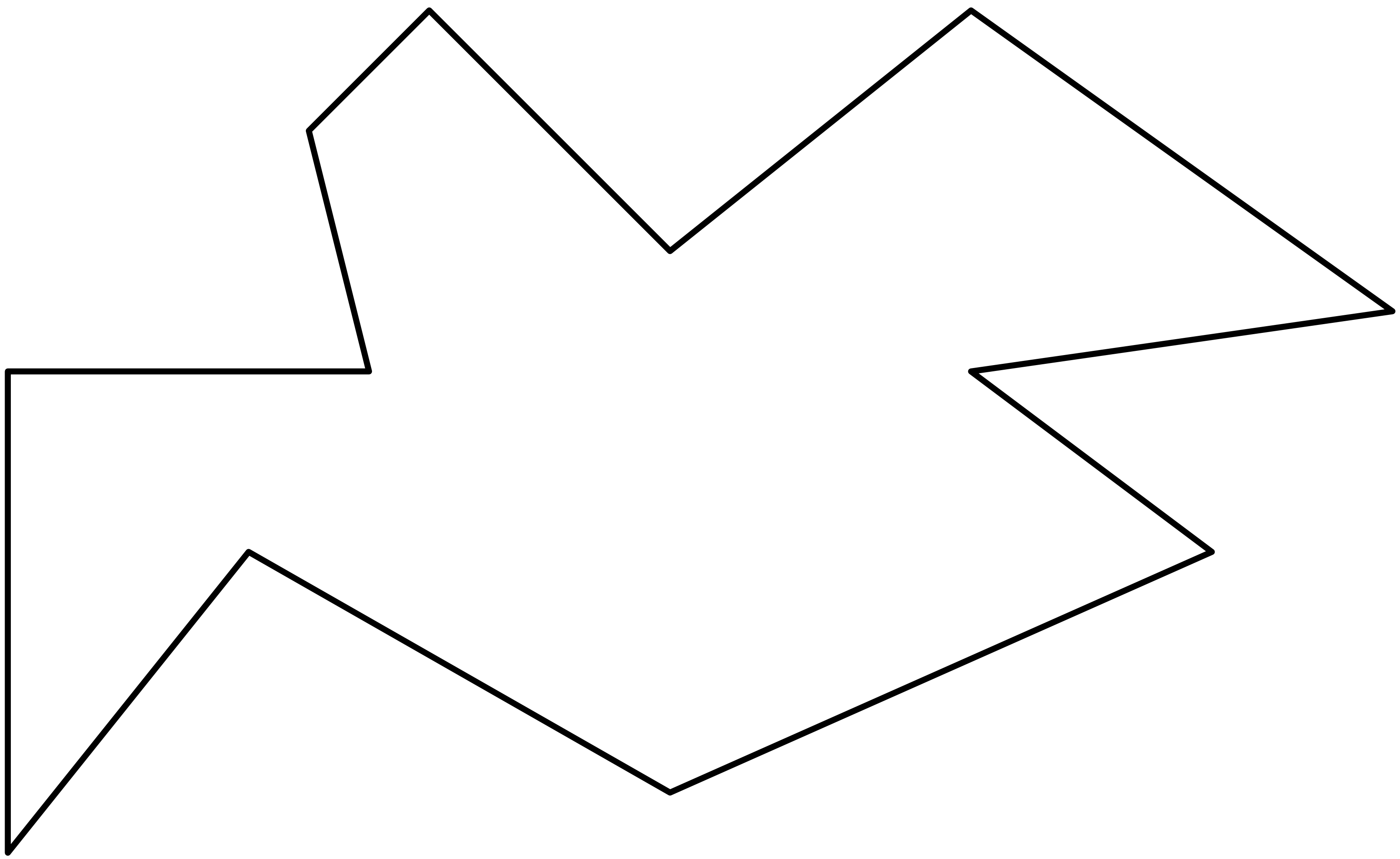
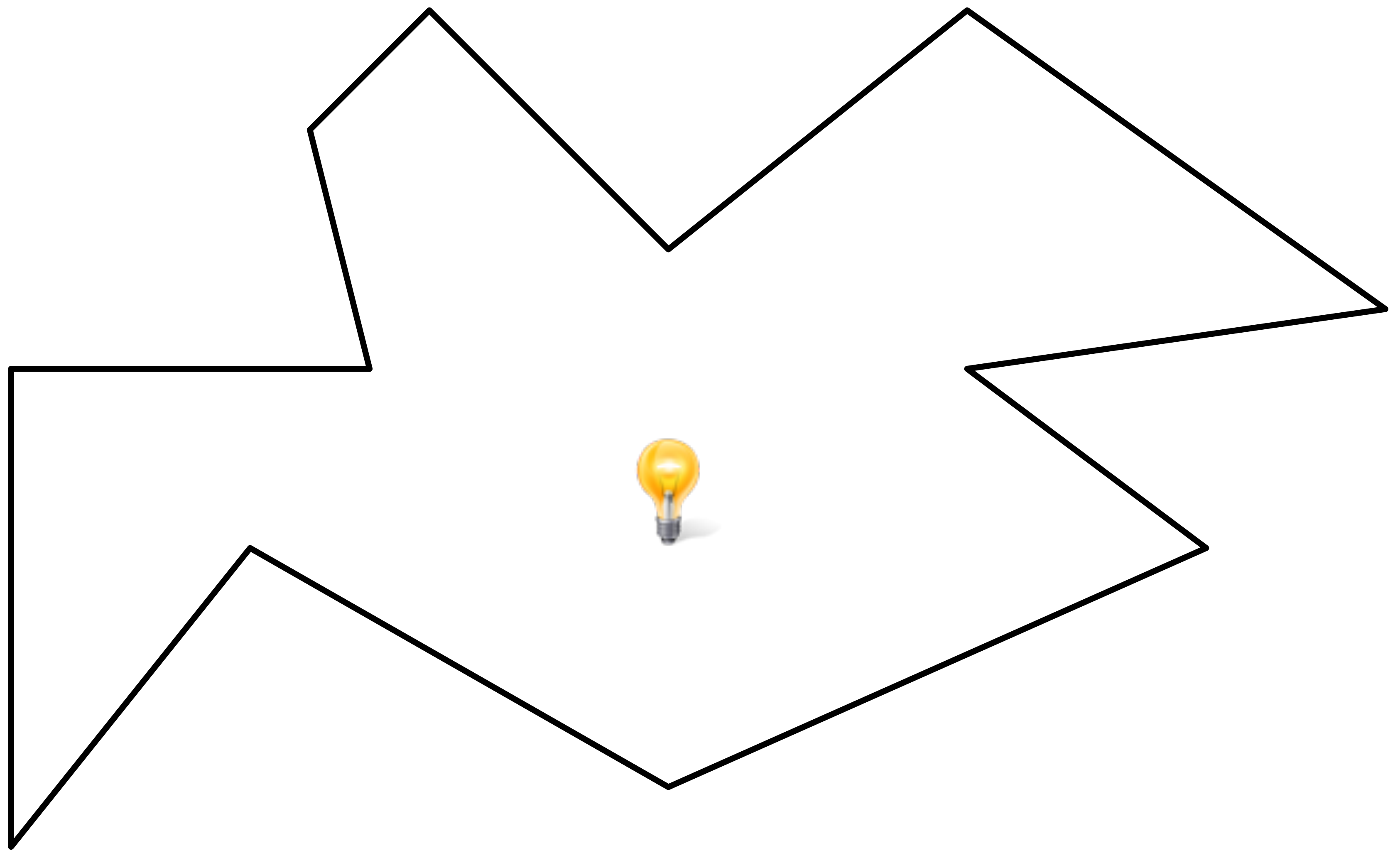
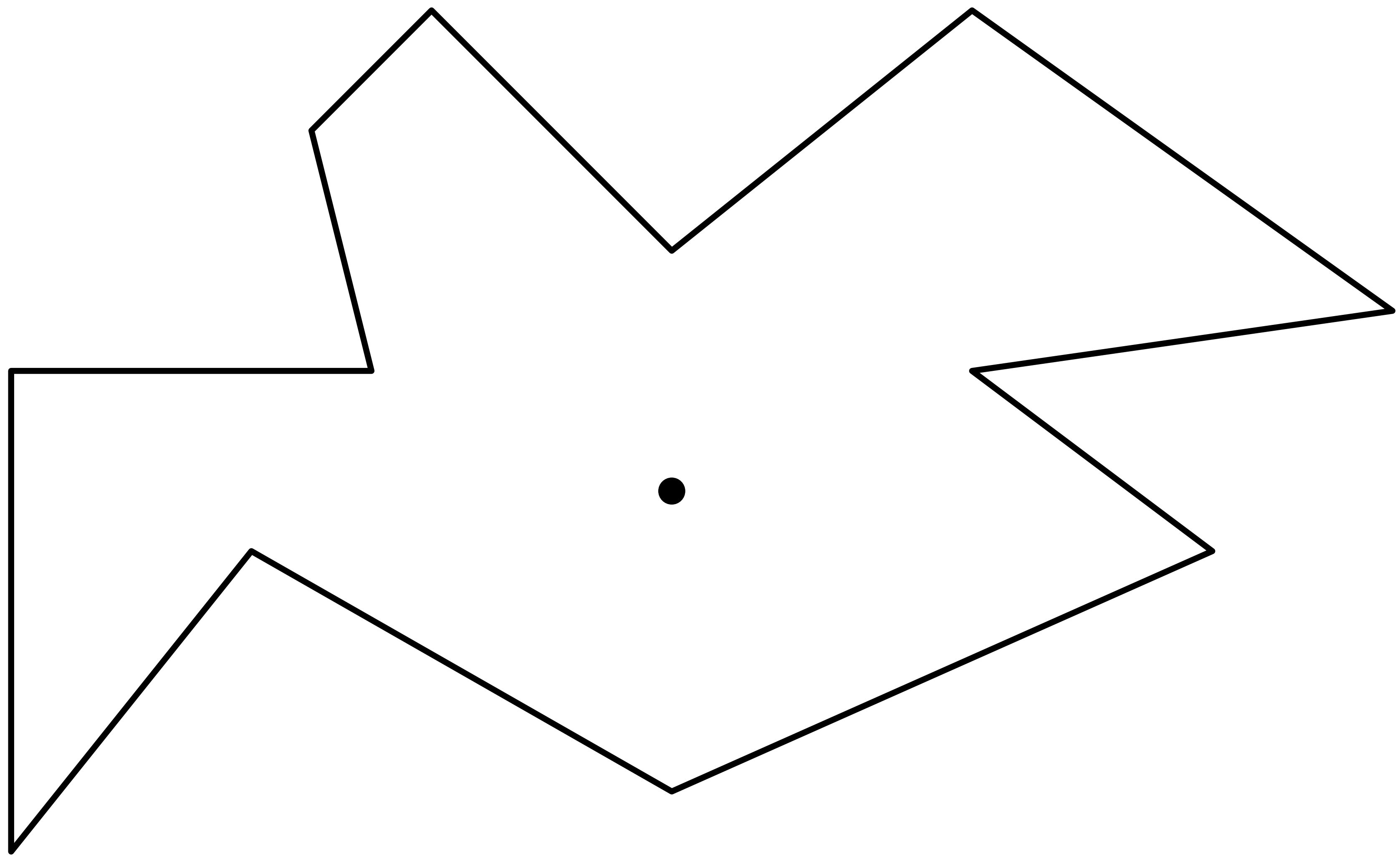
Diffuse Reflection Radius in a Simple Polygon

