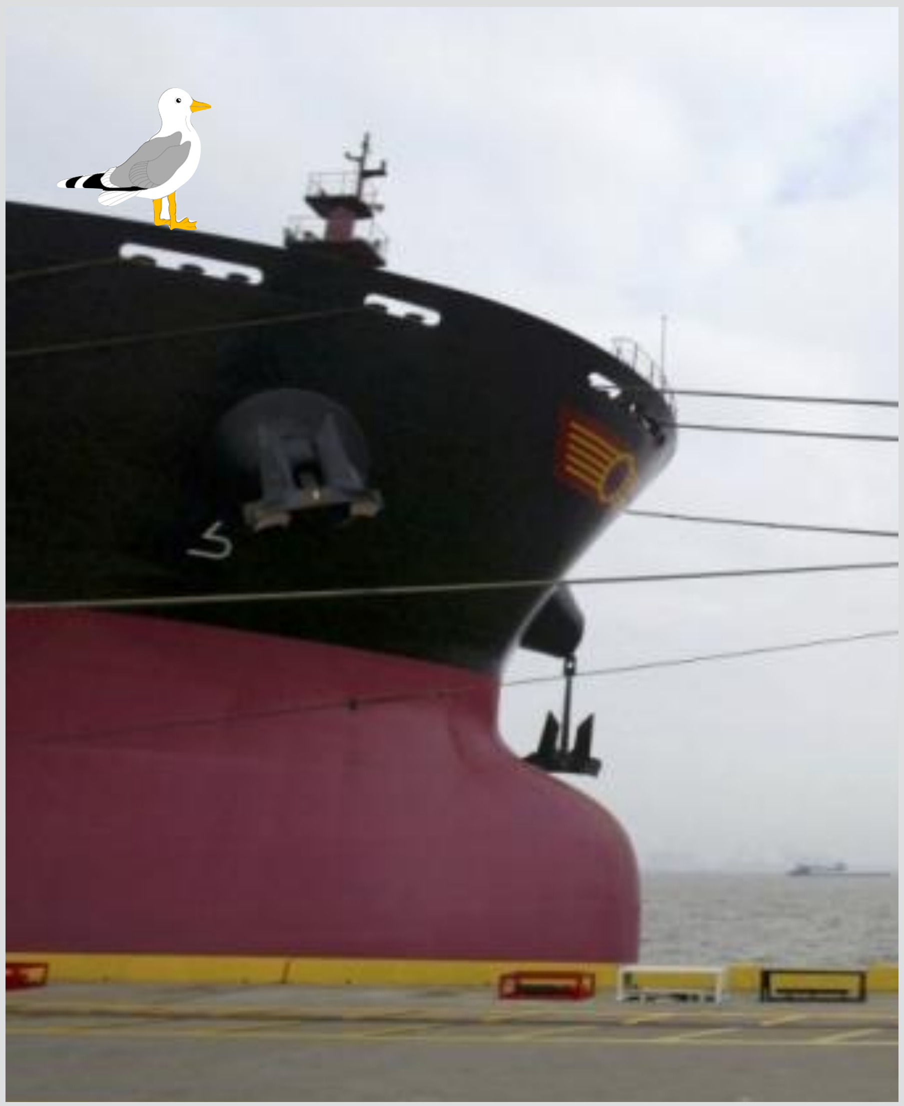
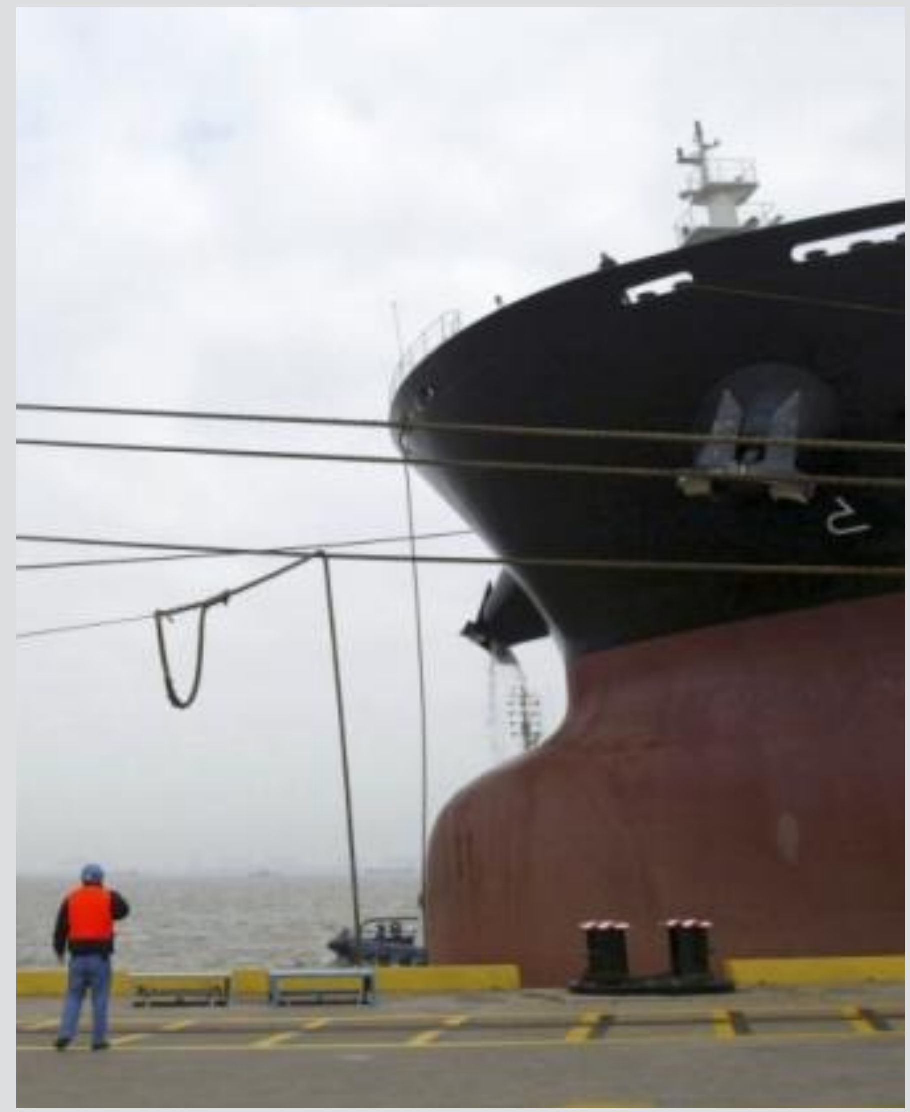
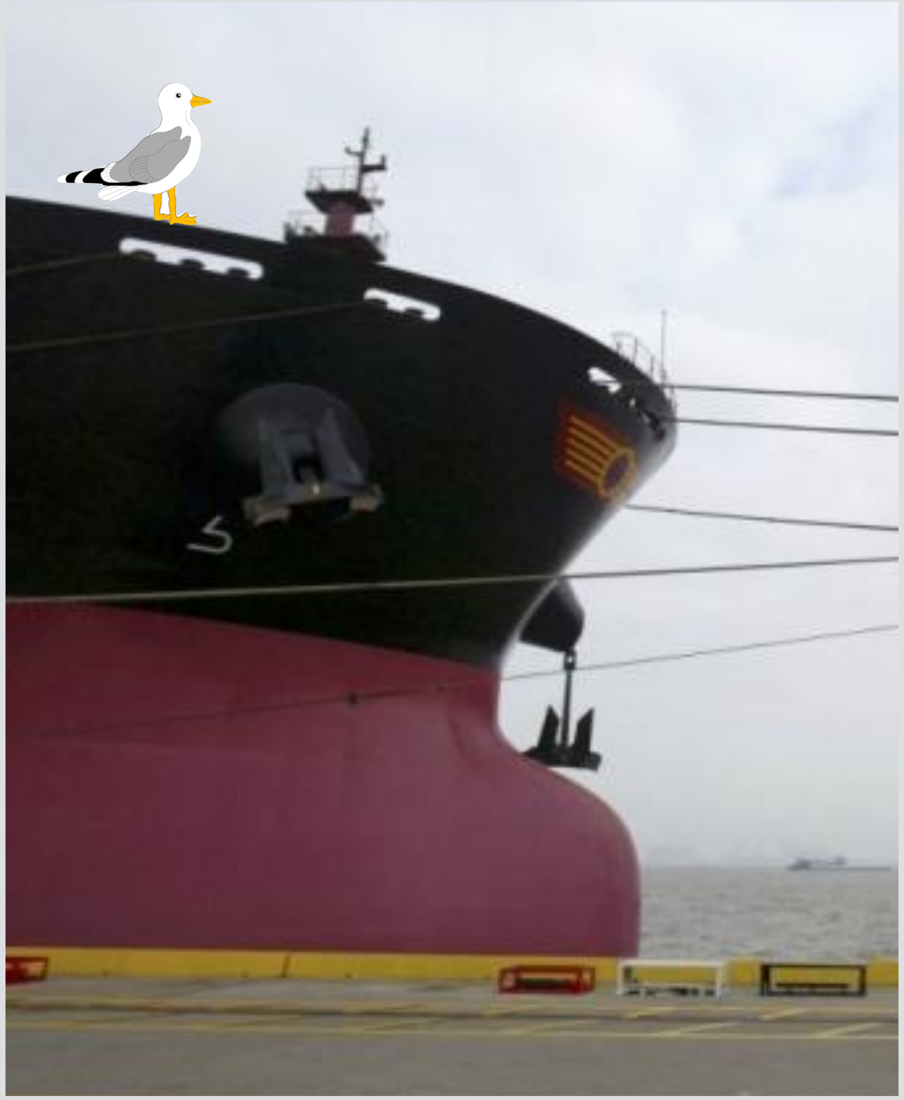
Size-dependent assembly

- Replace temperature τ with increasing temperature function $\tau : \mathbb{N} \to \mathbb{N}$.
- Assemblies α, β can bond if total bond strength is ≥ $\tau(\min(|\alpha|, |\beta|))$.
- Assembly is <u>stable</u> if every cut into connected subassemblies α, β has strength ≥ $\tau(\min(|\alpha|, |\beta|))$.

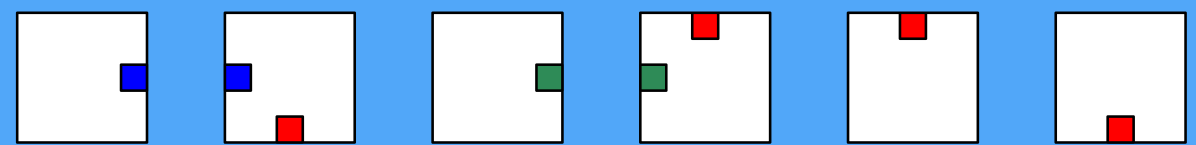




Unstable assemblies break along weak cuts.