Eli Fox-Epstein, Csaba D. Tóth, Andrew Winslow

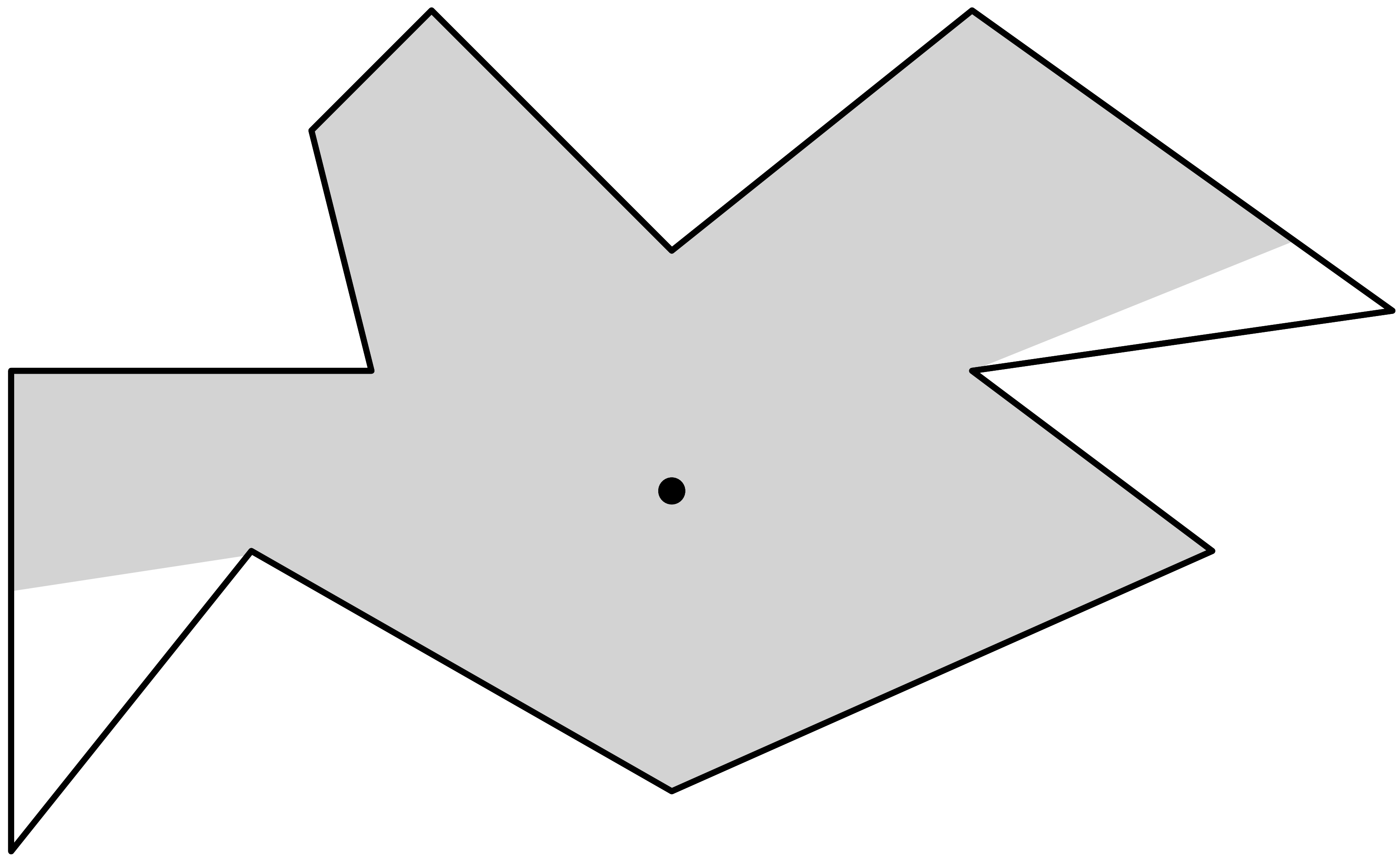
BROWN



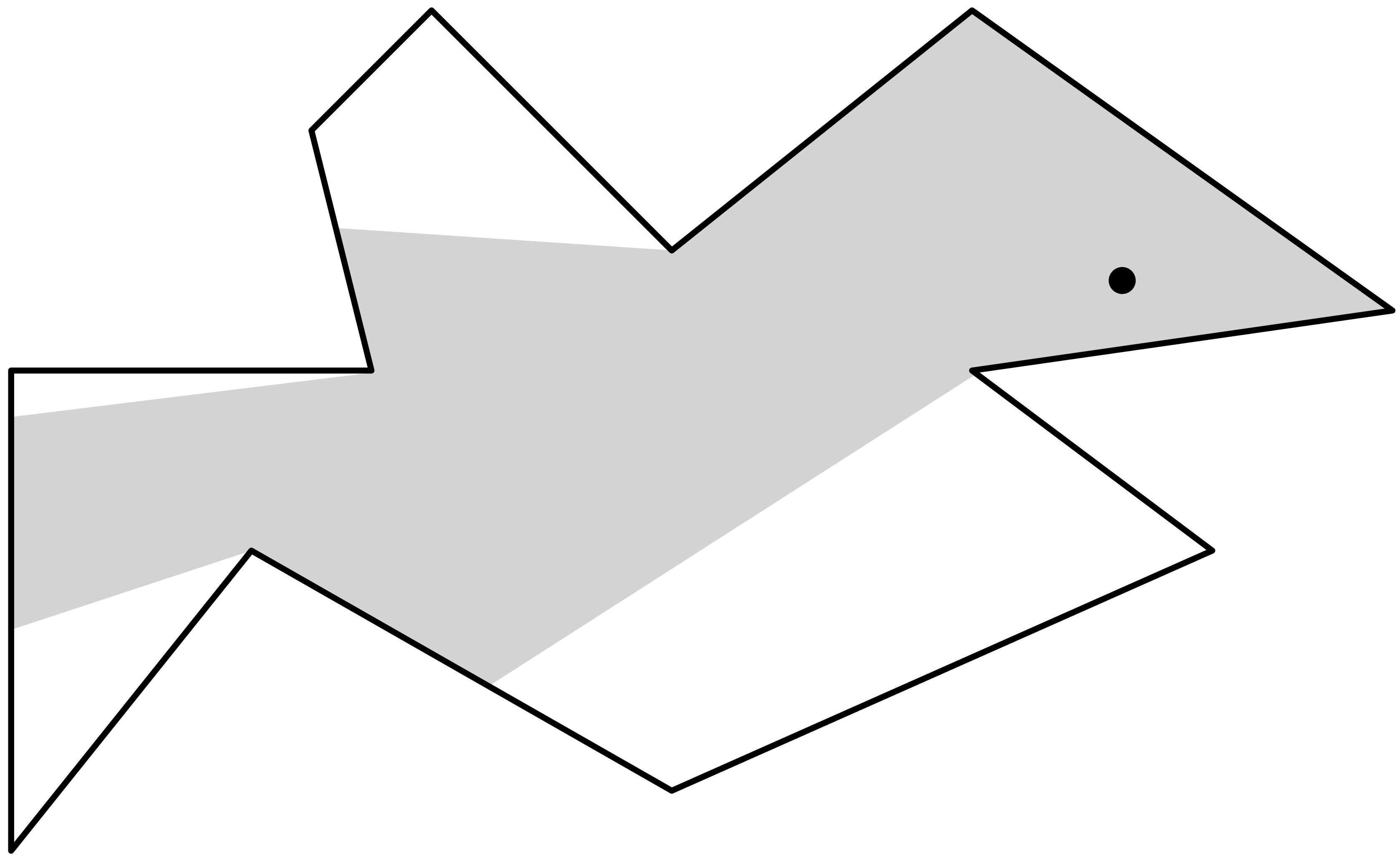




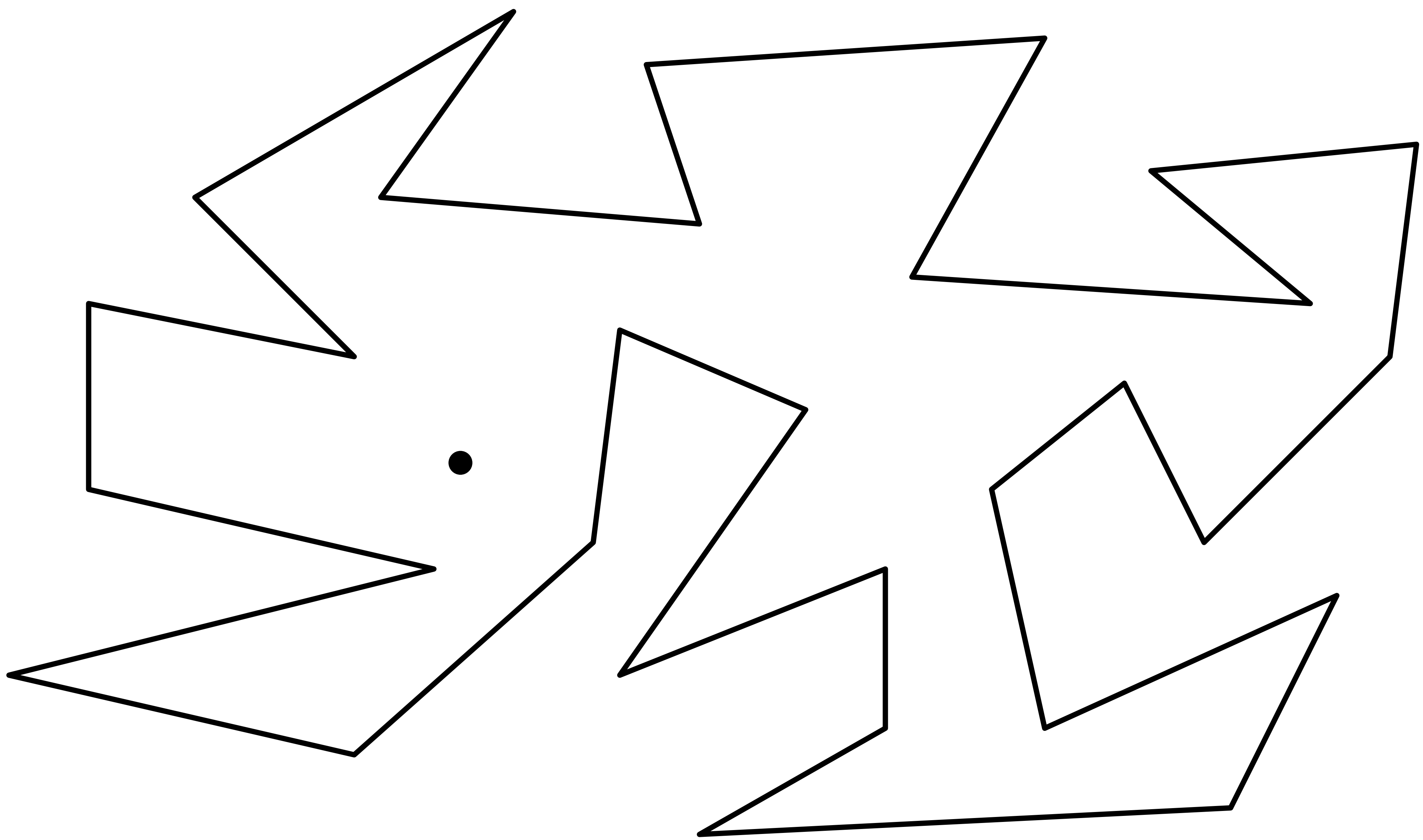
Direct Illumination



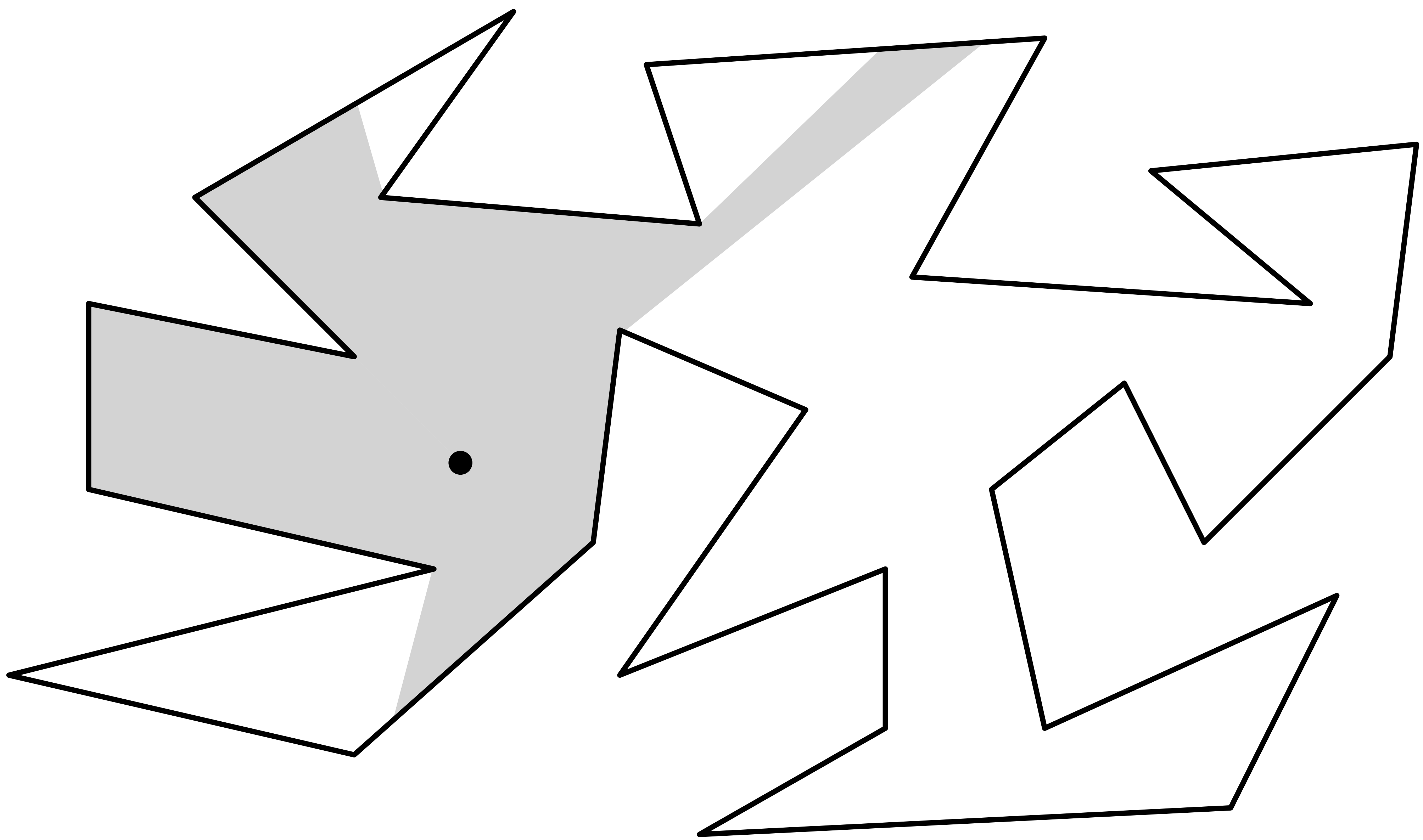
Direct Illumination



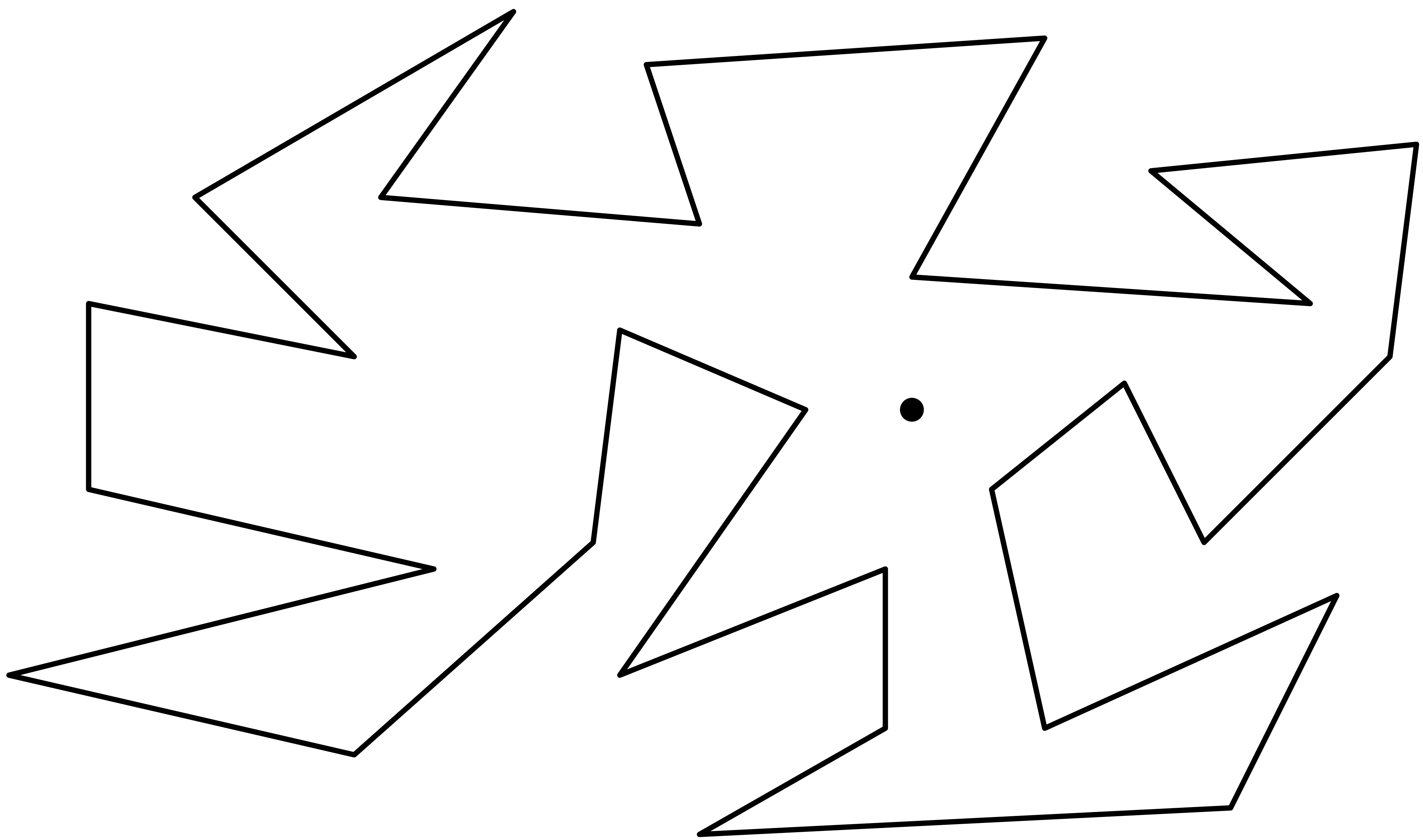
Direct Illumination



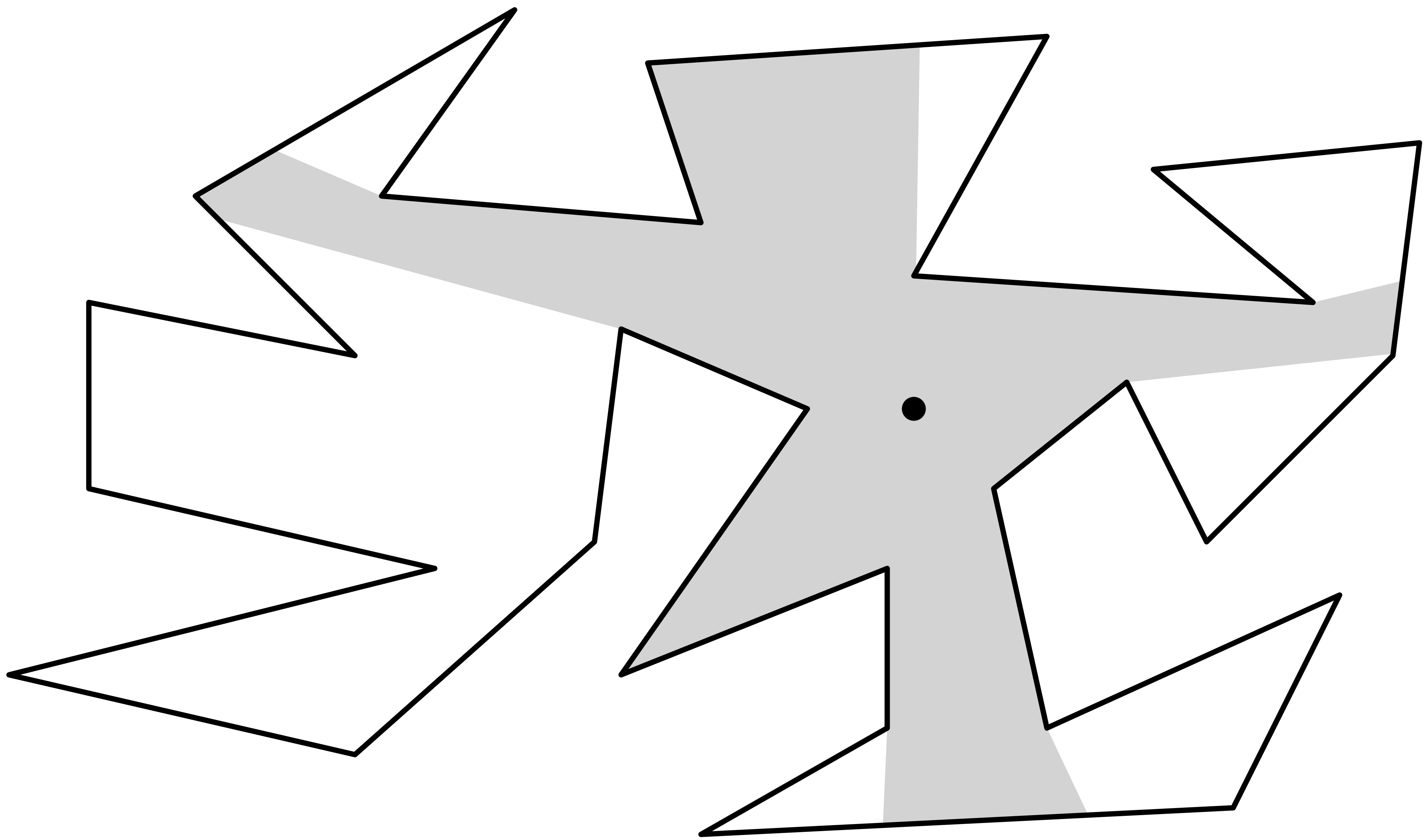
Direct Illumination



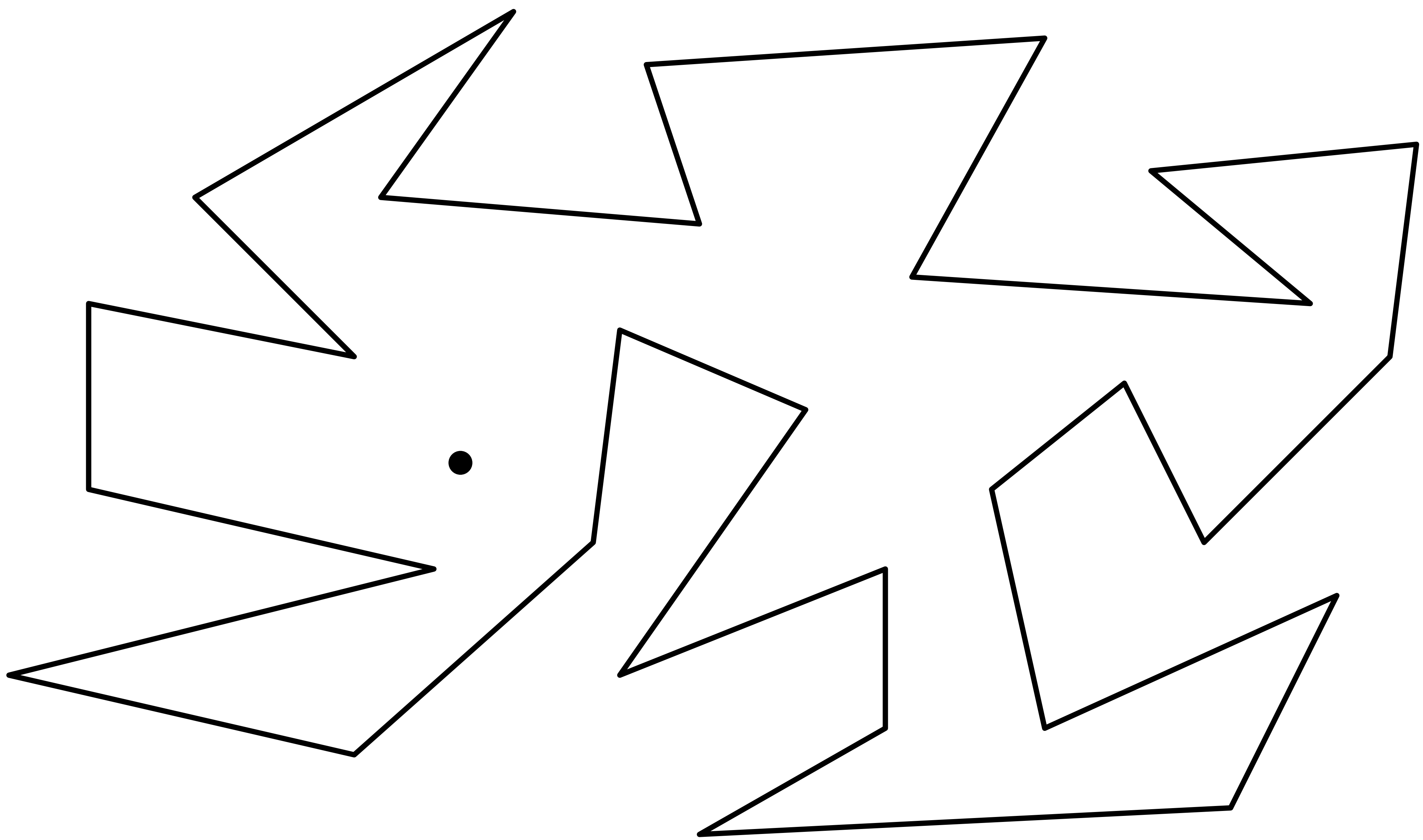
Direct Illumination



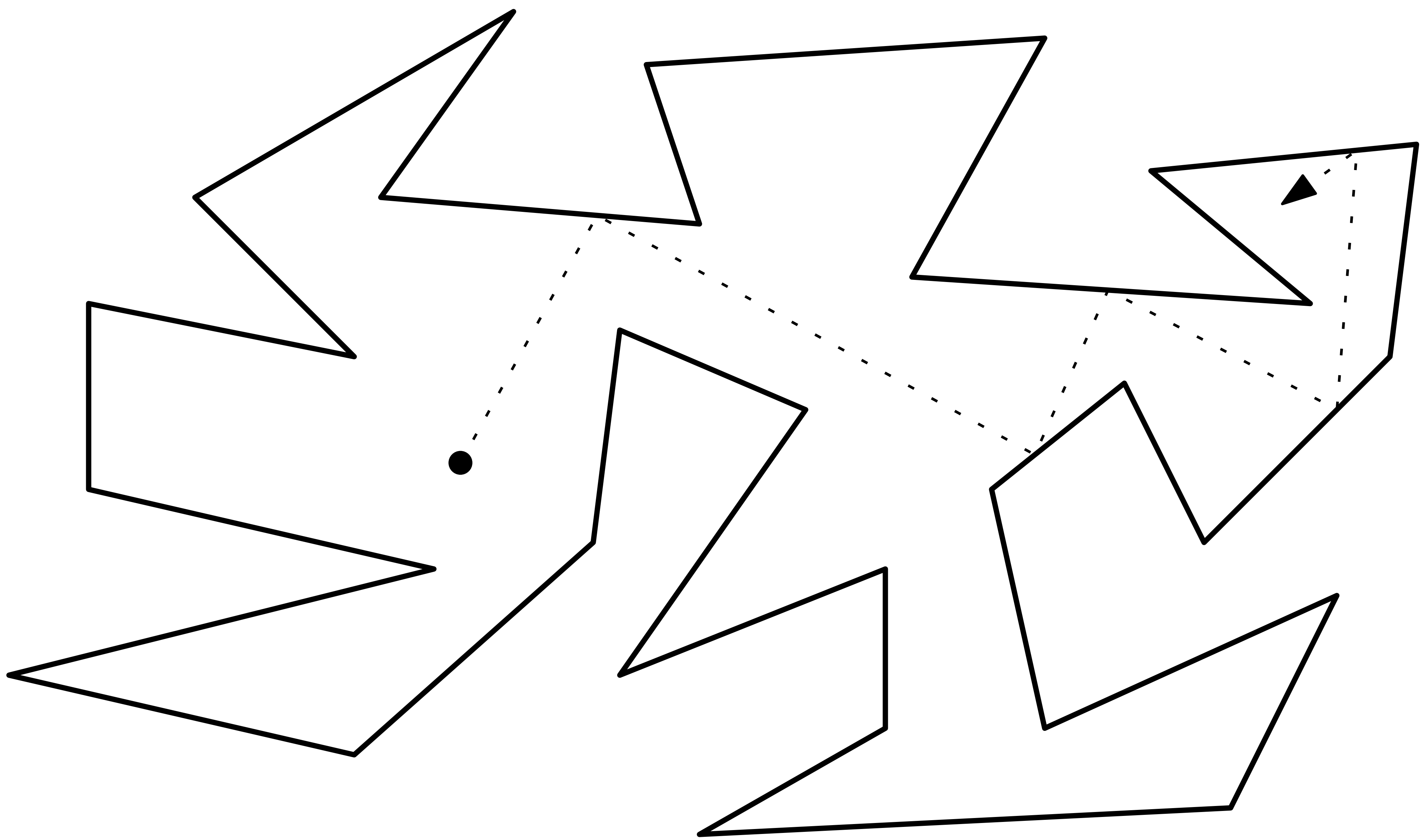
Direct Illumination



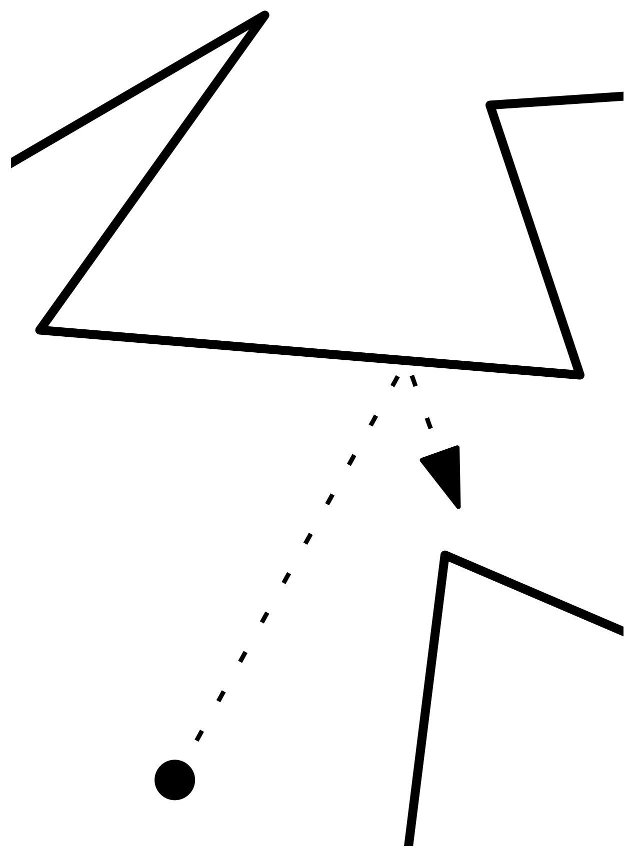
Illumination via Reflections



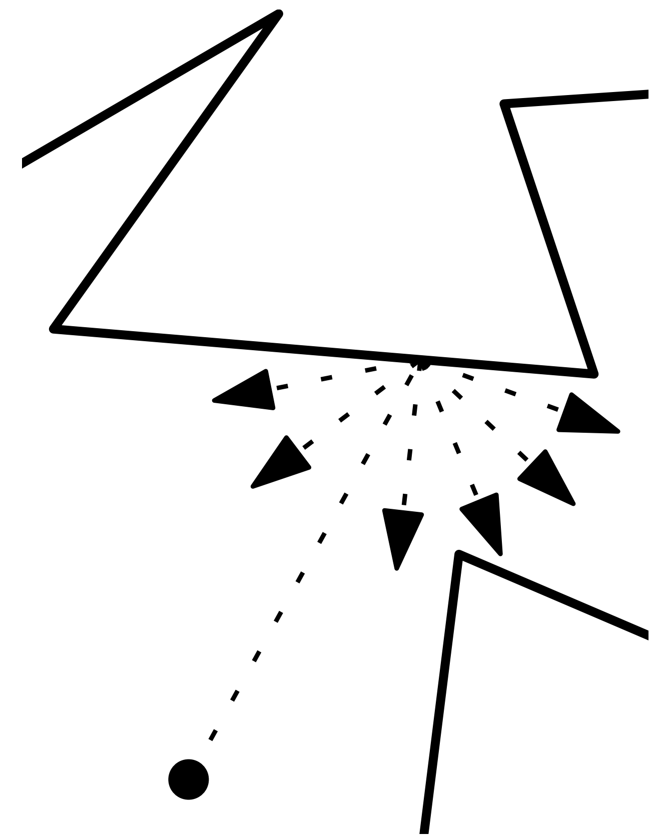
Illumination via Reflections



Illumination Types

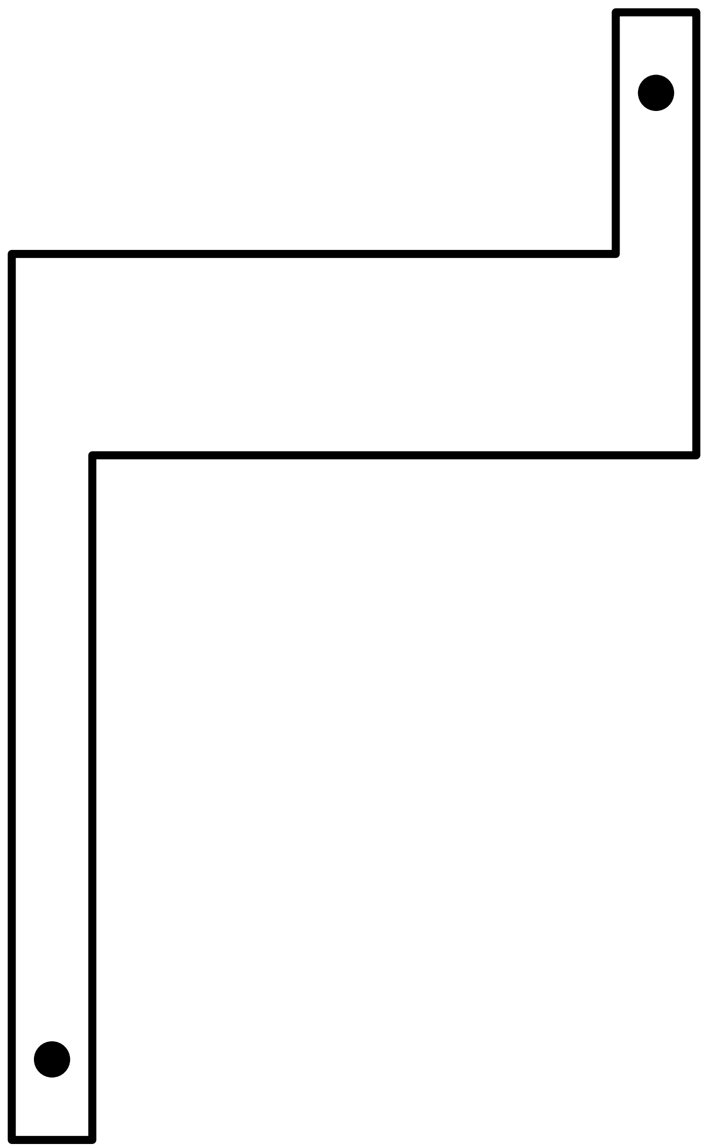


Specular

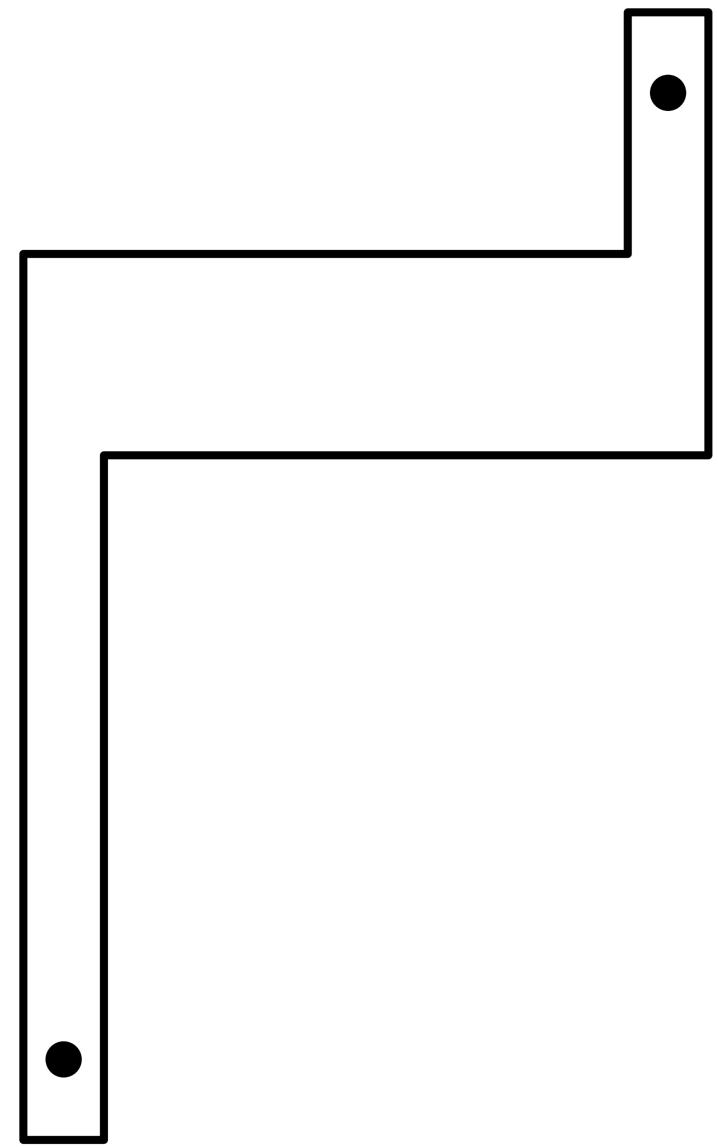


Diffuse

Illumination Types