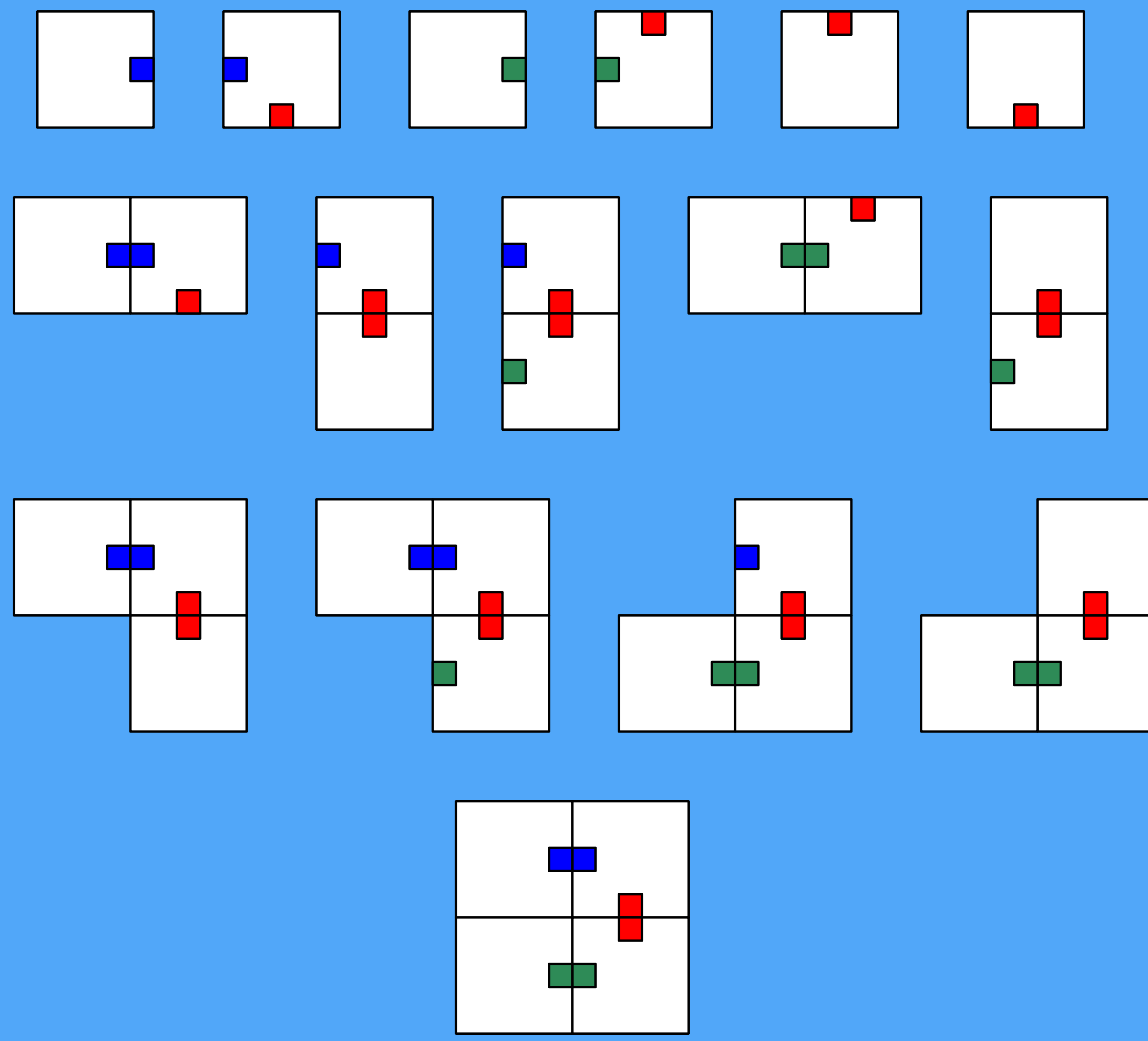


Size-Dependent Assembly



$$\tau(n) = \begin{cases} 1 : n \leq 1 \\ 2 : \text{otherwise} \end{cases}$$

Size-Dependent Assembly



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Questions

Can temperature functions do anything “useful”, e.g. build shapes more efficiently?

Breakage looks complicated.
How hard is deciding if an assembly is stable?

Questions and Prior Work

Can temperature functions do anything "useful", e.g. build shapes more efficiently?

For fixed τ, NxN: $\Theta(\log(N)/\log\log(N))$ tile types,
CxN: $\Theta(N^{1/C})$ tile types.

Breakage looks complicated.
How hard is deciding if an assembly is stable?

For fixed τ, polynomial-time (min-cut).

Questions and Answers

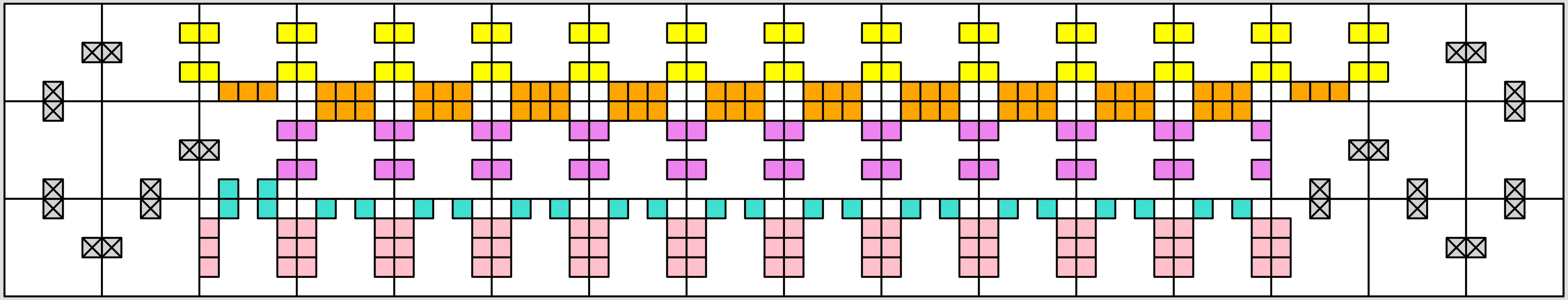
Can temperature functions do anything “useful”, e.g. build shapes more efficiently?

There exists a set of tiles T that assembles 3xN rectangle for each N ≥ 7, given appropriate $\tau(n)$.

Breakage looks complicated.
How hard is deciding if an assembly is stable?

coNP-complete.

3xN Rectangle Construction

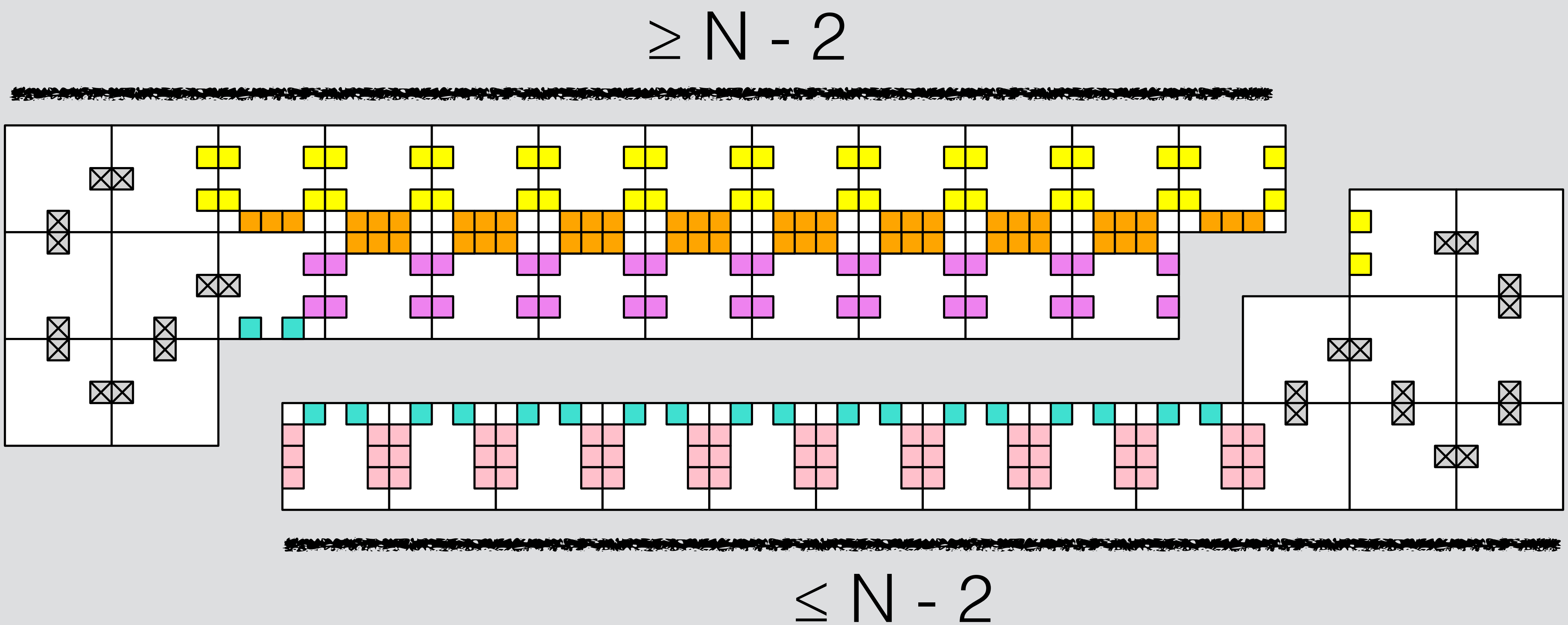


Terminal assembly

$$\tau(n) = \begin{cases} 3 : n \leq N - 6 \\ 4 : N - 5 \leq n \leq N + 3 \\ 5 : N + 4 \leq n \leq 2N - 2 \\ 8 : \text{otherwise} \end{cases}$$

Temperature function

3xN Rectangle Construction



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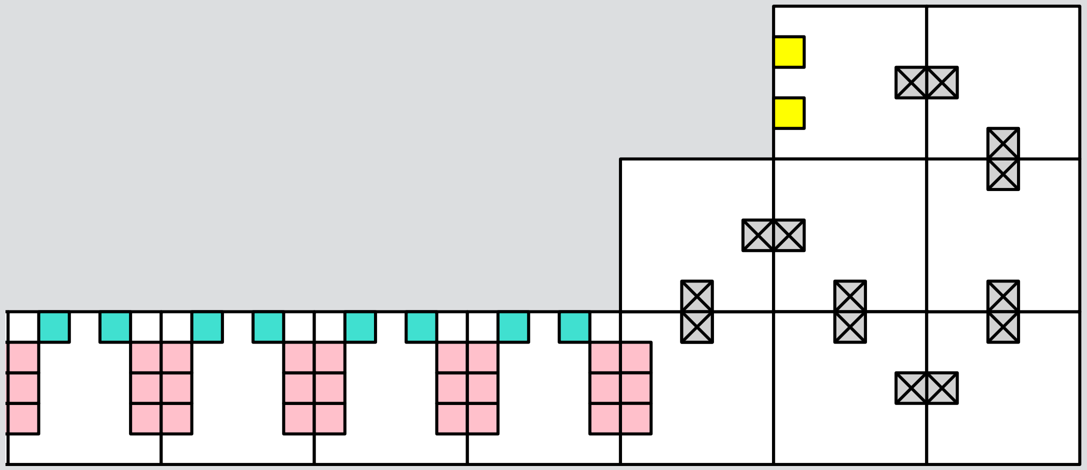