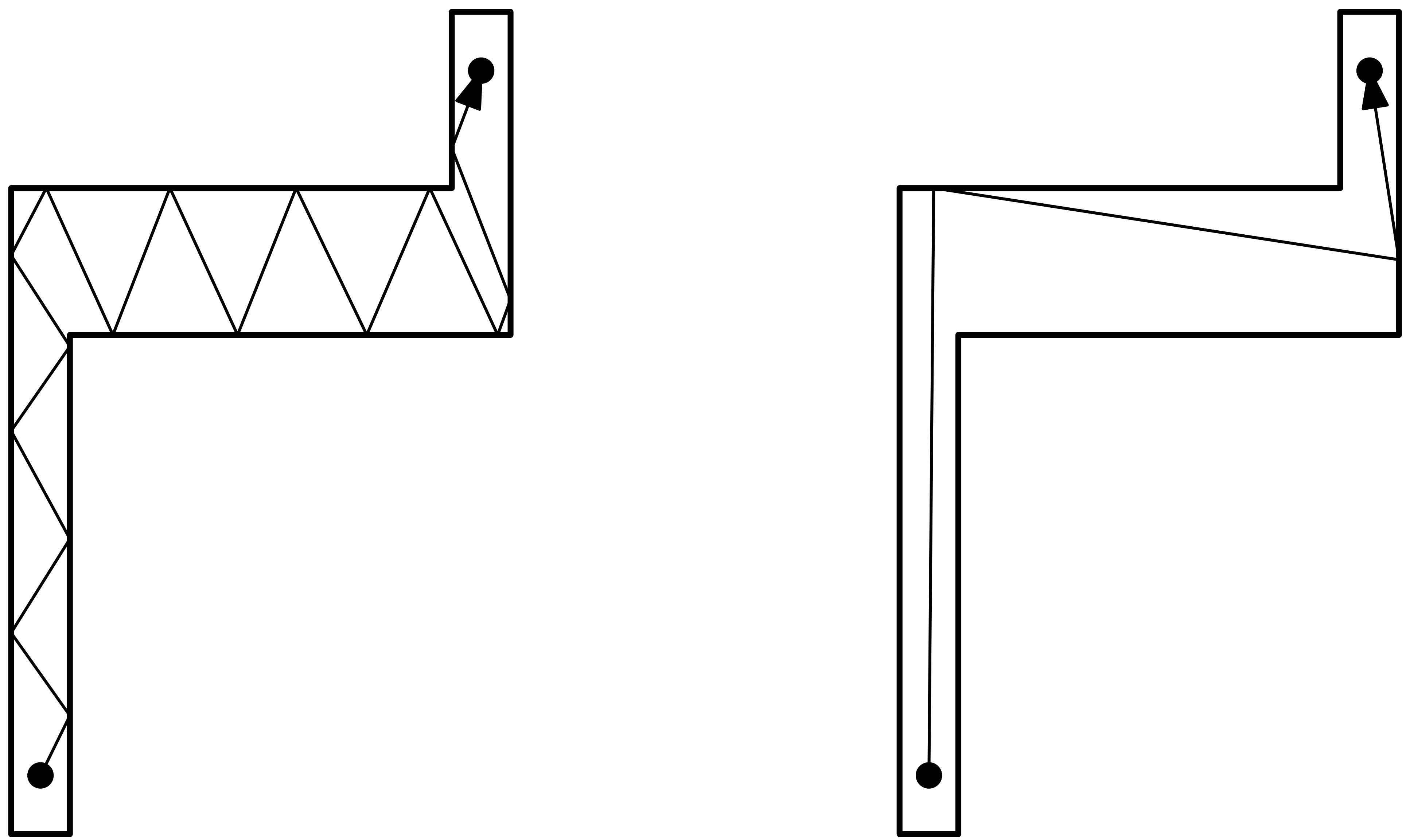


Specular



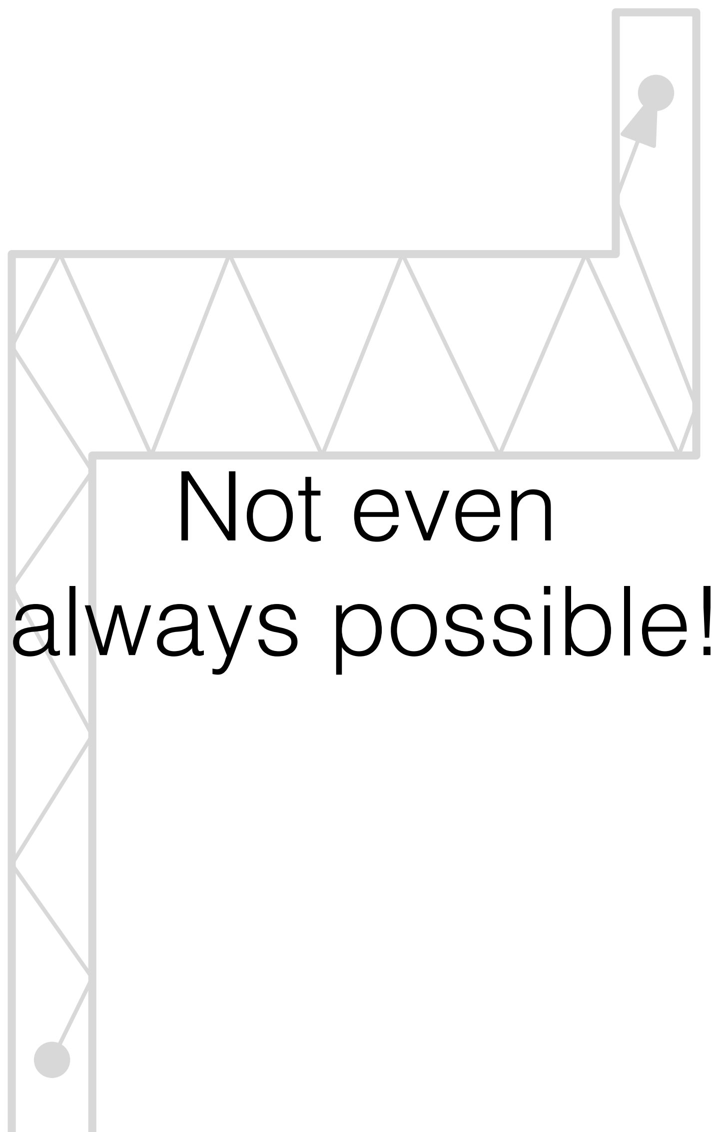
Diffuse

Illumination Types

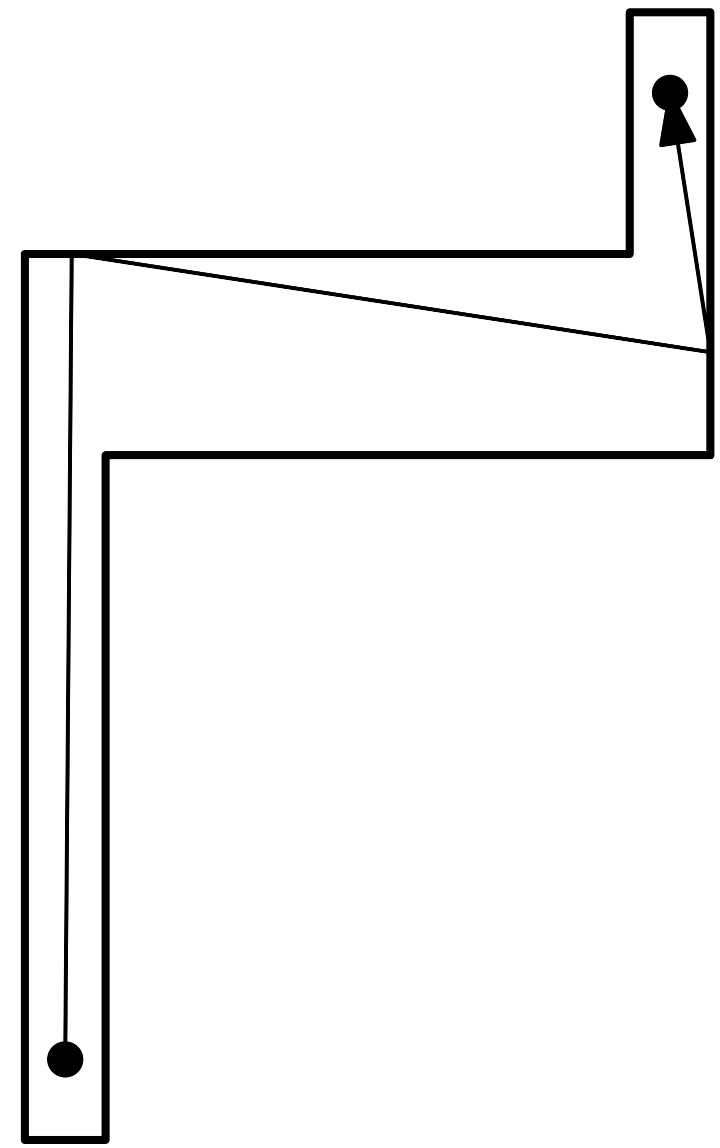


Specular Diffuse

Illumination Types

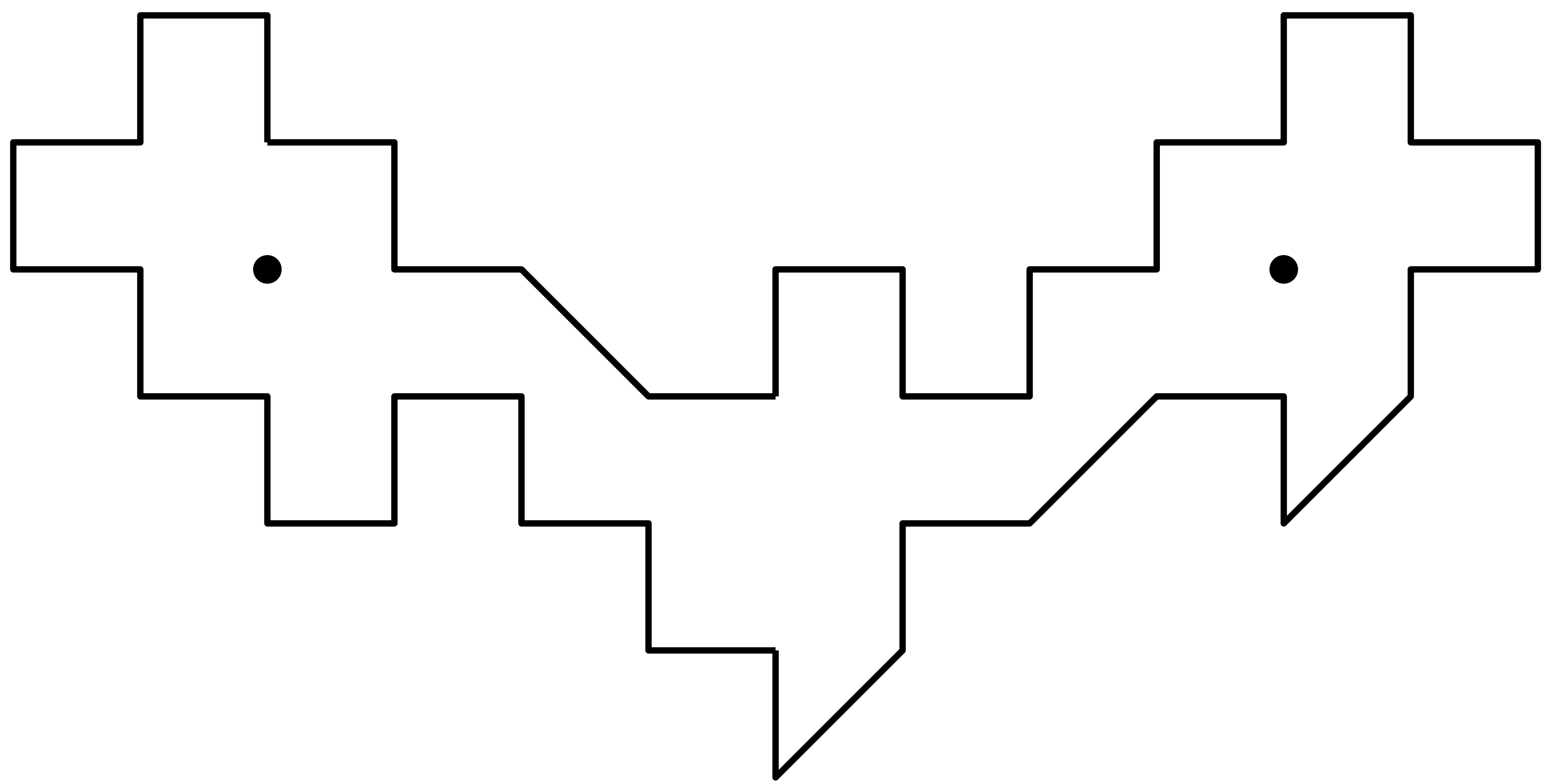


Specular



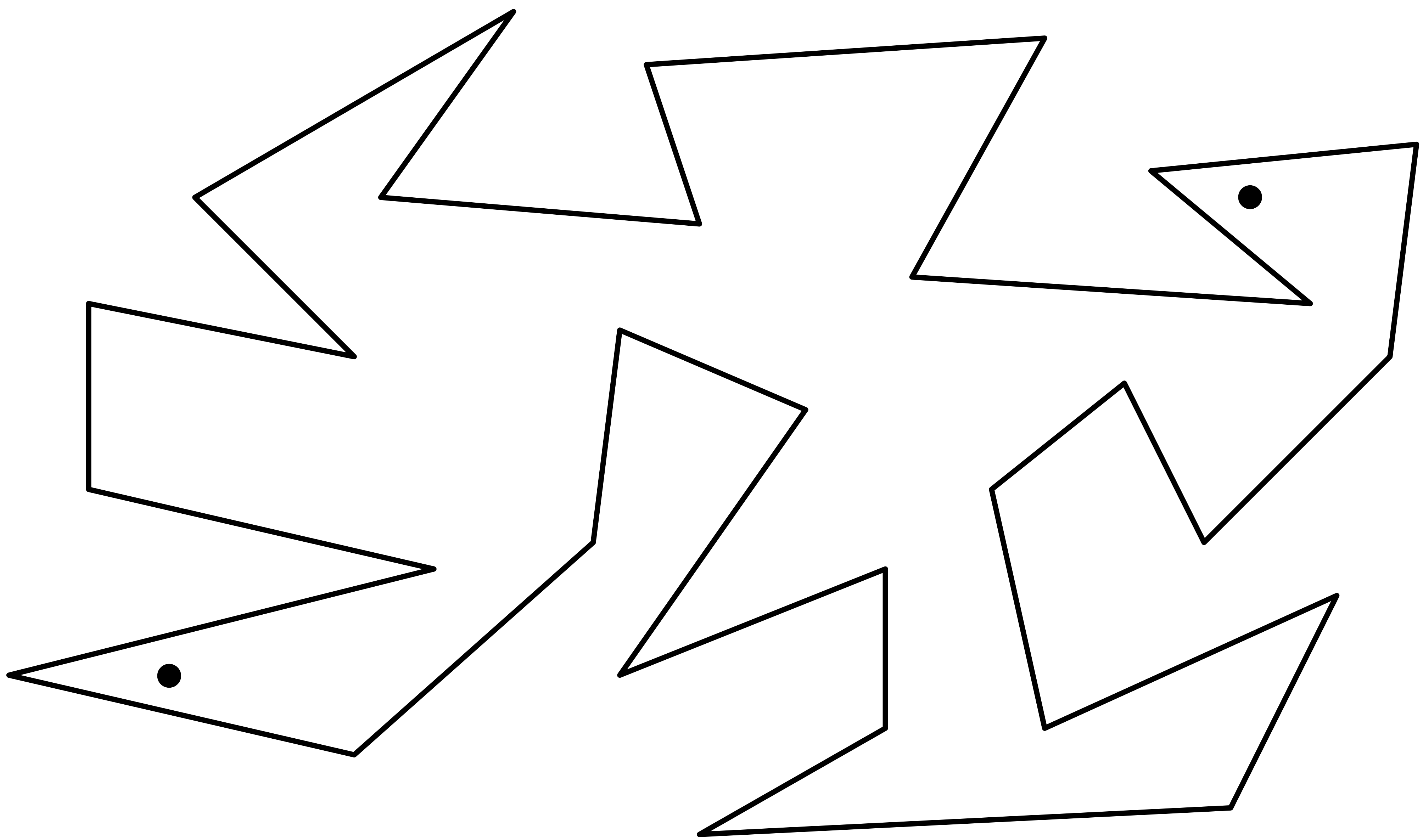
Diffuse

No specular illumination of one point by other!

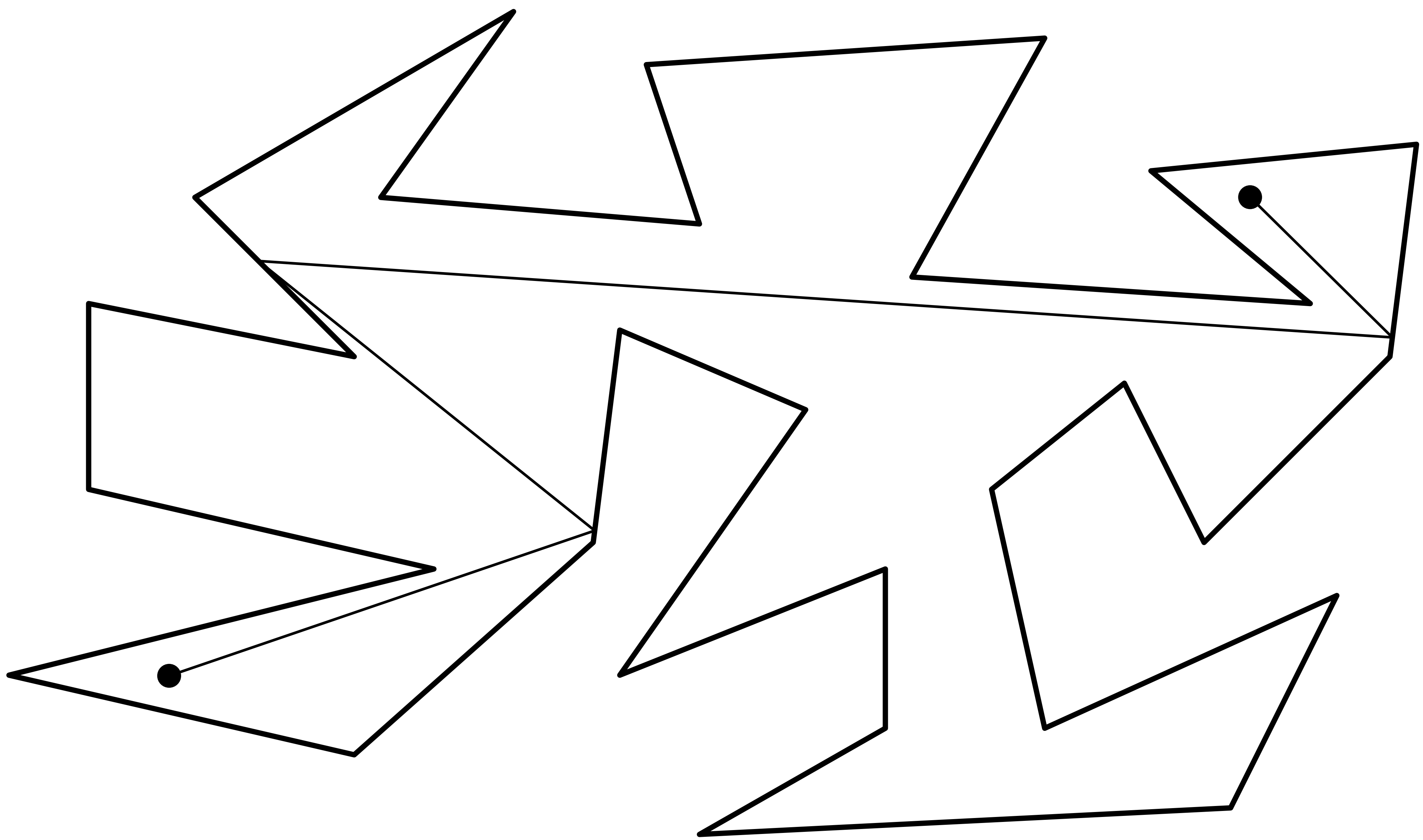


[Tokarsky 1995]

Diffuse Reflection Distance

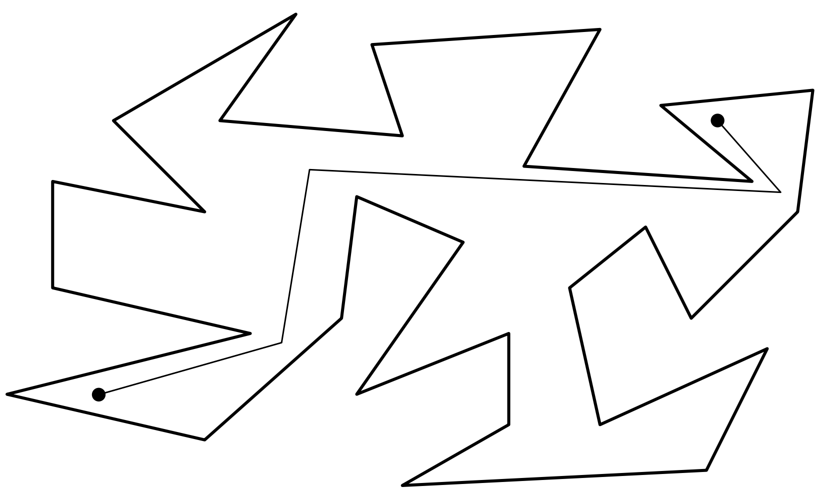


Diffuse Reflection Distance

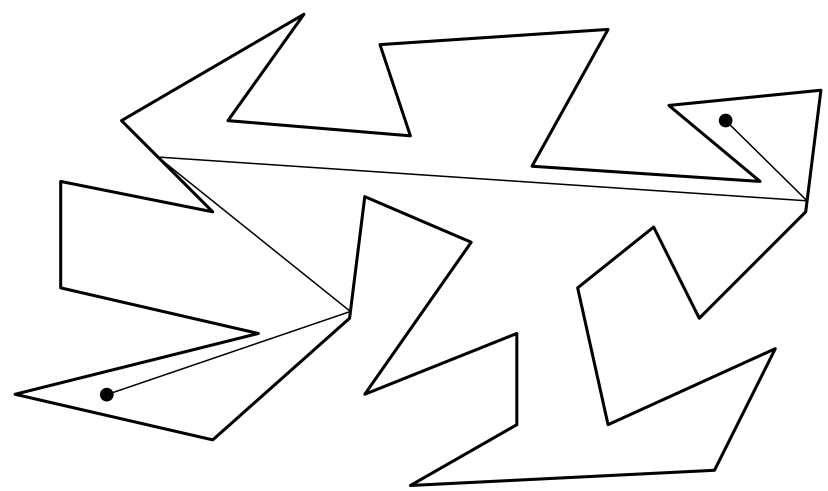


Diffuse reflection distance: # reflections

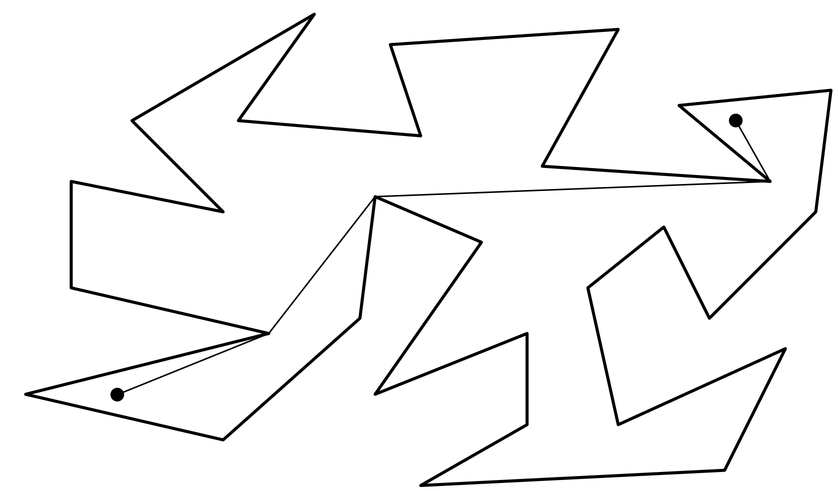
Distance Measures



Link distance

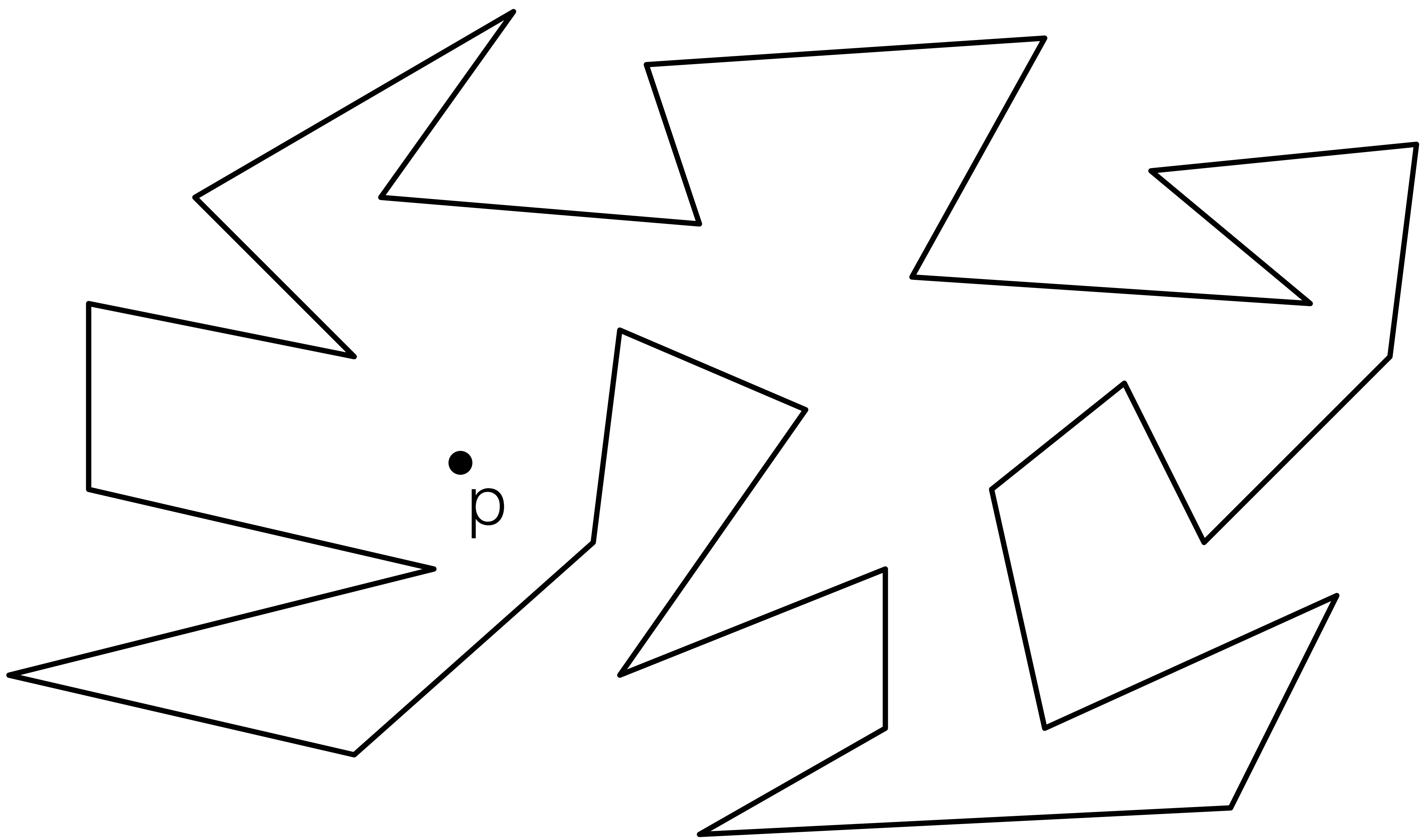


Diffuse reflection distance

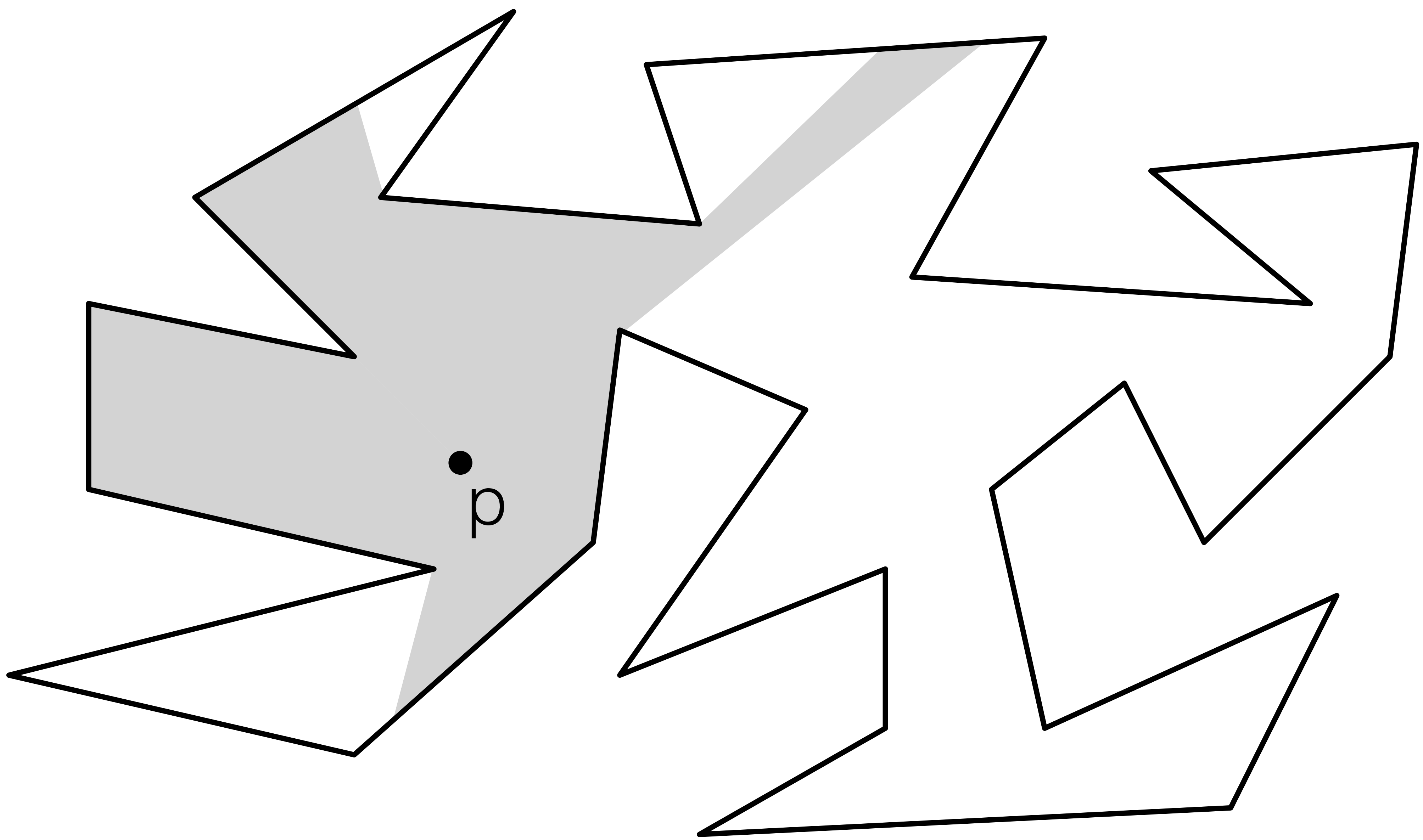


Geodesic distance

Diffuse Reflections

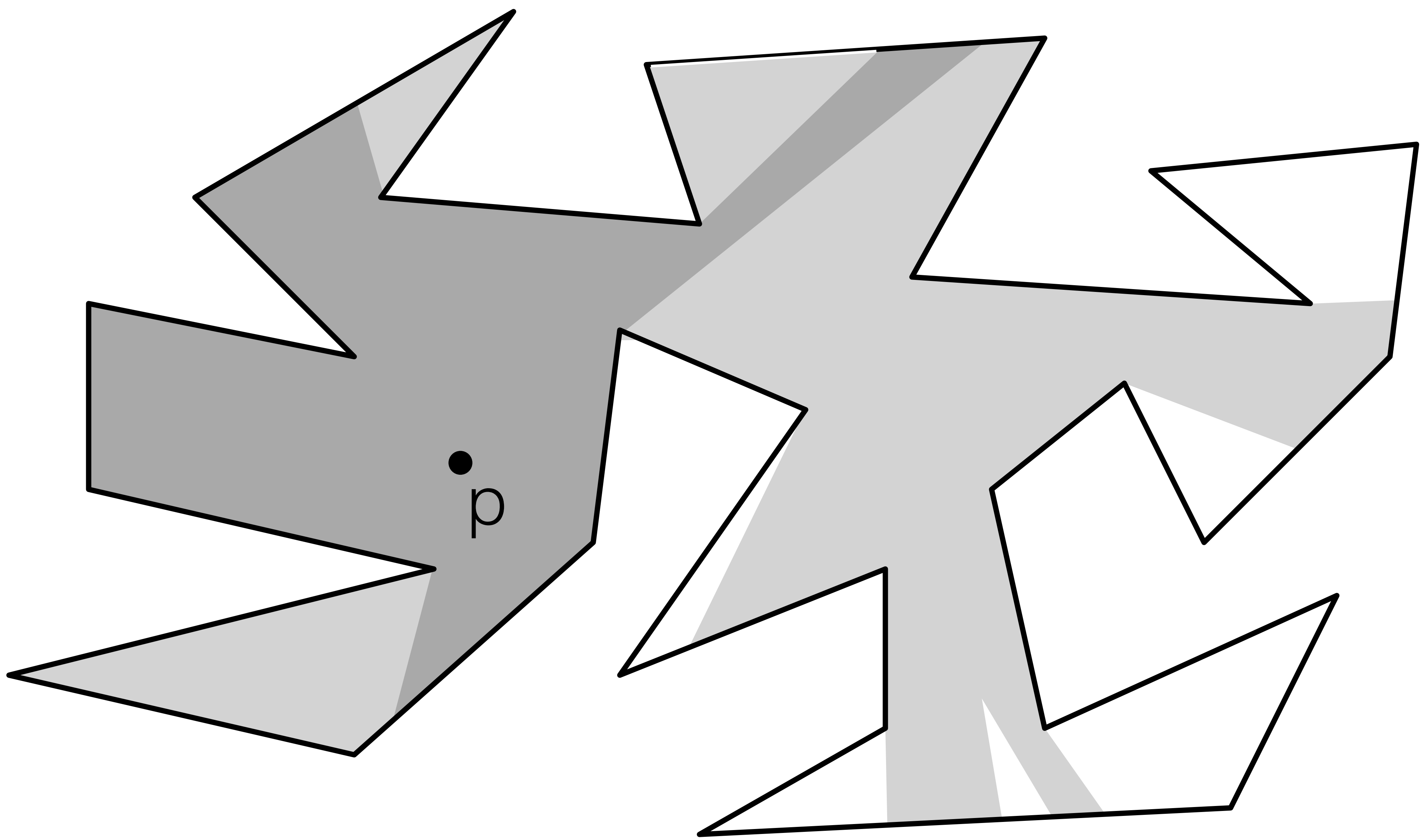


Diffuse Reflections



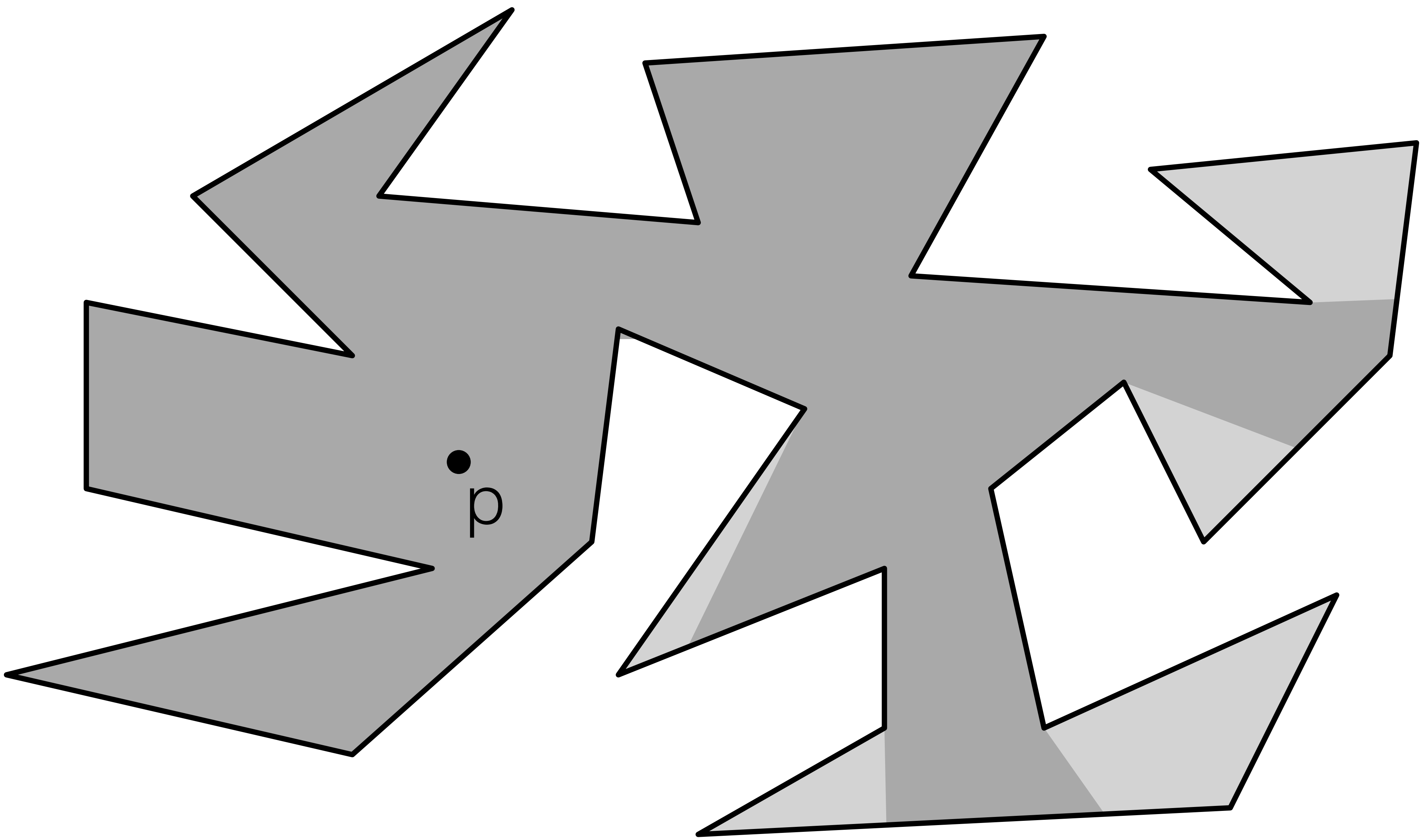
0 reflections

Diffuse Reflections



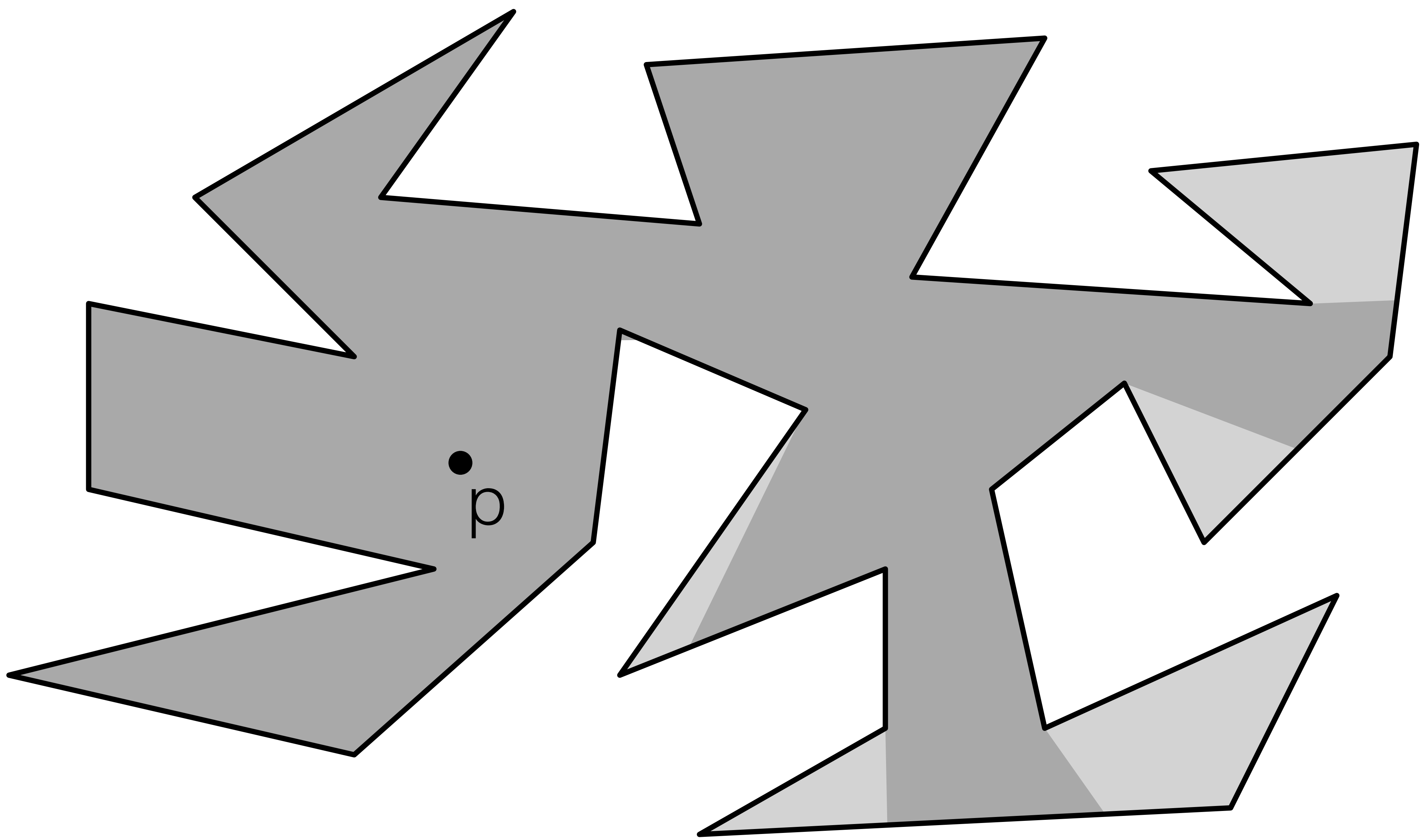
1 reflections

Diffuse Reflections



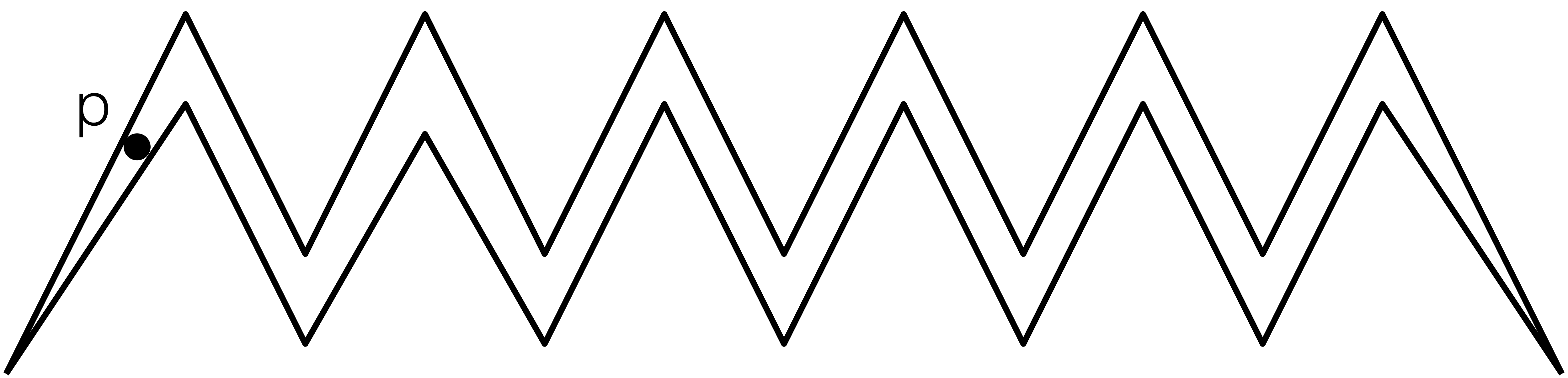
2 reflections

Diffuse Reflections

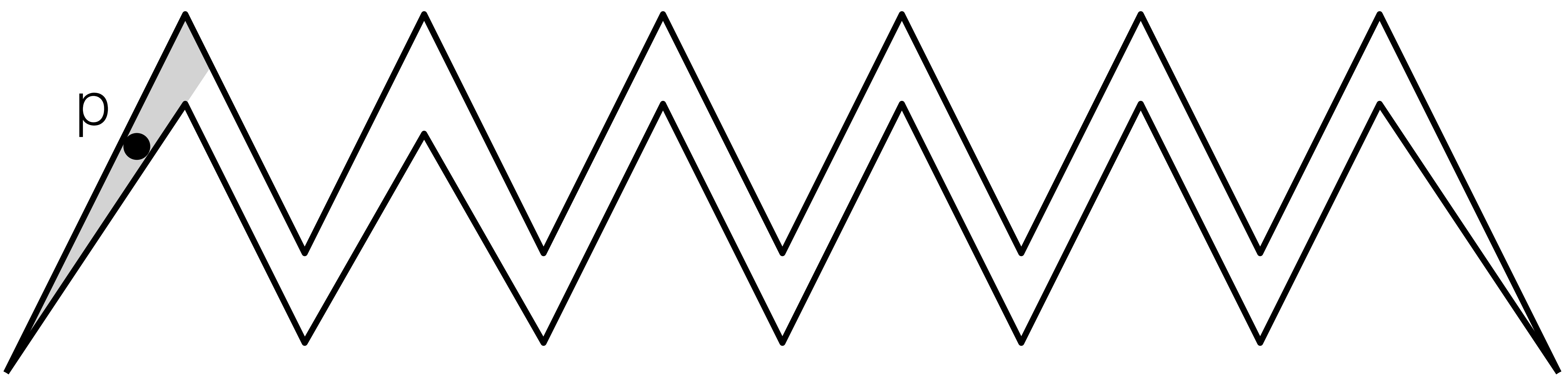


Polygon interior illuminated from p in 2 reflections.