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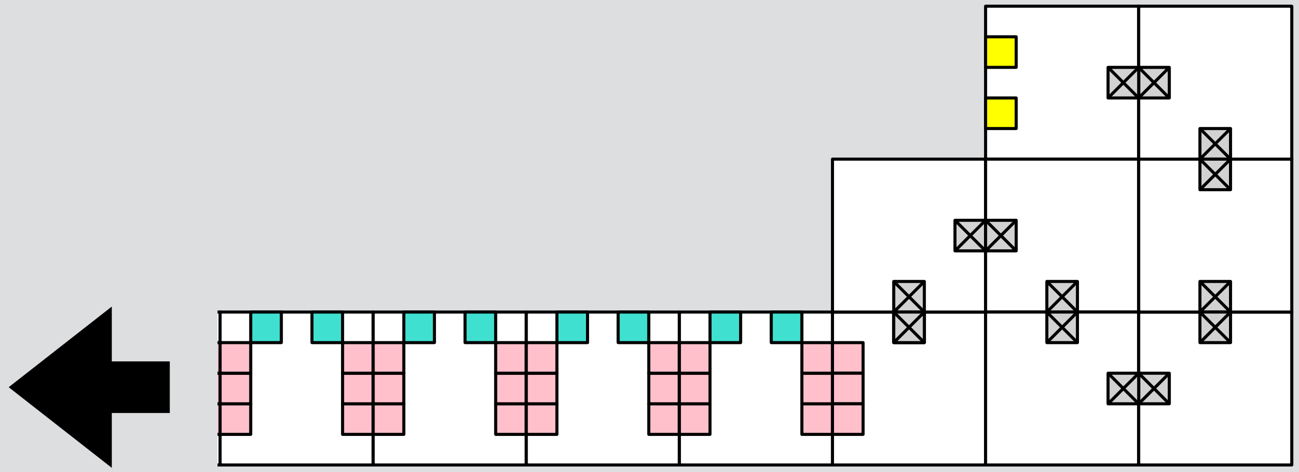

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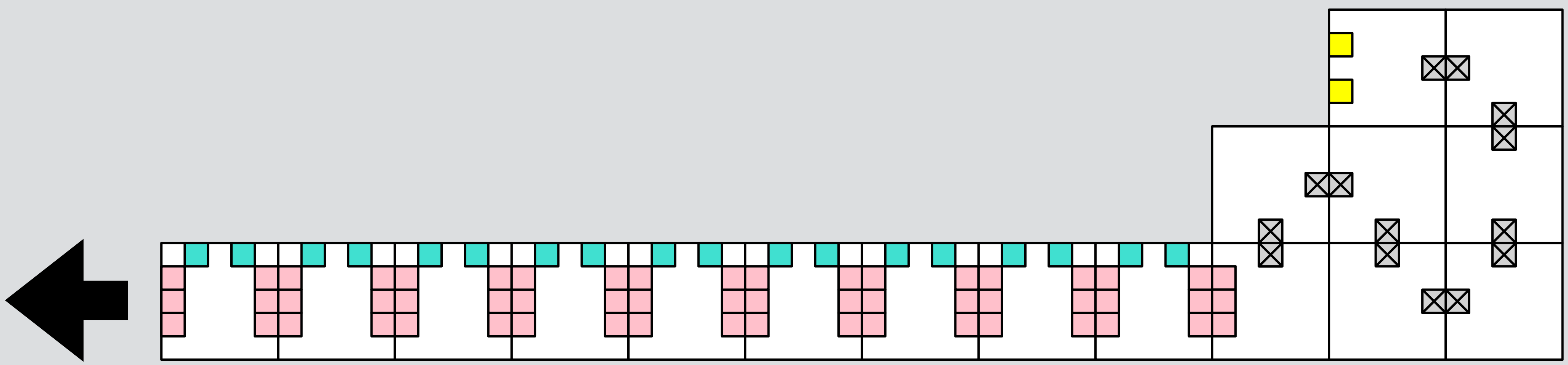

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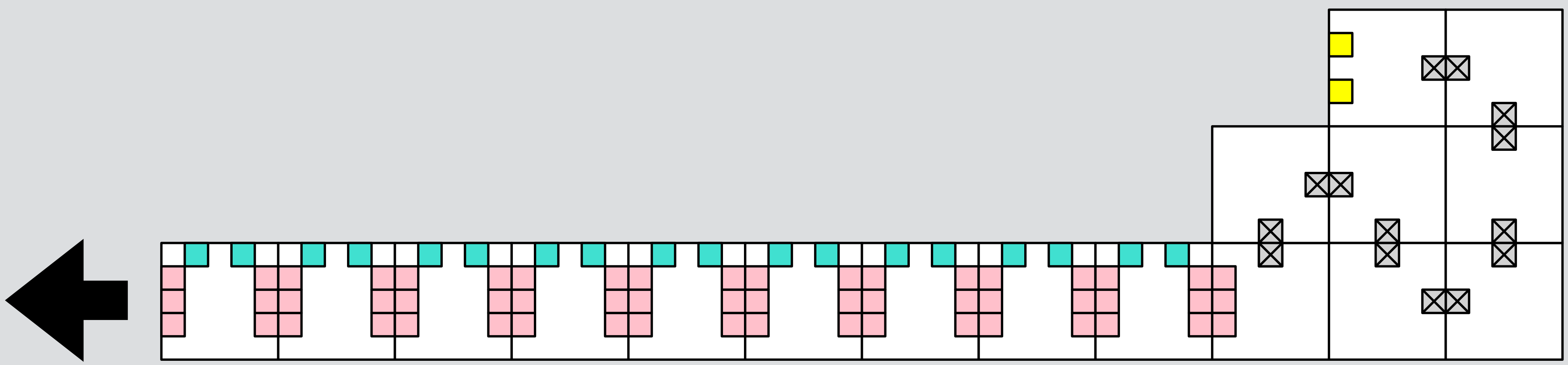

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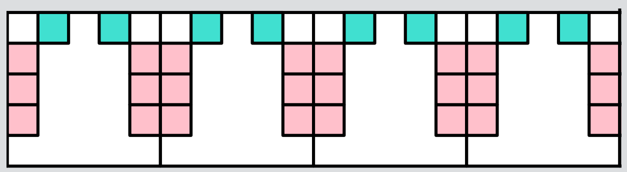

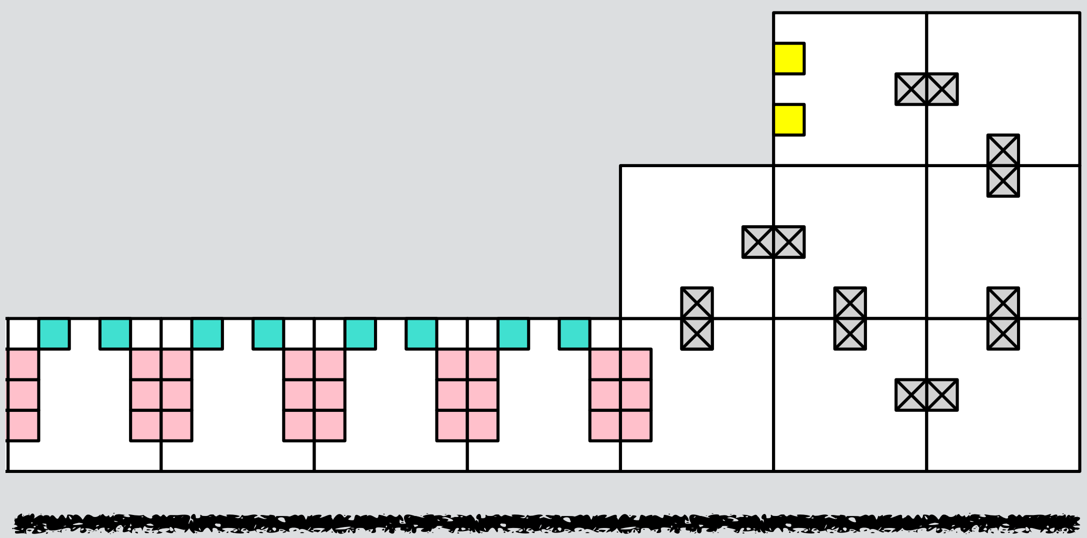

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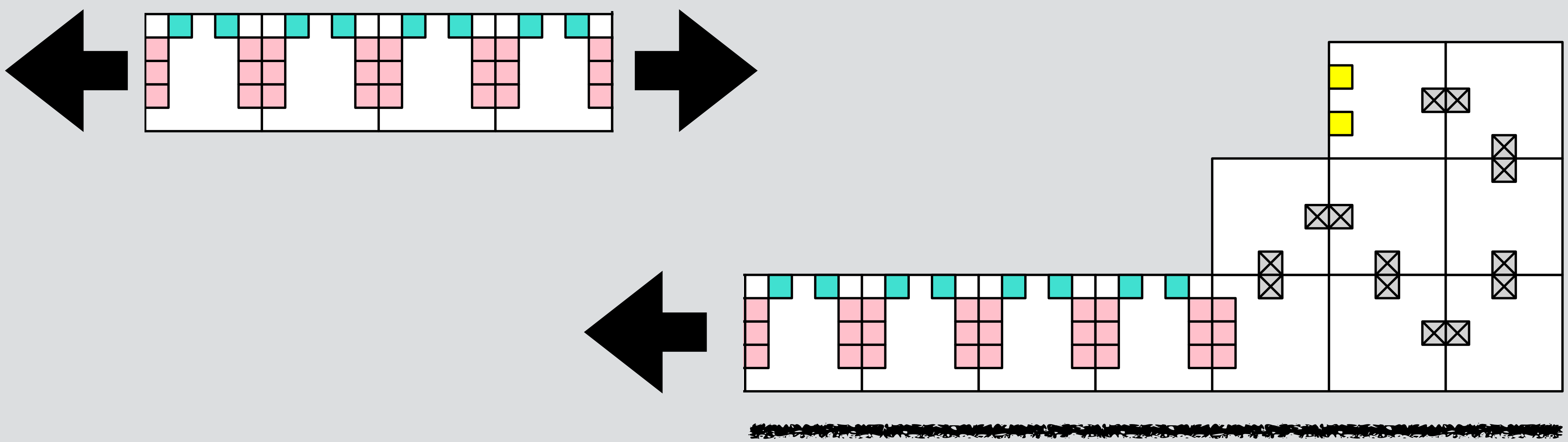
3xN Rectangle Construction



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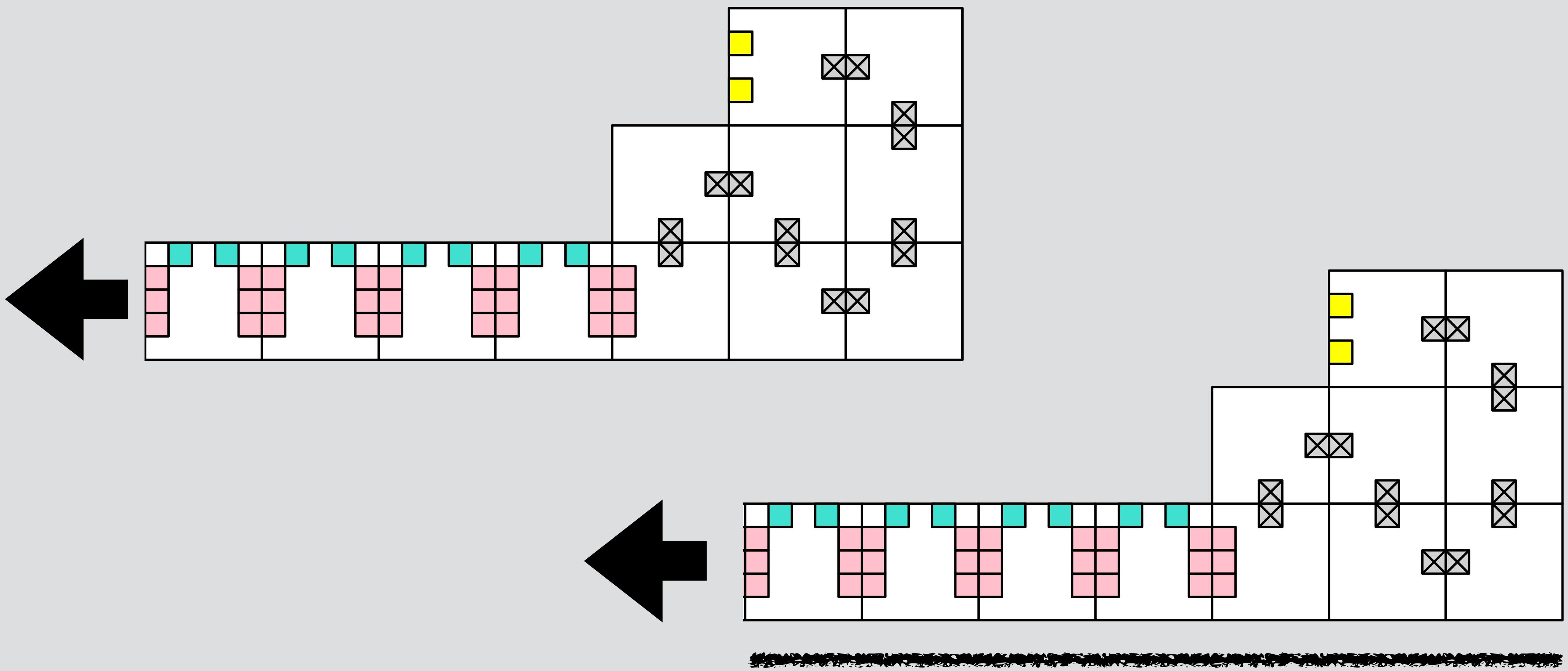

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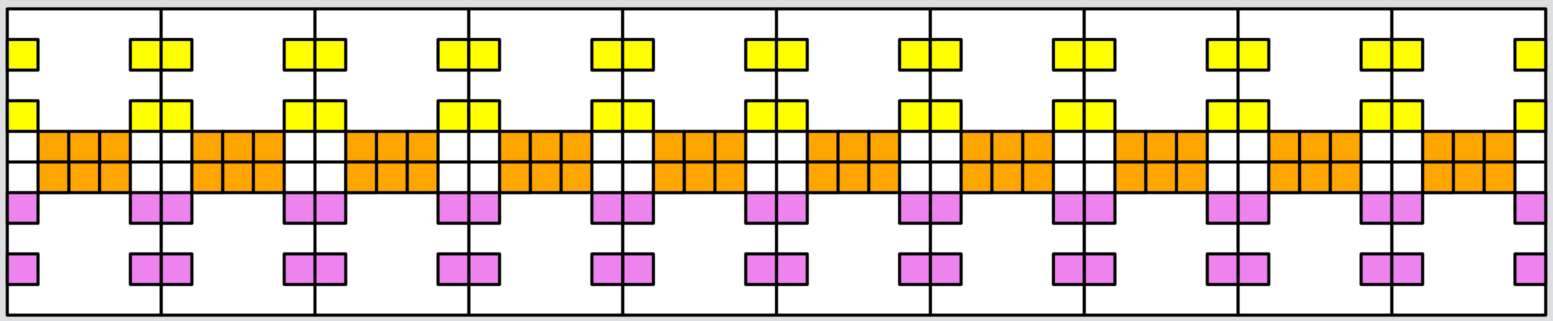

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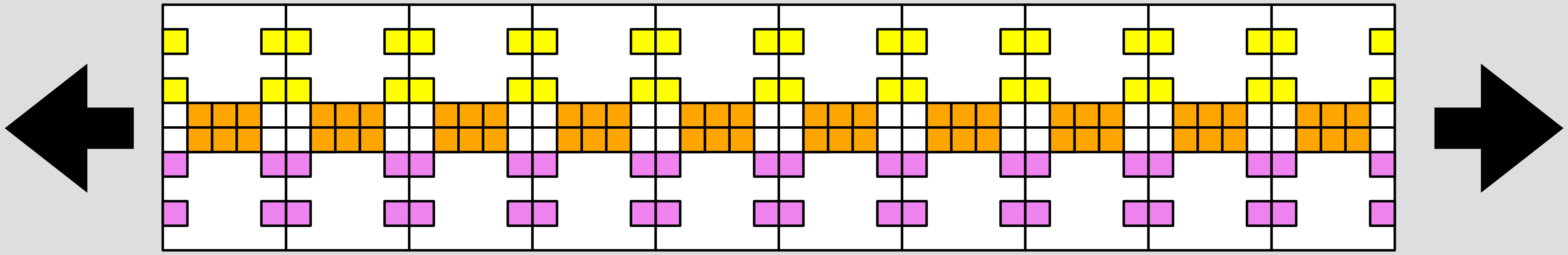

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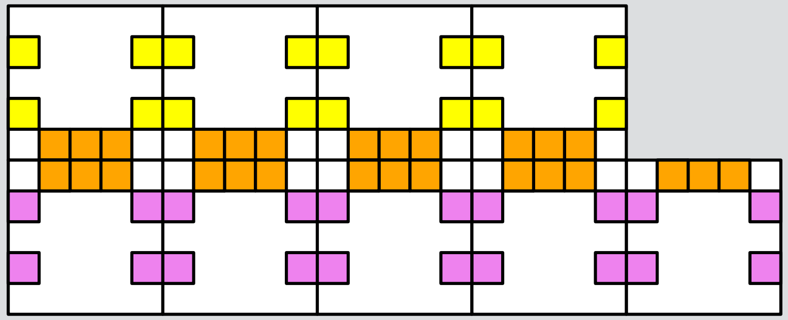

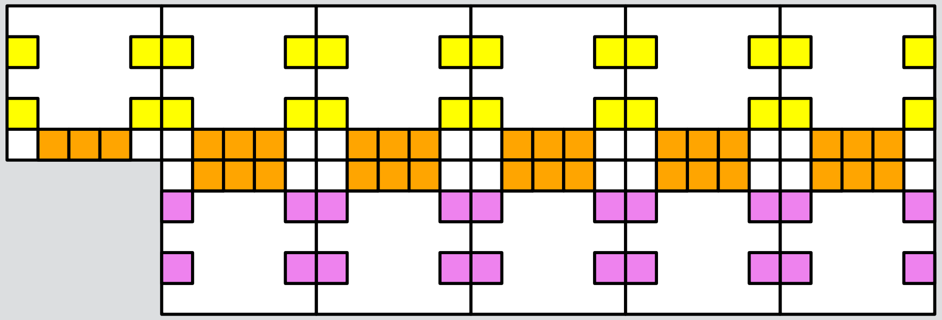

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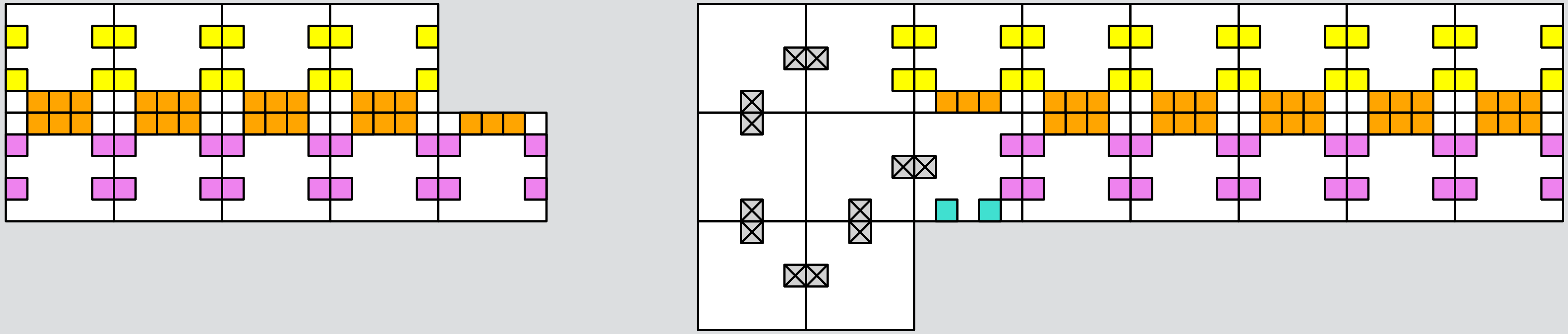

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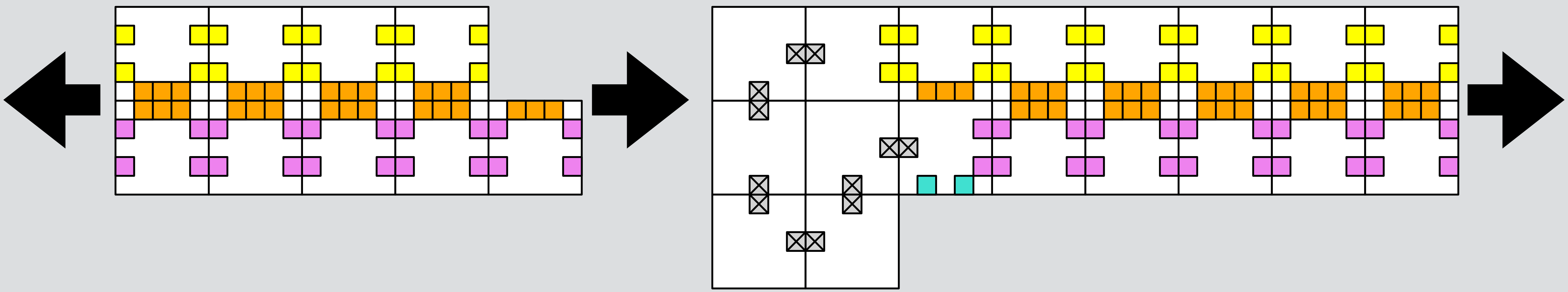