Diffuse Reflections

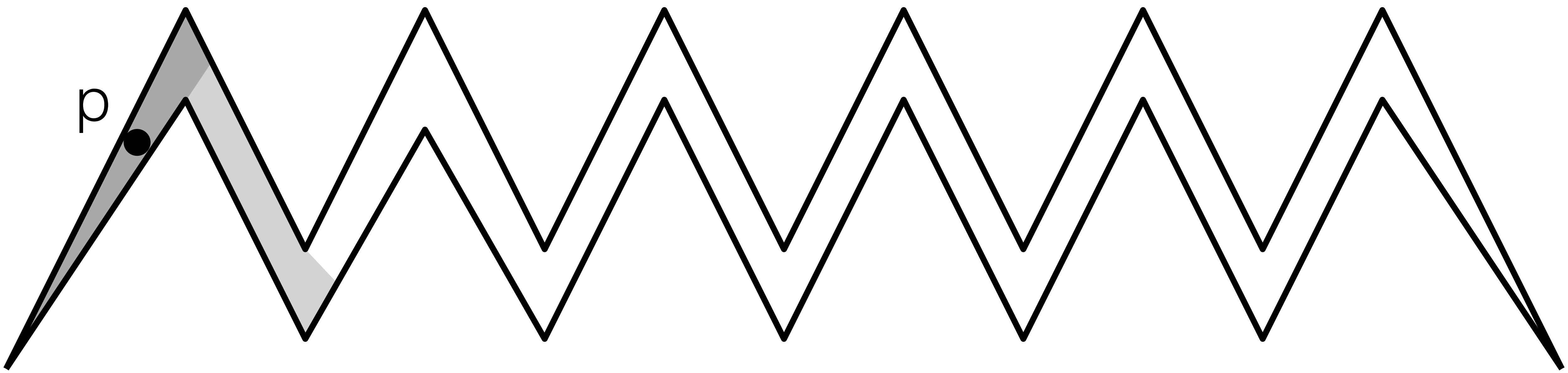


Diffuse Reflections



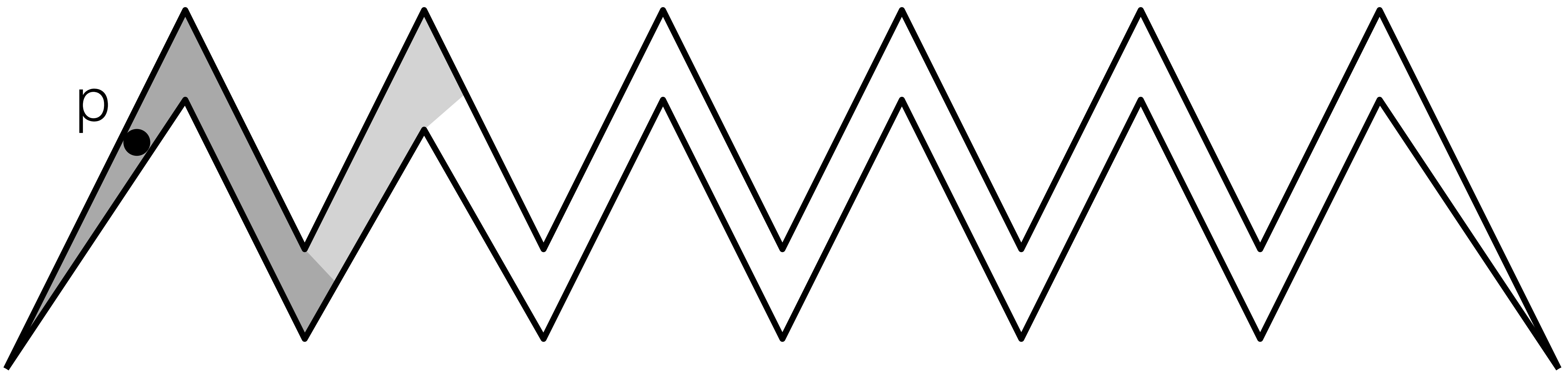
0 reflections.

Diffuse Reflections



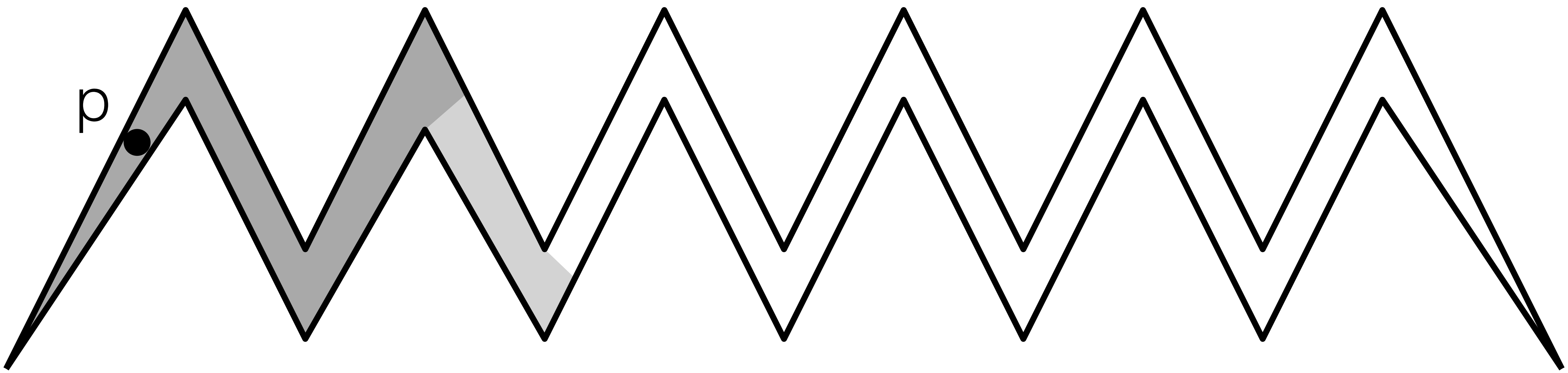
1 reflections.

Diffuse Reflections



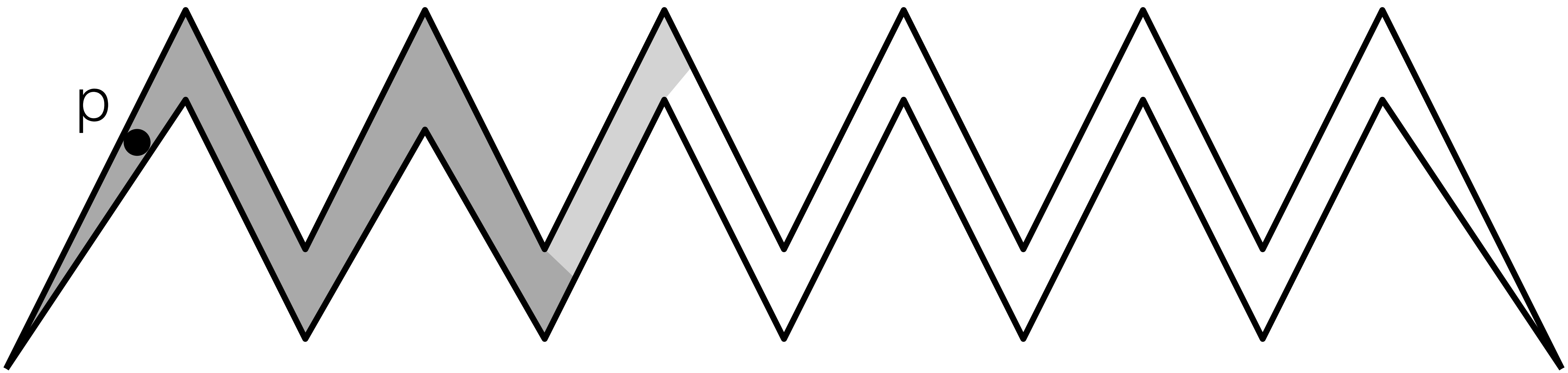
2 reflections.

Diffuse Reflections



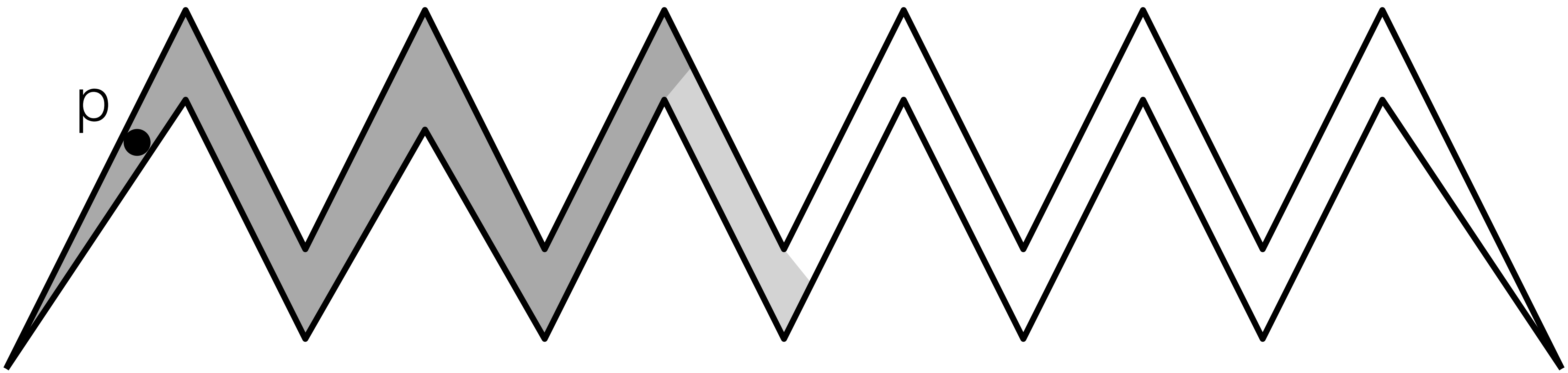
3 reflections.

Diffuse Reflections



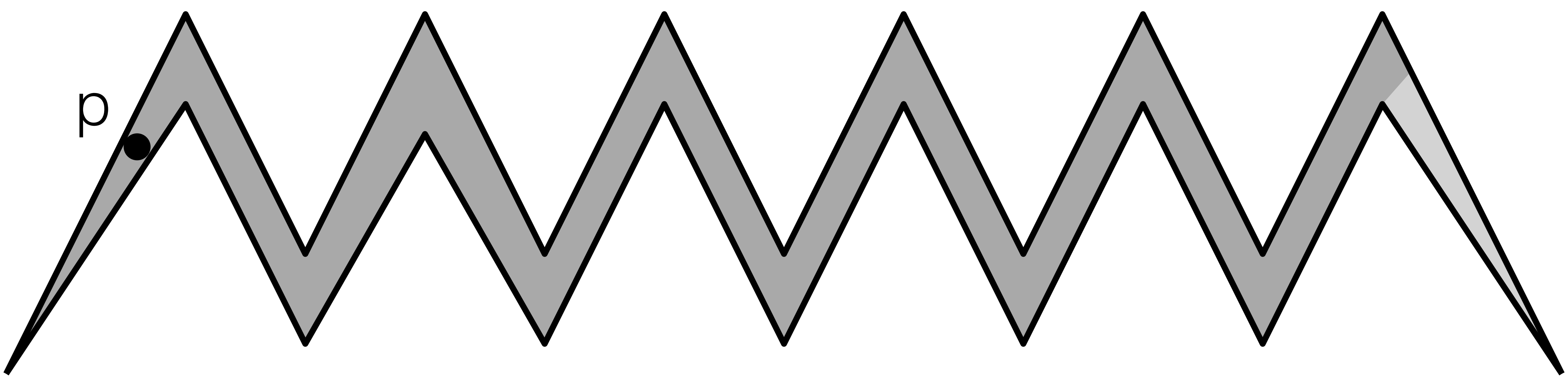
4 reflections.

Diffuse Reflections



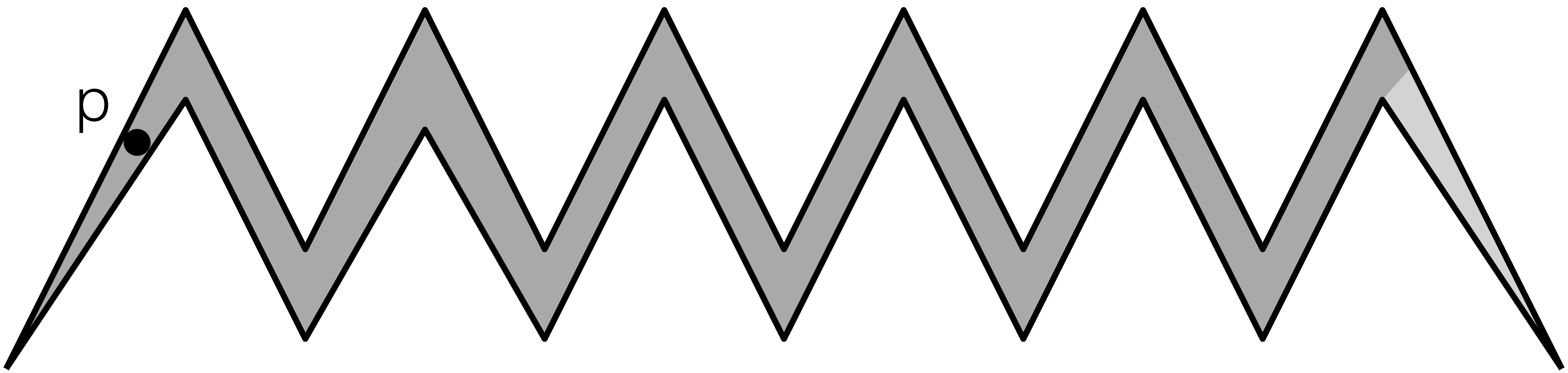
5 reflections.

Diffuse Reflections



Polygon interior illuminated from p in 11 reflections.

Diffuse Reflections



n-gon interior illuminated from p in n/2-1 reflections.

Diffuse Reflection Diameter and Radius

- *Diameter:* # diffuse reflections to illuminate an n-gon from <u>any</u> point.

- *Radius:* # diffuse reflections to illuminate an n-gon from <u>some</u> point.

Diffuse Reflection Diameter

- *Diameter:* # diffuse reflections to illuminate an n-gon from <u>any</u> point.

- Worst-case diffuse reflection diameter ≥ n/2-1.

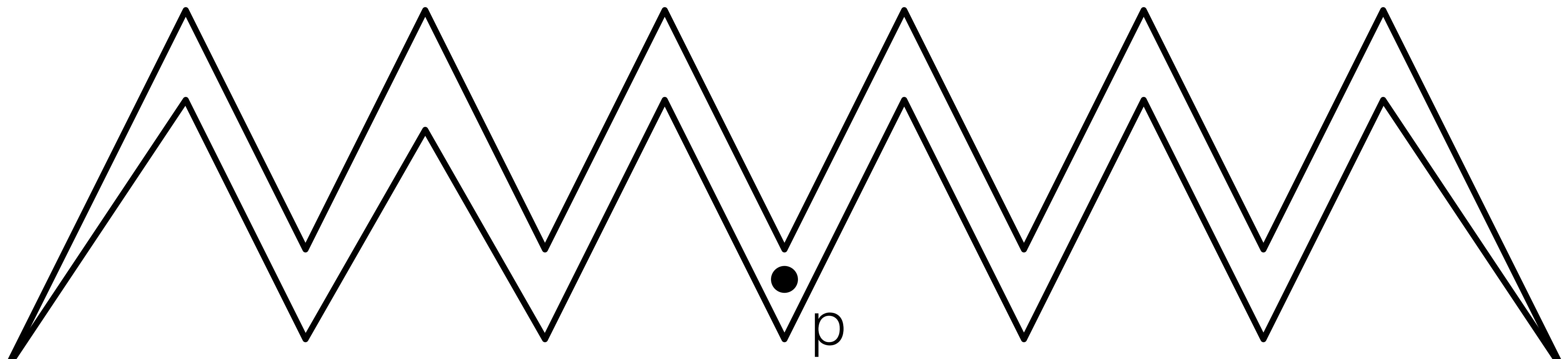
Diffuse Reflection Diameter

- *Diameter:* # diffuse reflections to illuminate an n-gon from <u>any</u> point.

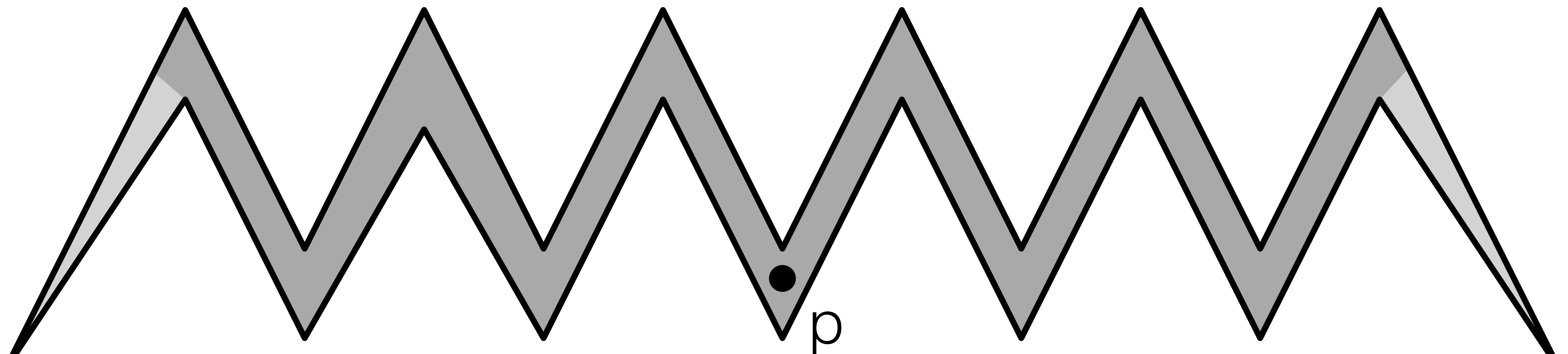
- Worst-case diffuse reflection diameter $\geq n/2-1$.