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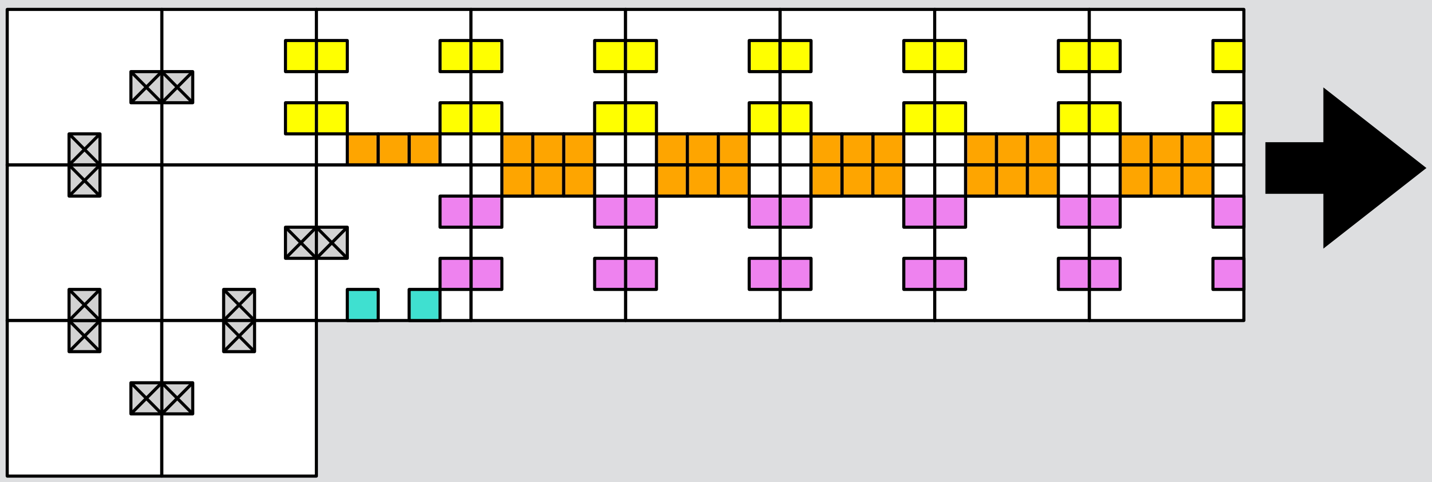

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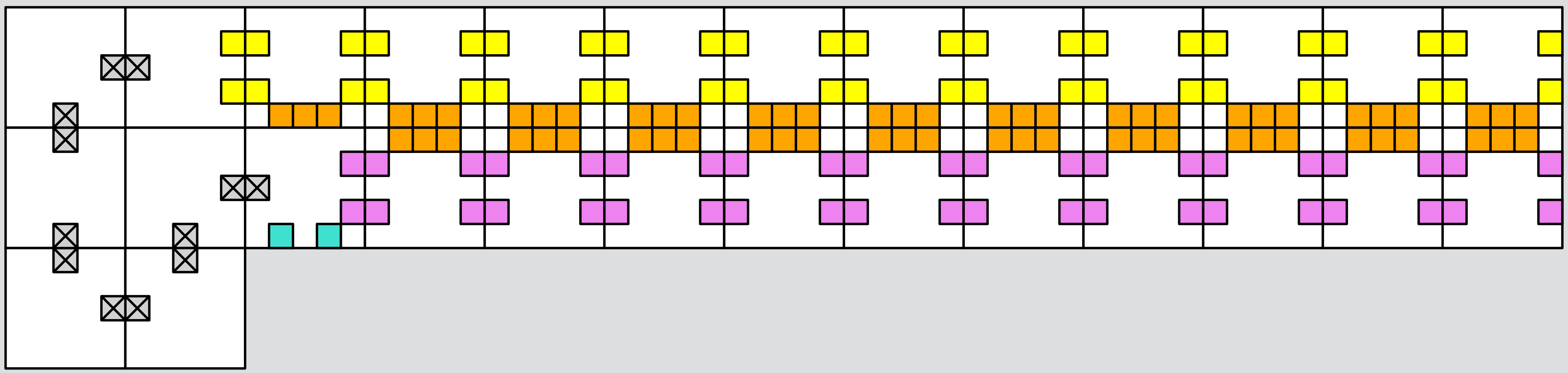

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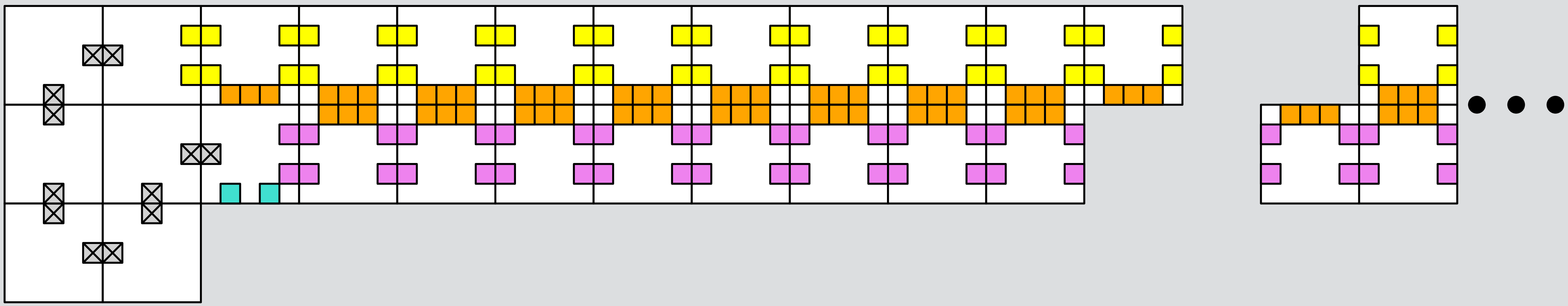

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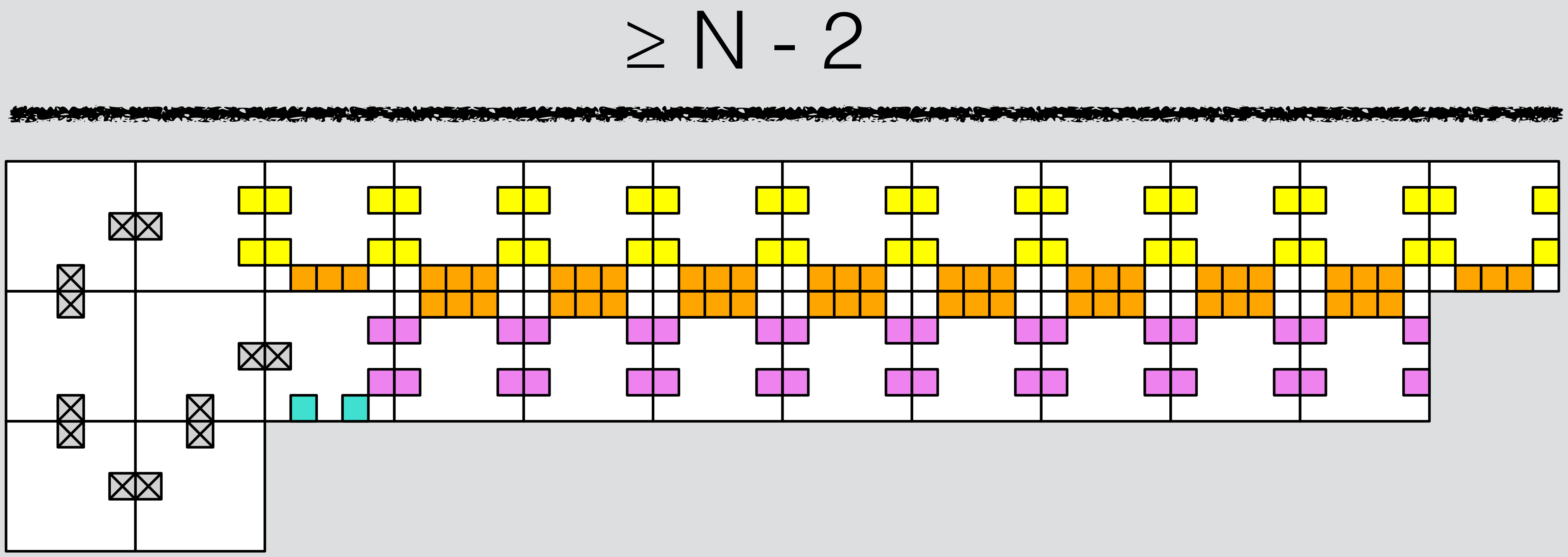

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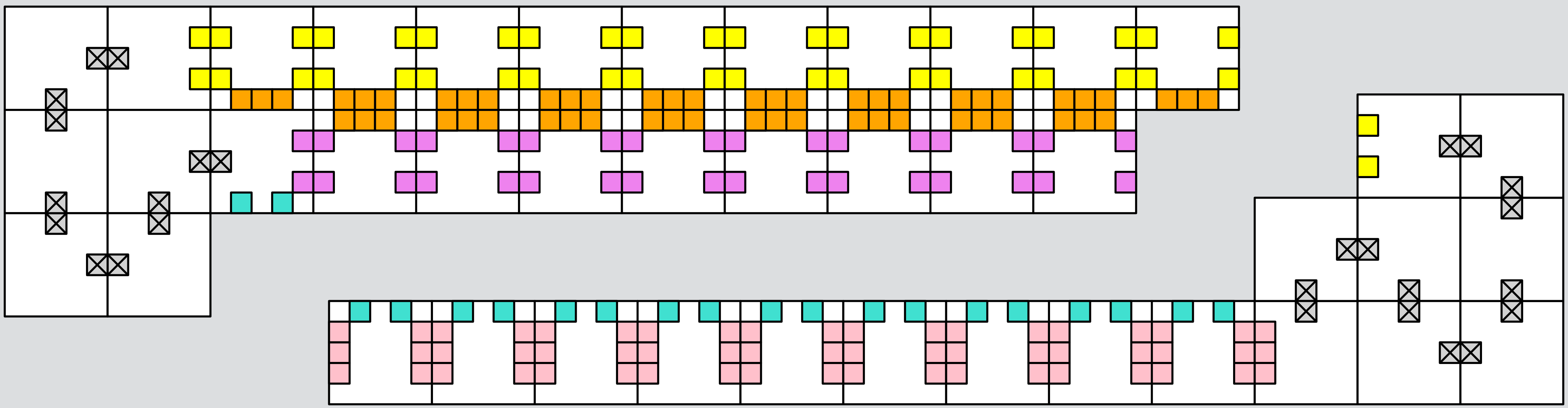

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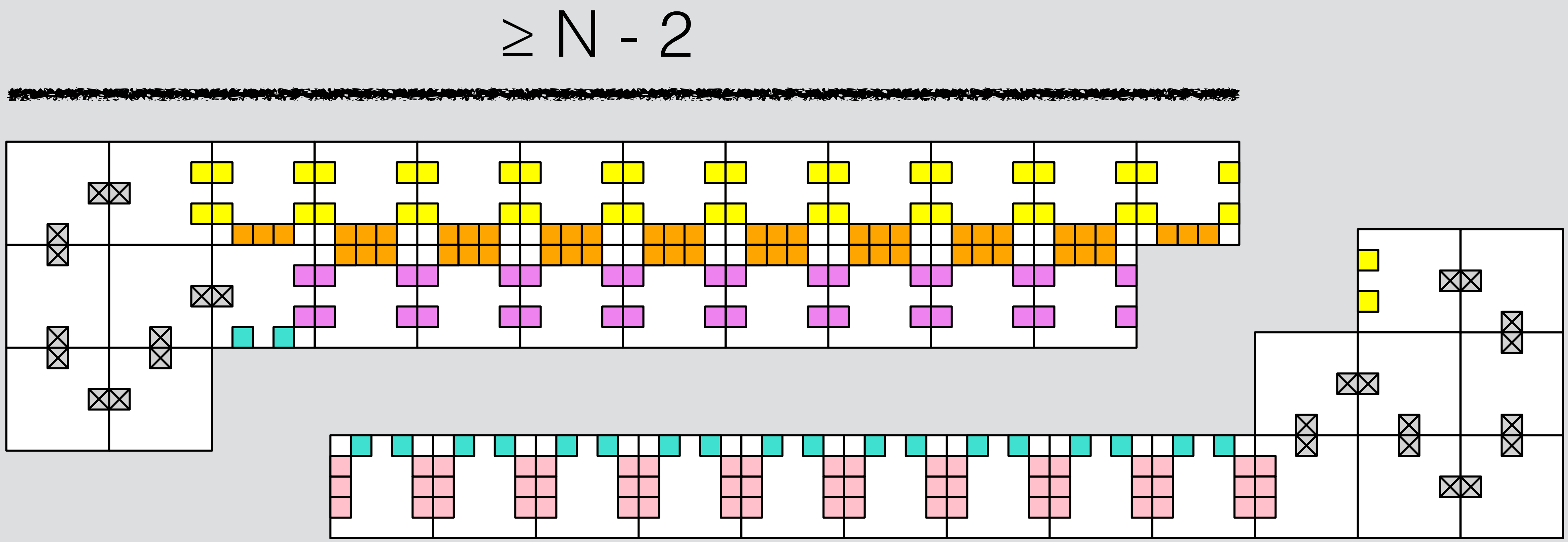

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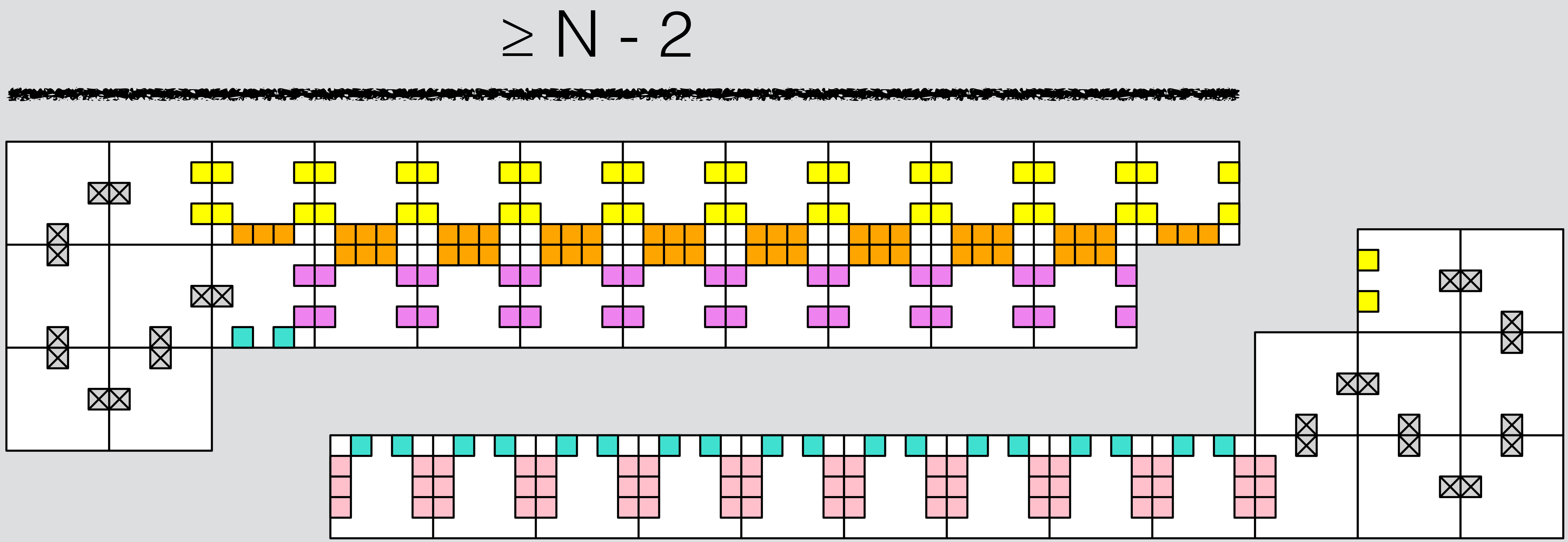

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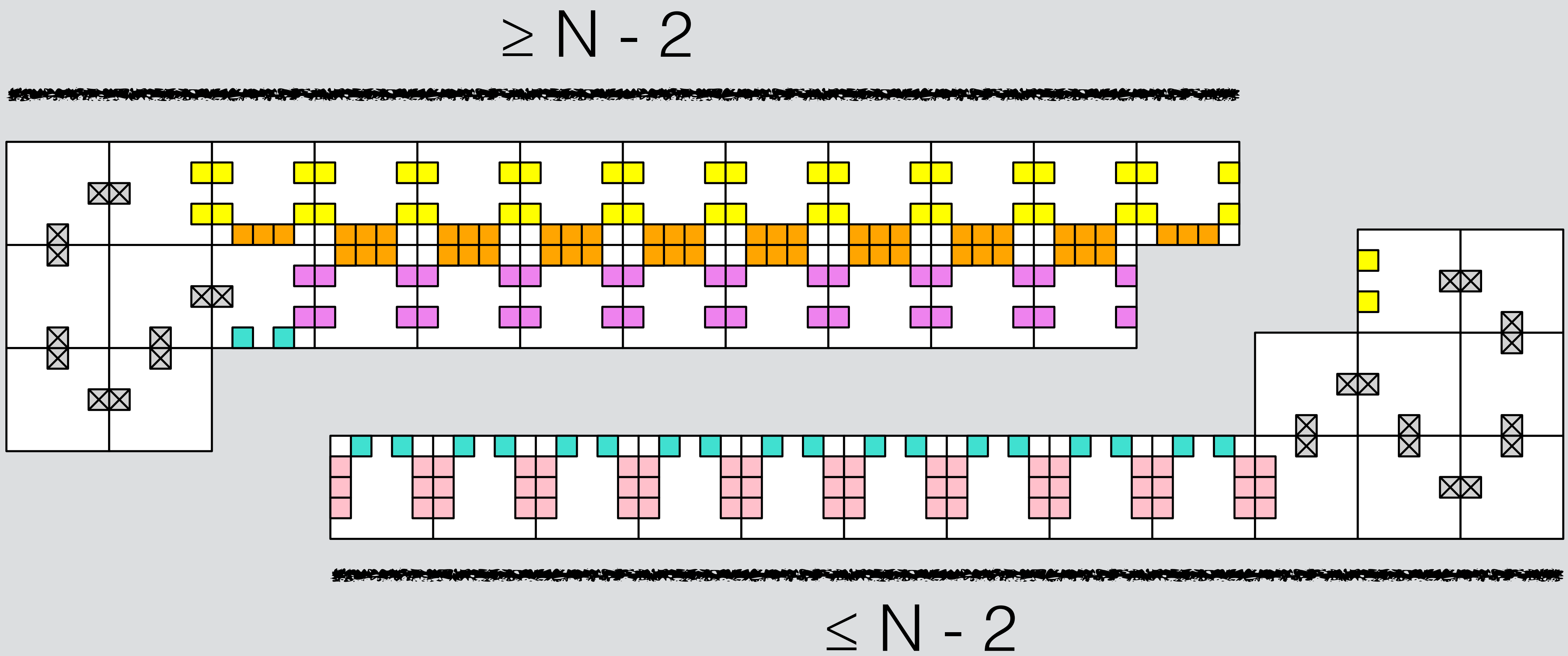

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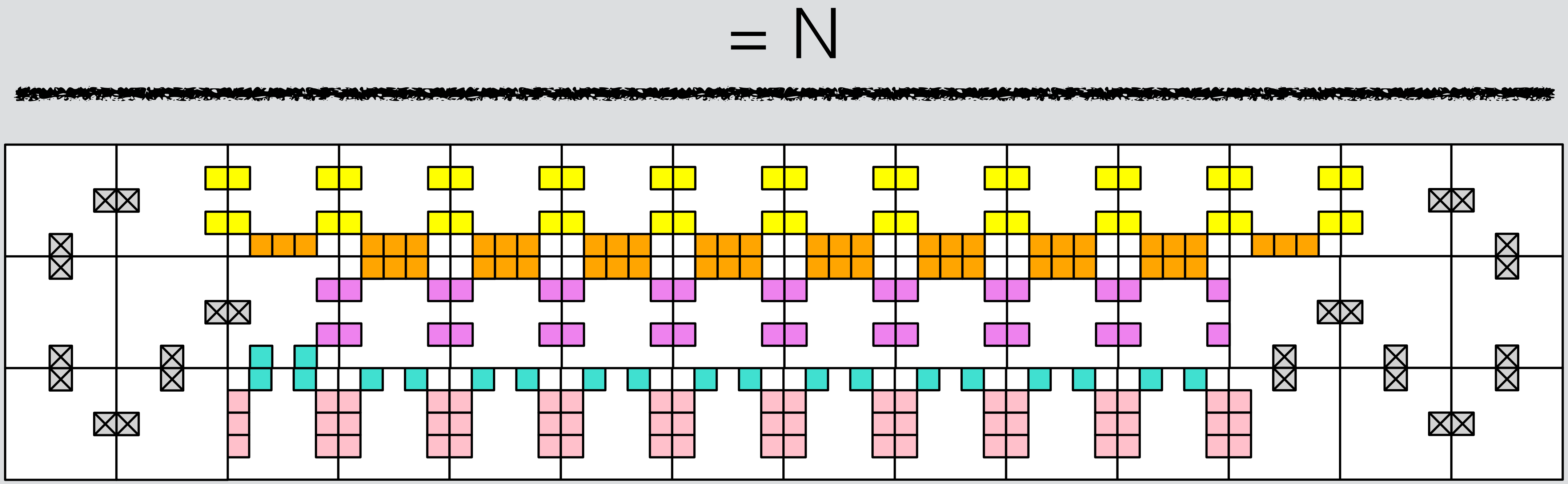