- Worst-case diffuse reflection diameter $\leq n/2-1$.
  [Barequet, Cannon, F.-E., Hescott, Souvaine, T., W. 2013]

Diffuse Reflections



Diffuse Reflections



n-gon interior illuminated from p in (n-2)/4 reflections.

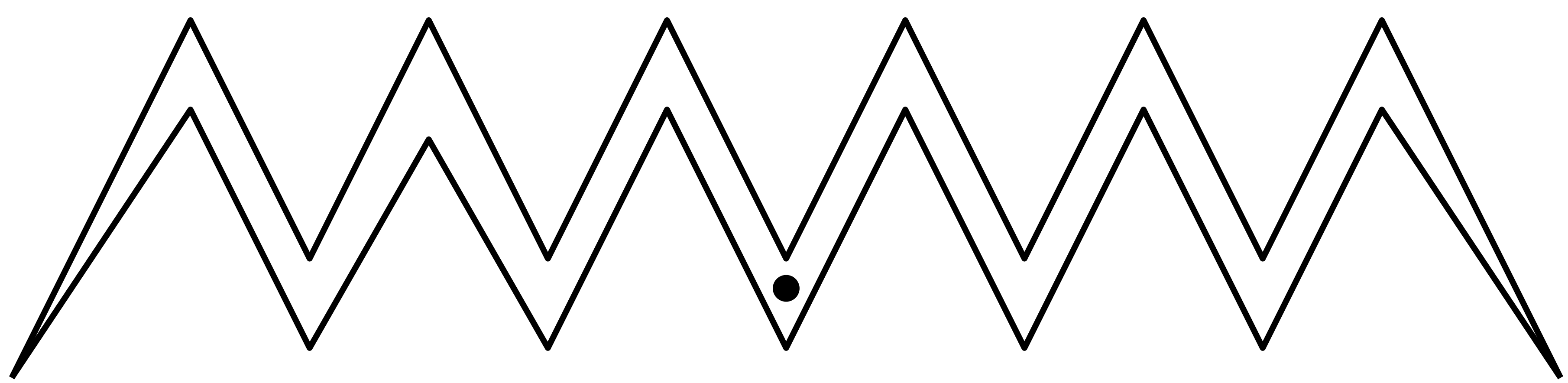
Diffuse Reflection Radius

- *Radius:* # diffuse reflections to illuminate an n-gon from <u>some</u> point.

Diffuse Reflection Radius

- *Radius:* # diffuse reflections to illuminate an n-gon from <u>some</u> point.

- Worst-case diffuse reflection radius ≥ (n-2)/4.



Diffuse Reflection Radius

- *Radius:* # diffuse reflections to illuminate an n-gon from <u>some</u> point.
- Worst-case diffuse reflection radius ≥ (n-2)/4.
- Worst-case diffuse reflection radius ≤ (n-2)/4. [Here]

Diffuse Reflection Radius

- *Radius:* # diffuse reflections to illuminate an n-gon from <u>some</u> point.

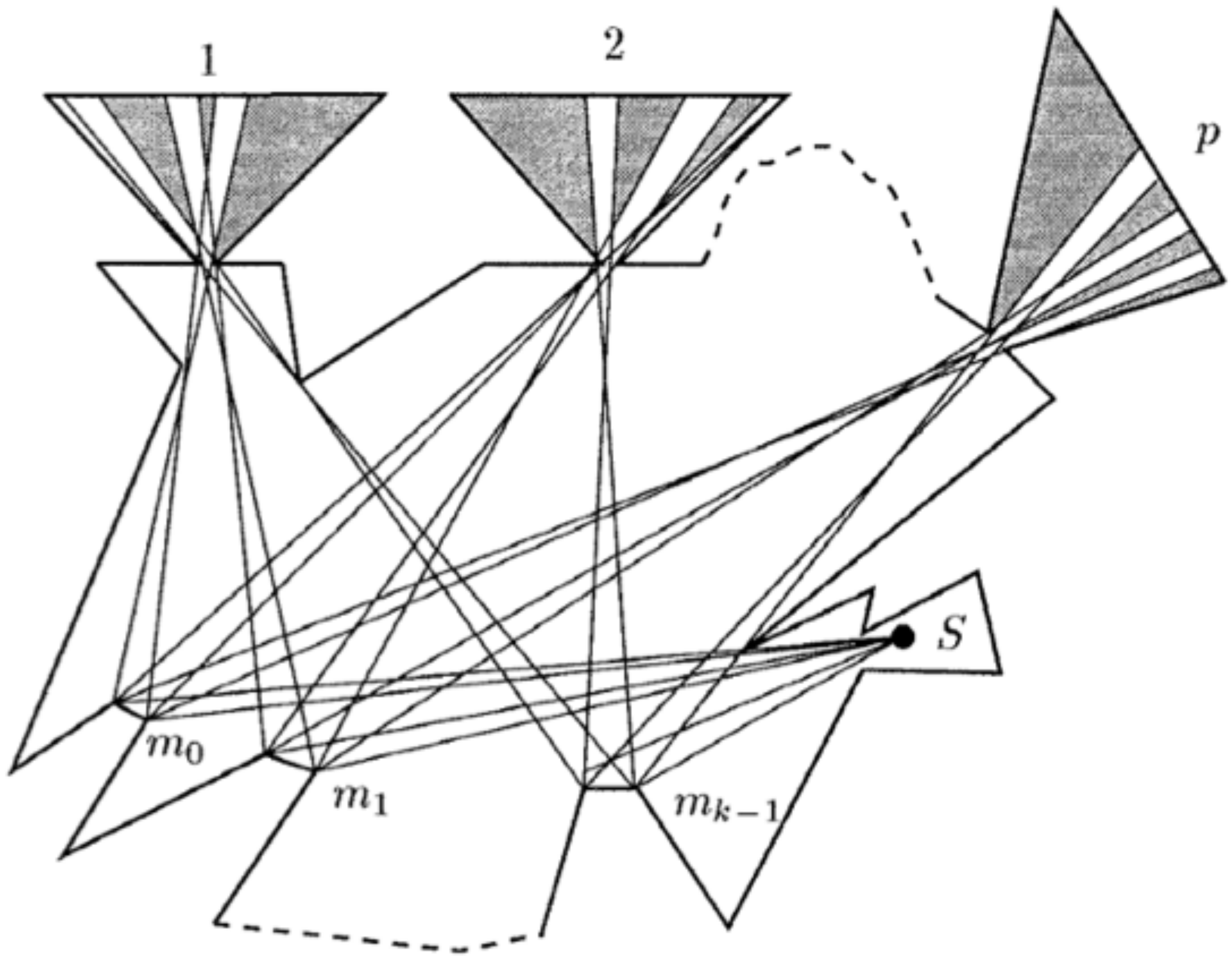
- Worst-case diffuse reflection radius $\geq (n-2)/4$.

- Worst-case diffuse reflection radius $\leq (n-2)/4$. [Here]

    - Also a $O(n*\log(n))$ algorithm to compute a witness illumination point.

Why is reasoning about diffuse reflection hard?

Visibility region after *one reflection* can have complexity Θ(n^2)



[Aronov, Davis, Dey, Pal, Prasad 1998]

Diffuse reflection visibility region

- Complexity $\Theta(n^2)$ after one reflection. [Aronov, Davis, Dey, Pal, Prasad '98]
- Non-simple after two reflections. [Pal, Sarkar '03]
- $\Theta(n)$ holes after three reflections. [Brahma, Pal, Sarkar '04]

- Complexity $O(n^{2k})$ after k reflections. [Aronov, Davis, Dey, Pal, Prasad, '98]
- Complexity $O(n^{2(k+1)/2+1})$ [Prasad, Pal, Dey '98]
- Complexity $O(n^9)$ [Aronov, Davis, Iacono, Yu '06]

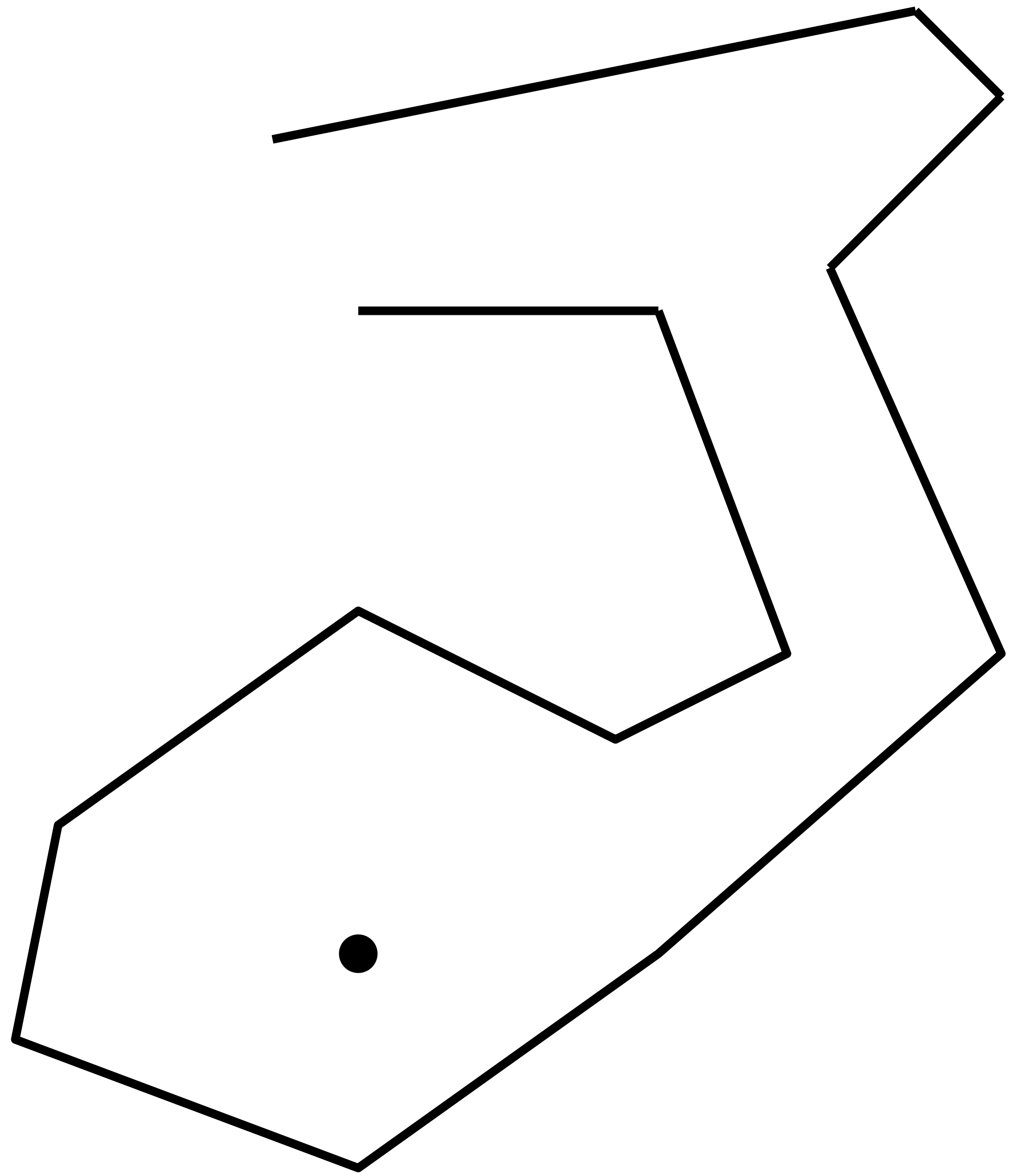
Diffuse reflection diameter and radius

- Worst-case diffuse reflection diameter ≤ n/2-1.
  [Barequet, Cannon, F.-E., Hescott, Souvaine, T., W. 2013]
- Worst-case diffuse reflection radius ≤ (n-2)/4.
  [Here]
- Use a common approach.
  - Use a subset of diffuse reflection visibility region.
  - Count total # new edges seen each reflection step.
  - Show this is at least 2 (diameter) or 4 (radius).

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.



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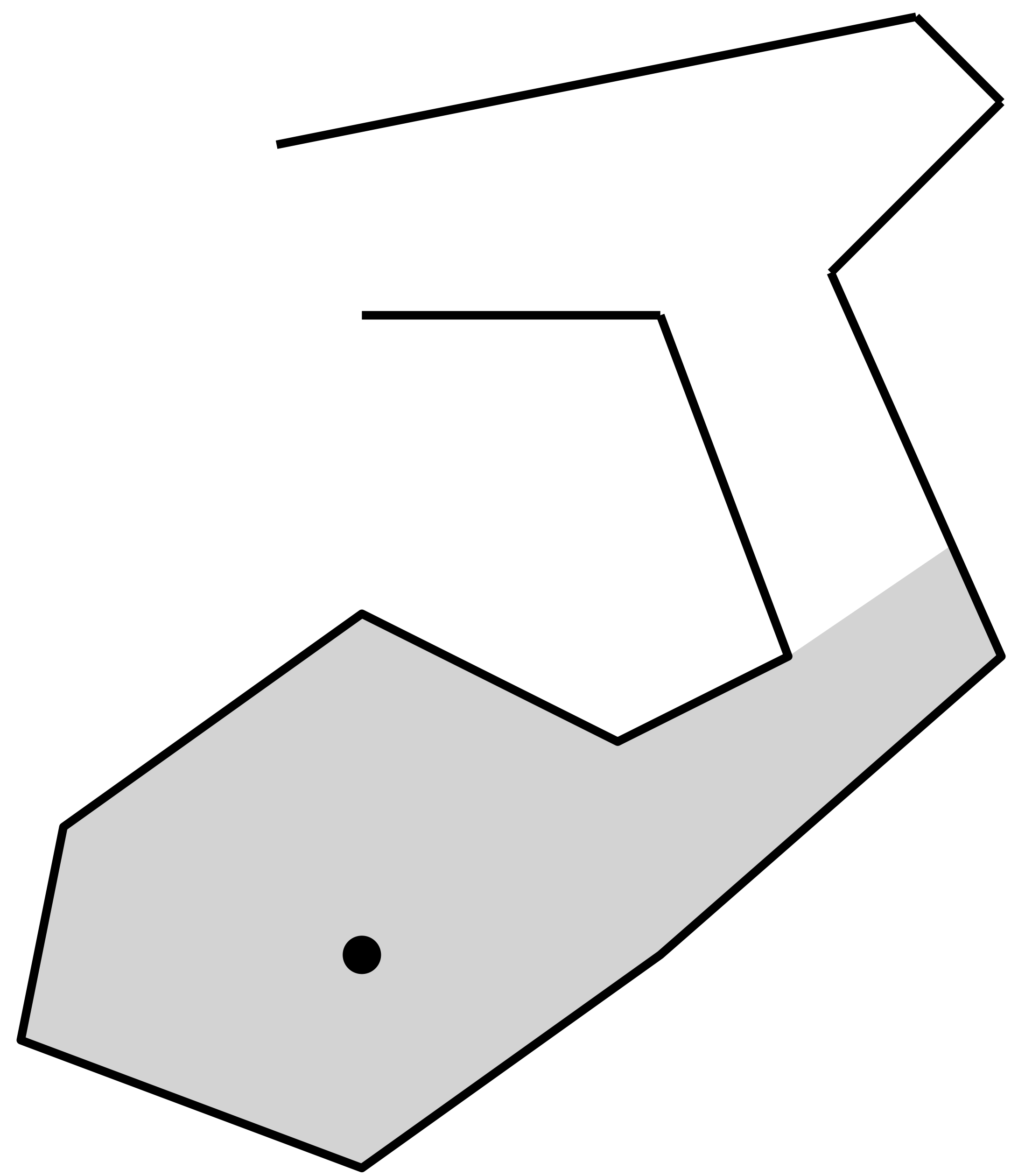

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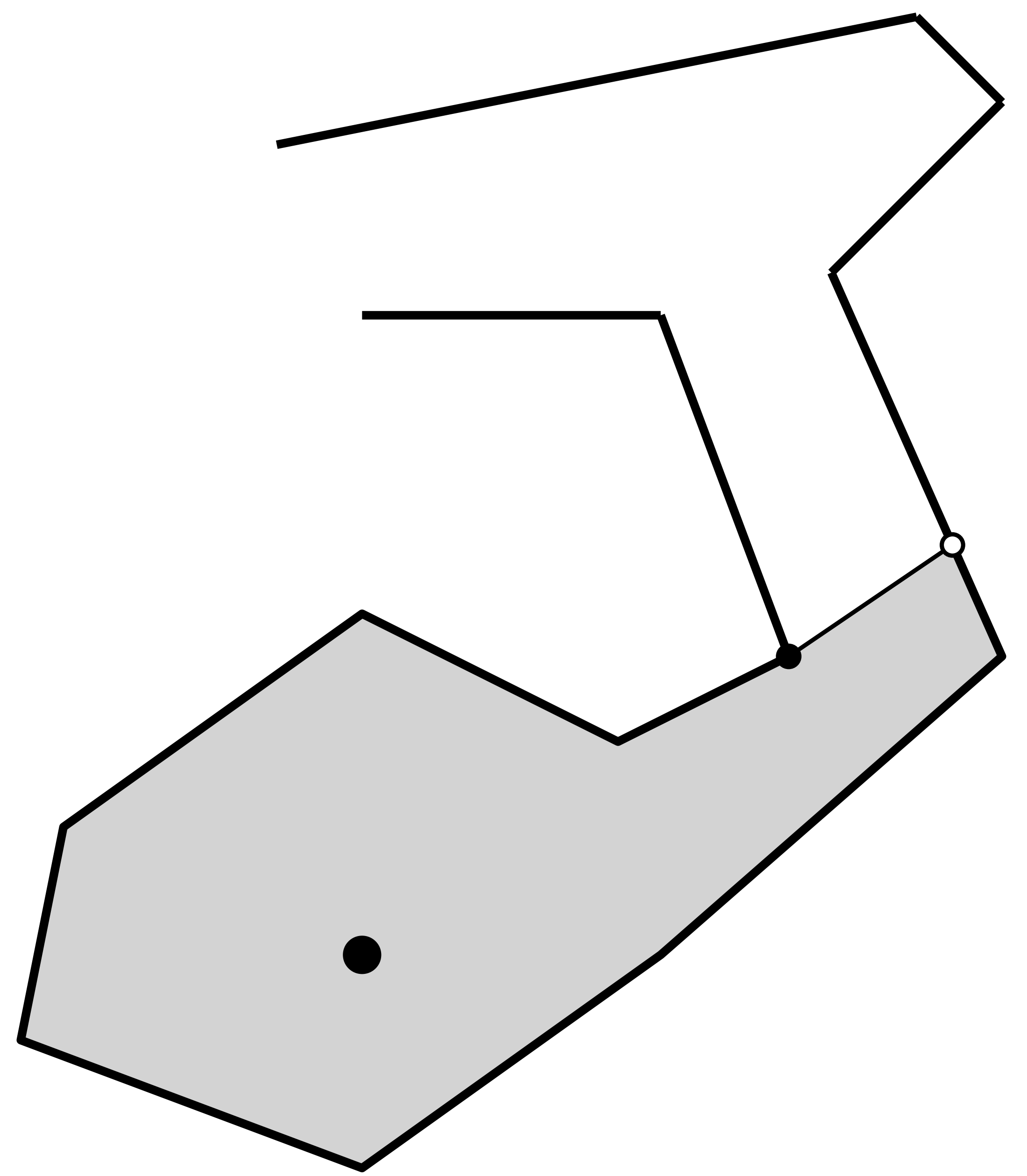