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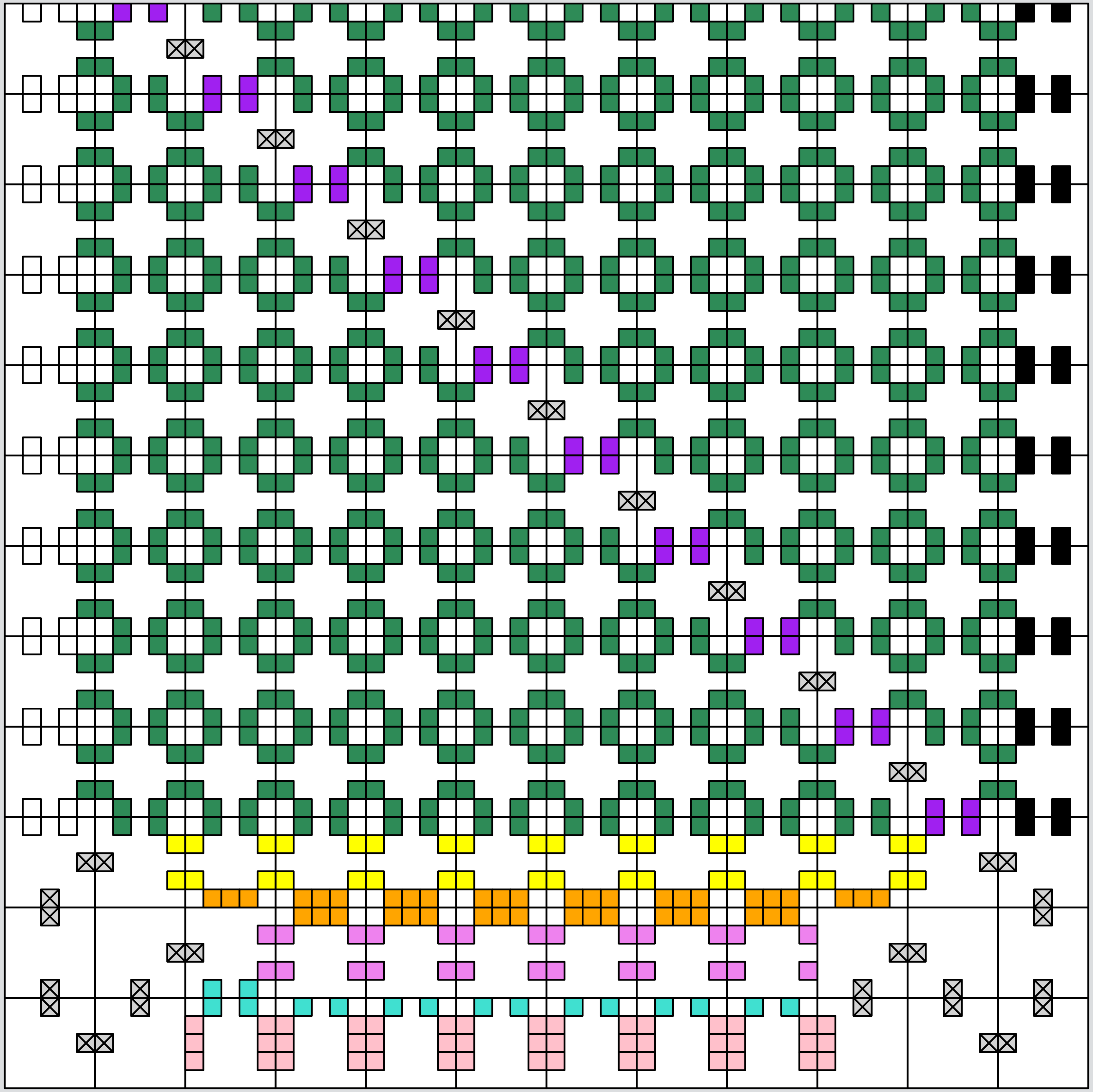
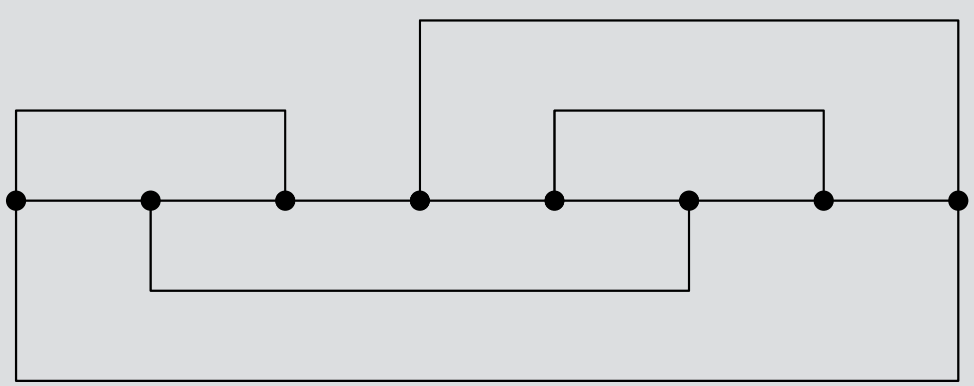
NxN Square Construction

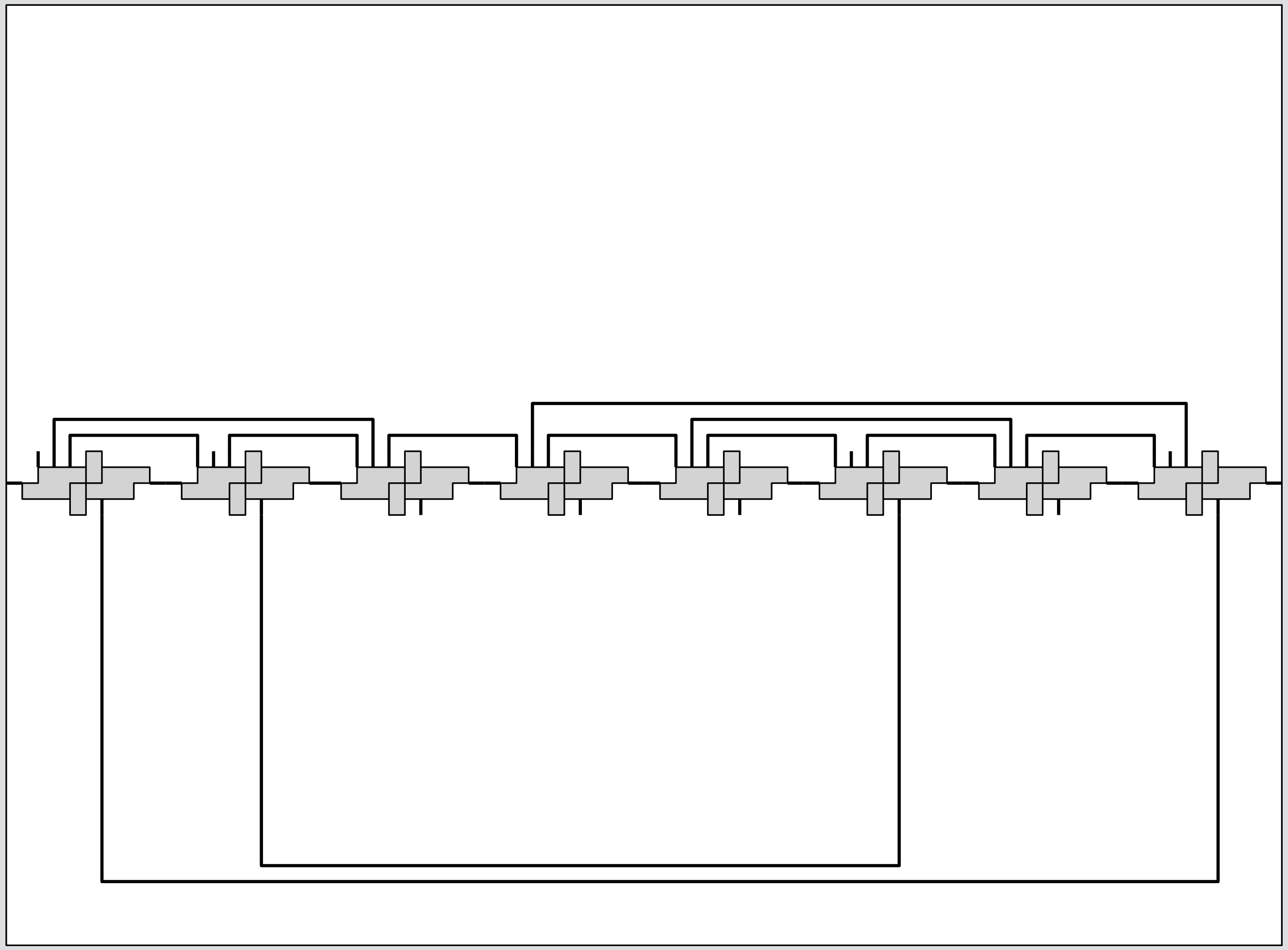


Same τ(n)

coNP-hardness Reduction

Reduce from independent set in planar cubic Hamiltonian graphs ([Fleischer et al. 2010])





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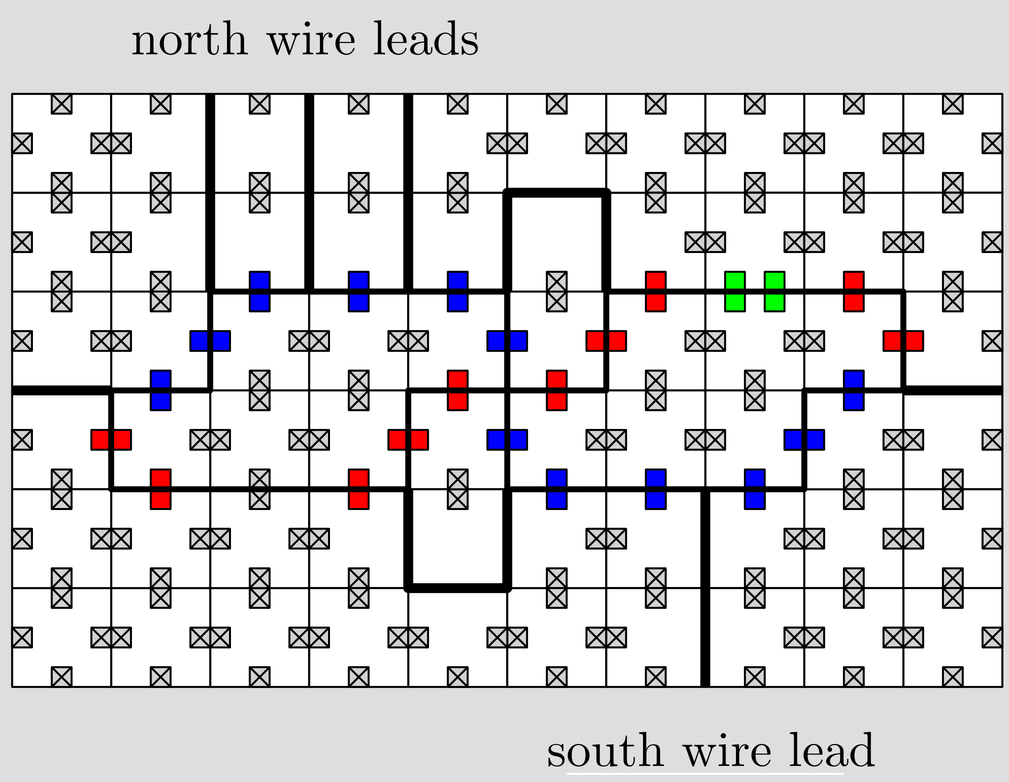

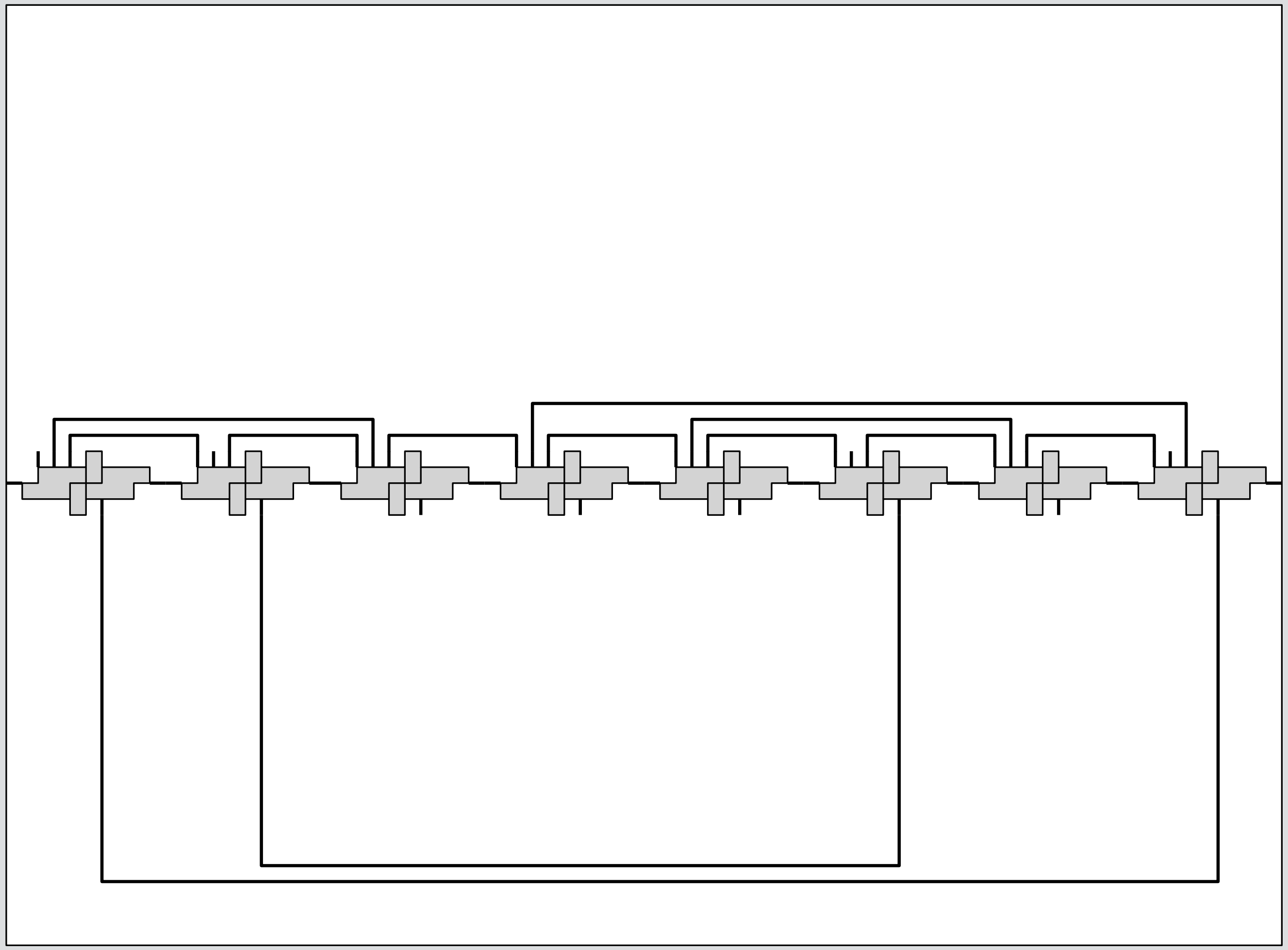

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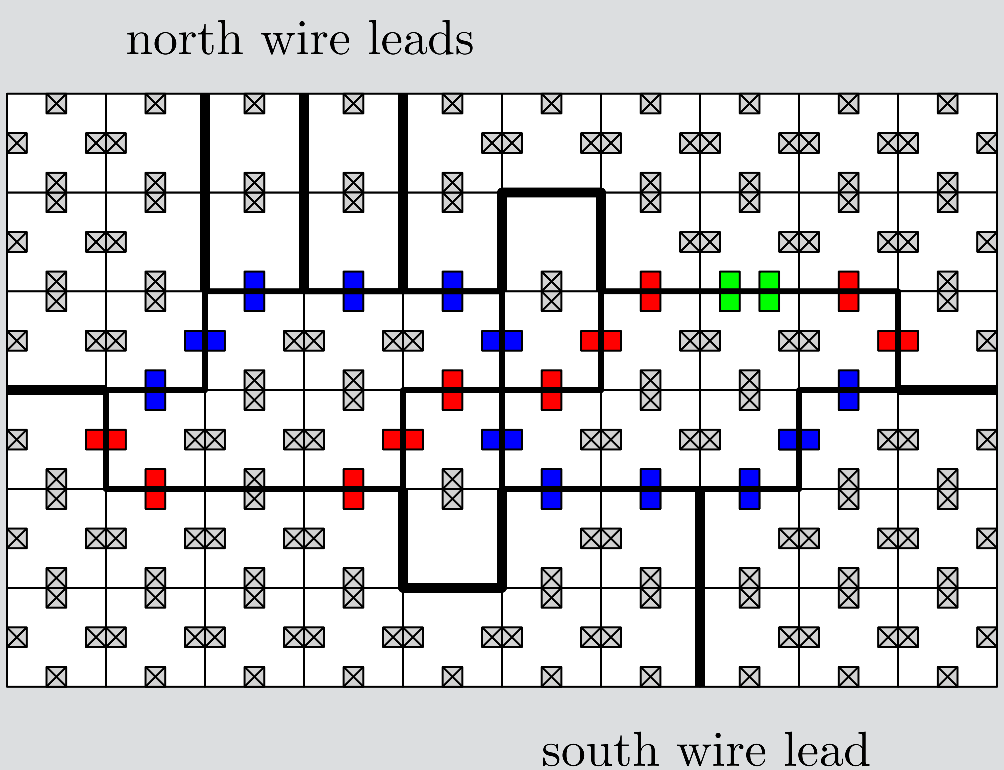
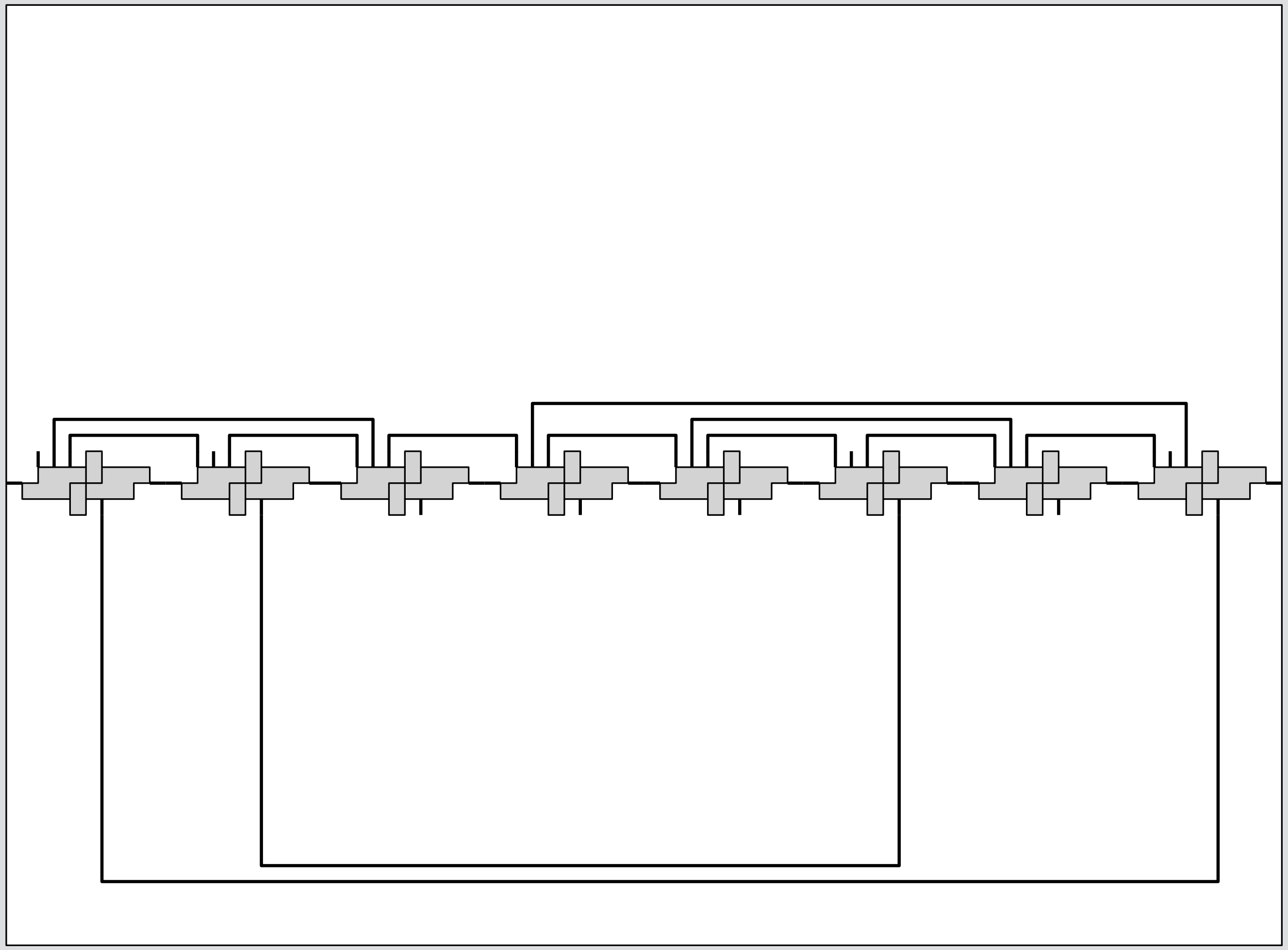




$$\tau(n) = \begin{cases} 1 : n < s/2 \\ 11|V| - k + 1 : \text{otherwise} \end{cases}$$

Conclusion

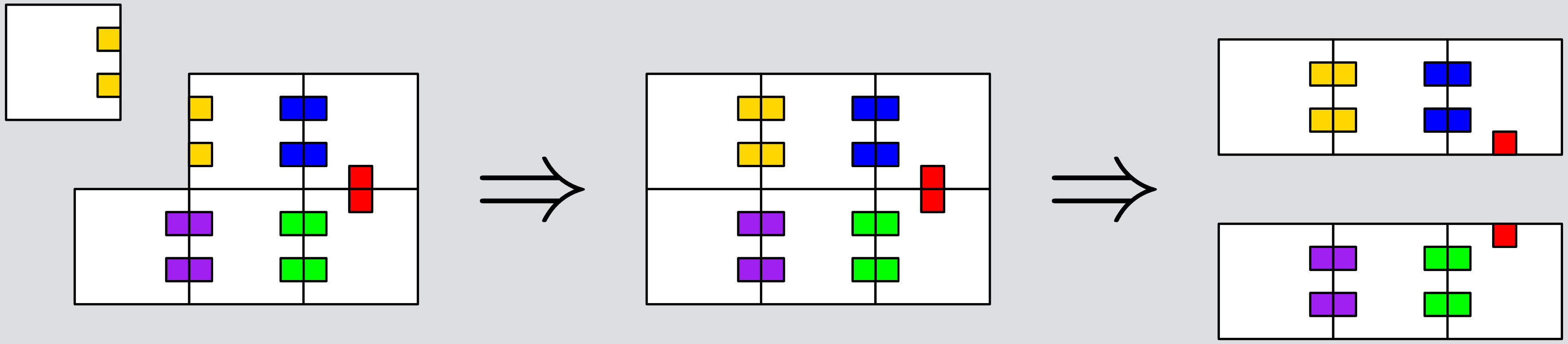
Temperature functions can yield sophisticated behavior even in simple systems.

Positive and negative: systems are provably more efficient, but (coNP-)harder to design.

Open: positive results with realistic temperature functions. What does “realistic” even mean?

Size-Dependent Tile Self-Assembly:

Constant-Height Rectangles and Stability



Sándor Fekete, Robert Schweller, Andrew Winslow

Technische Universität Braunschweig

UTRGV The University of Texas Rio Grande Valley

ULB Université Libre de Bruxelles