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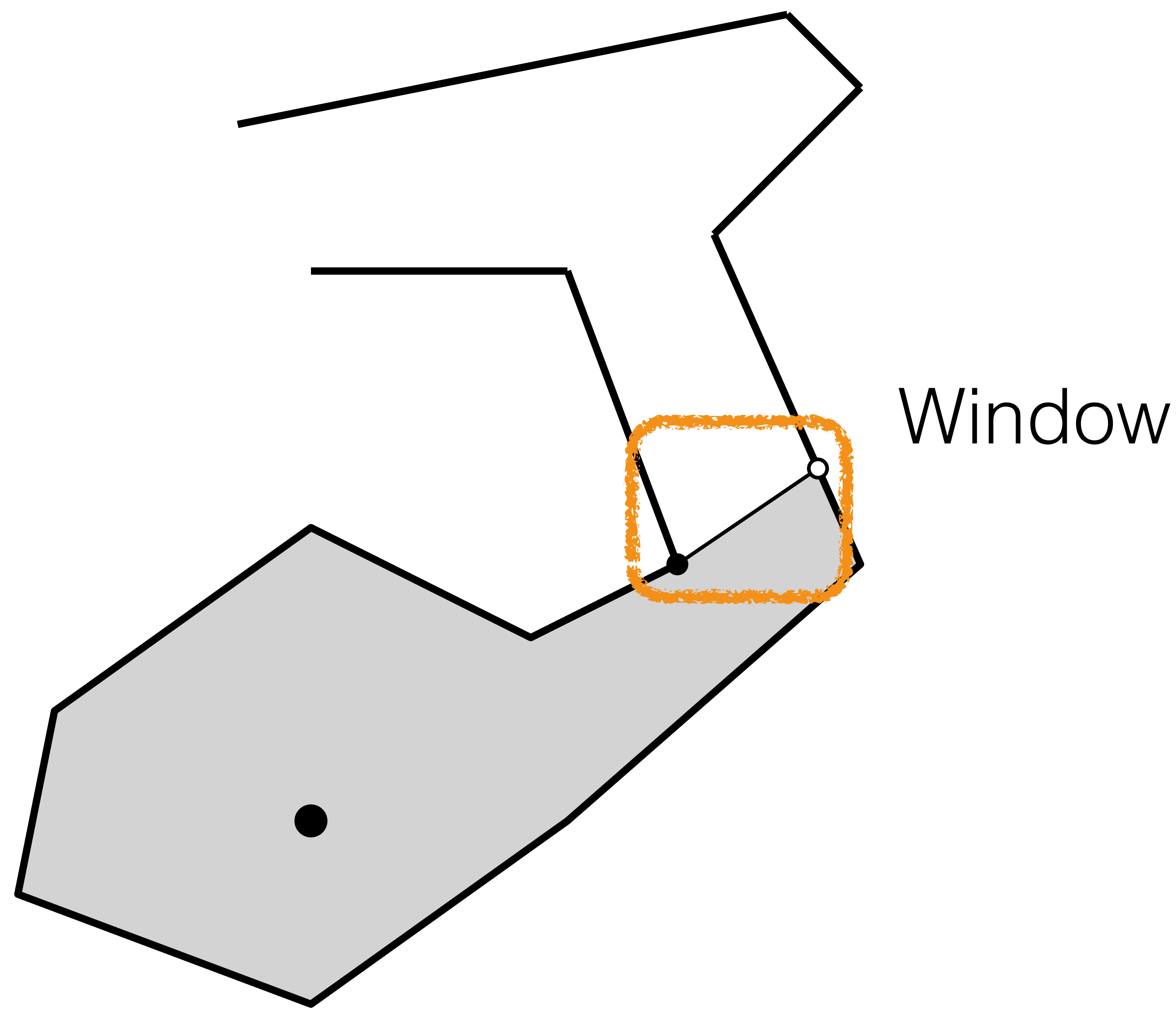

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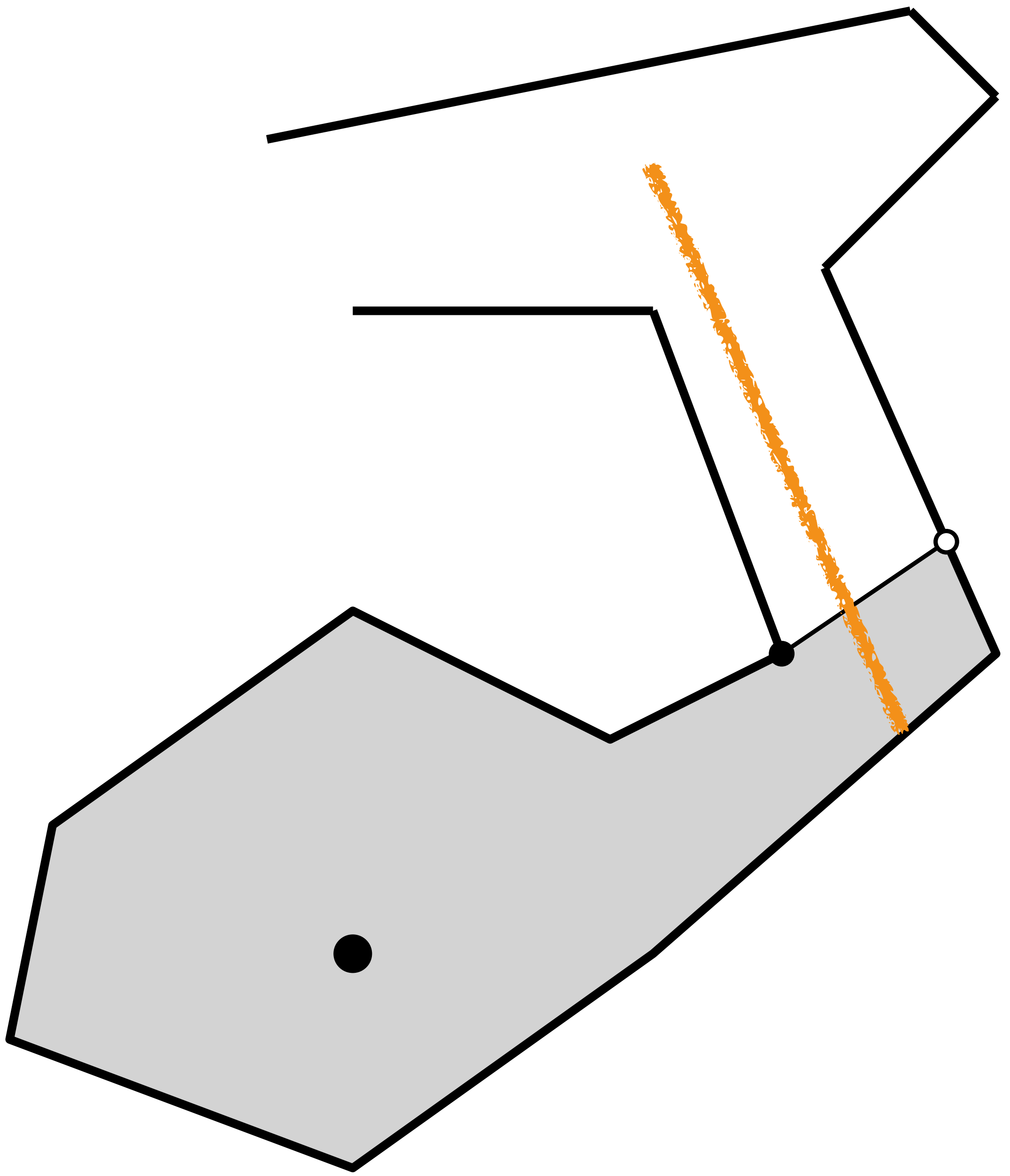

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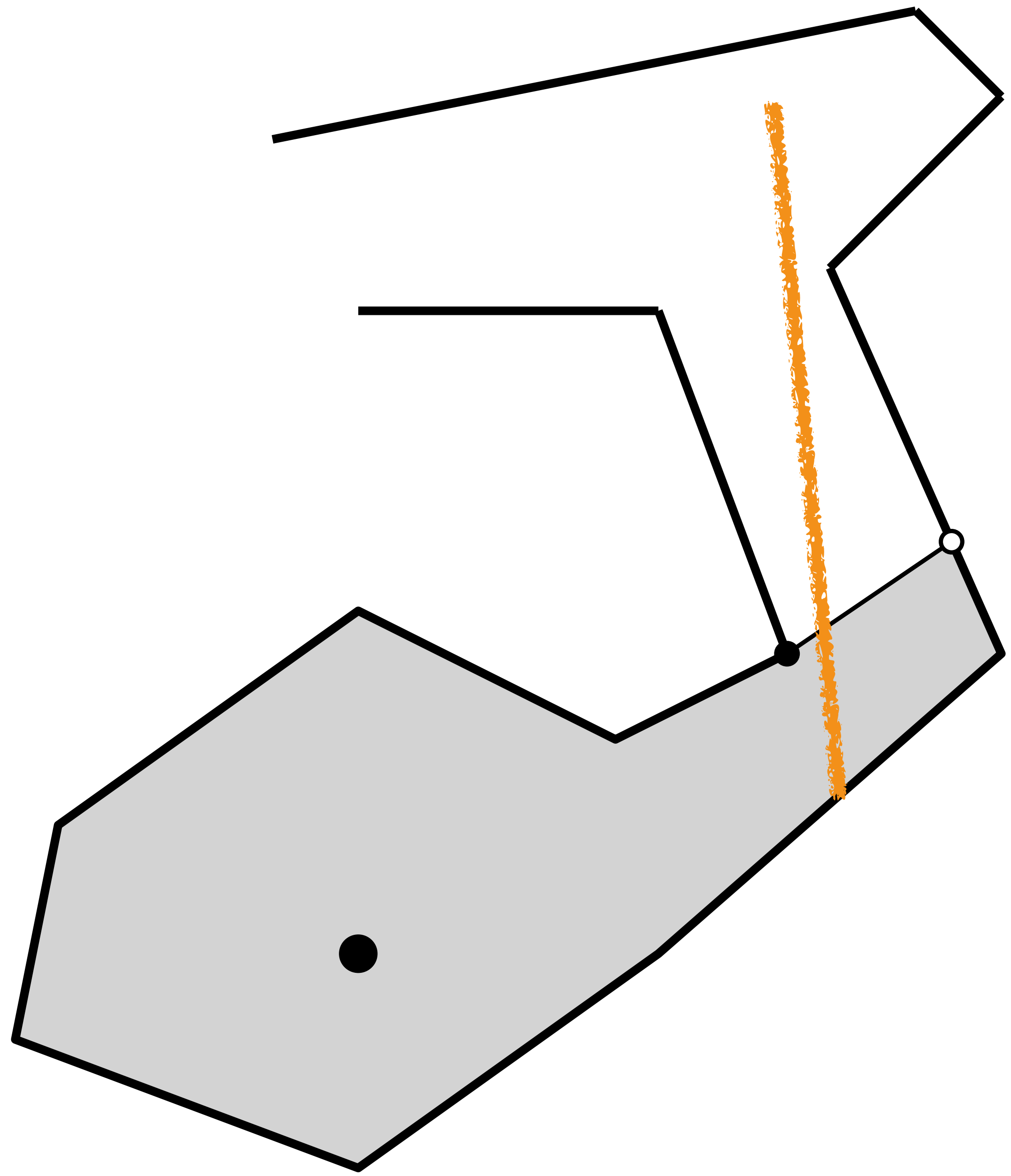

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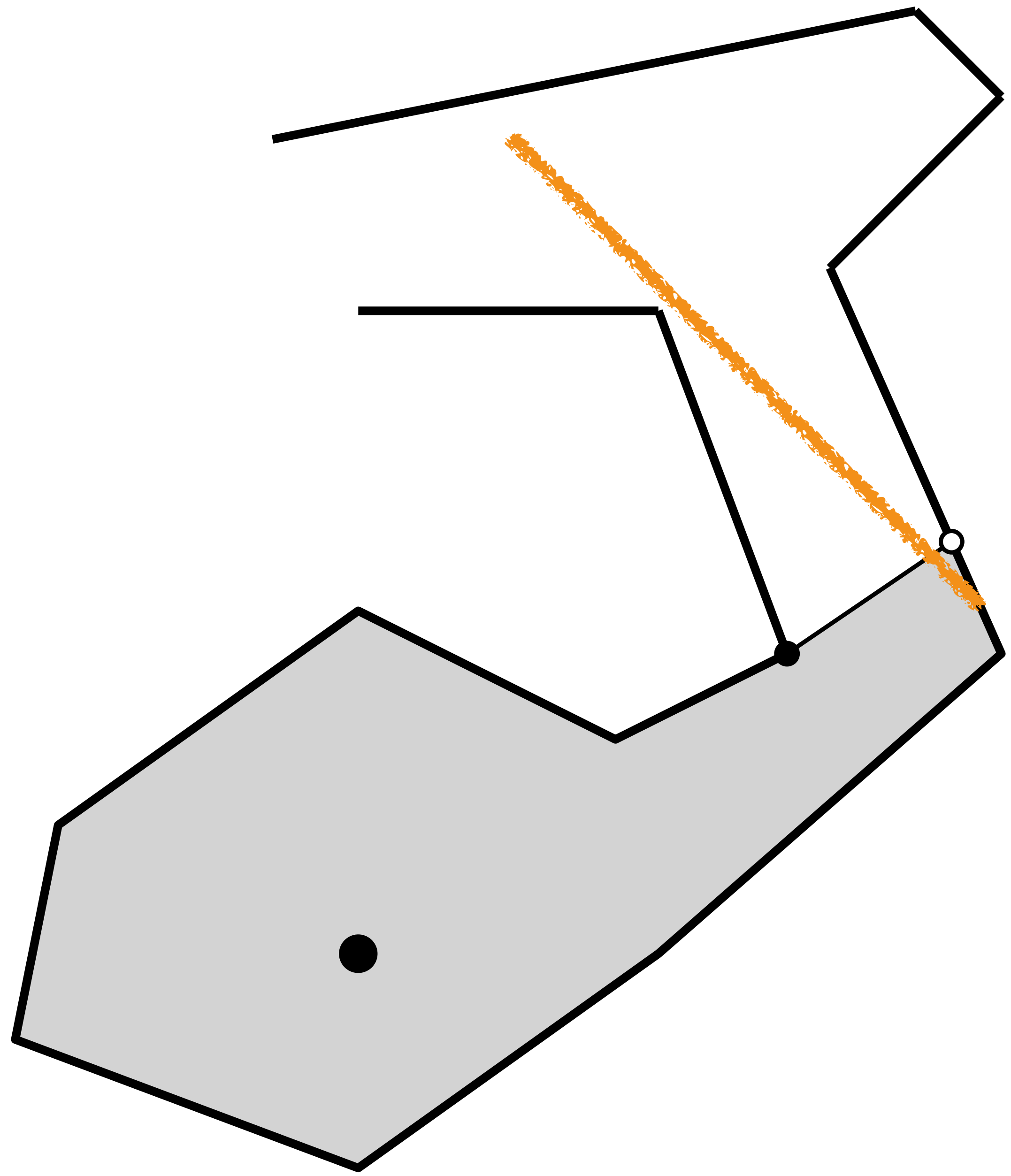

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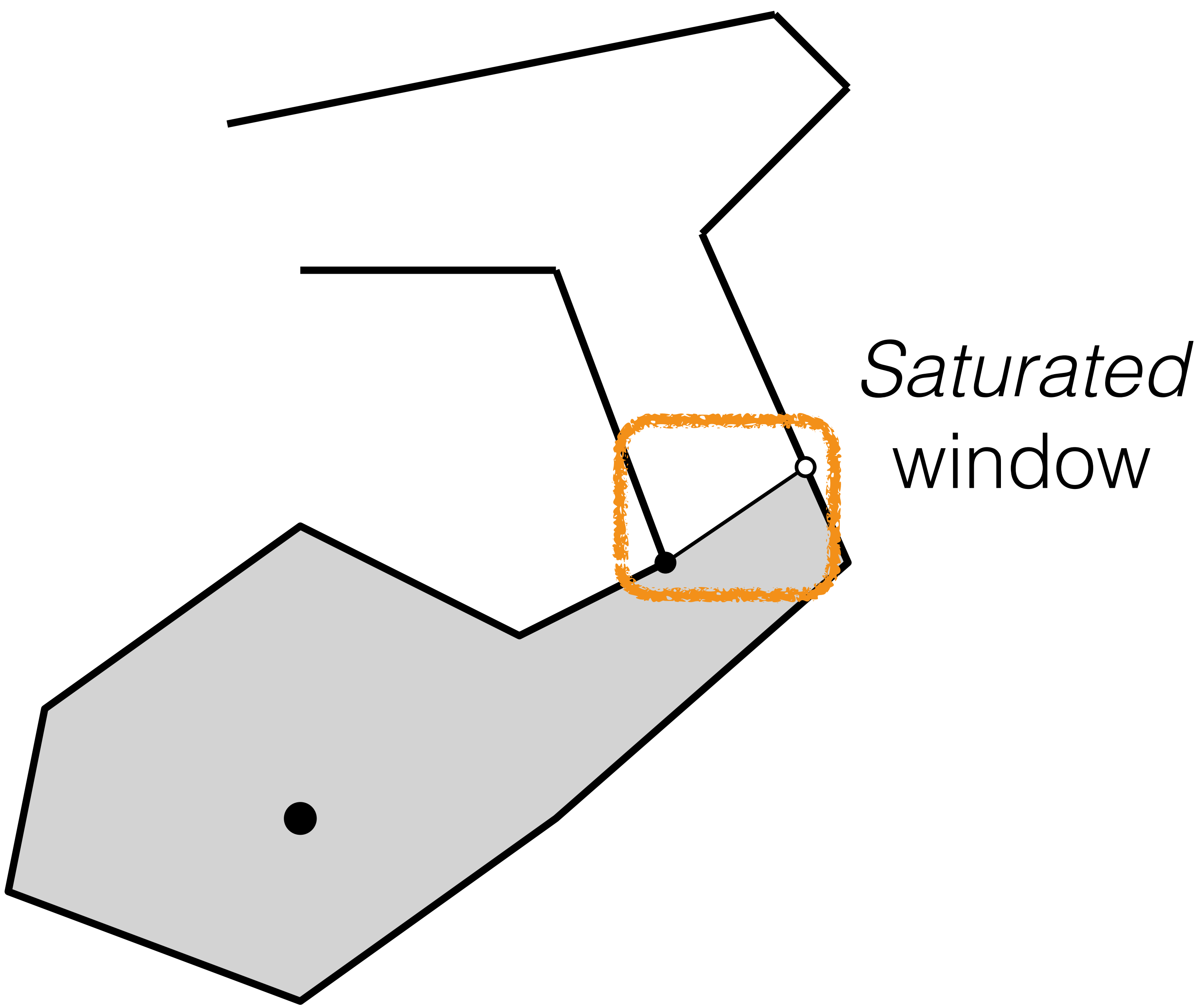

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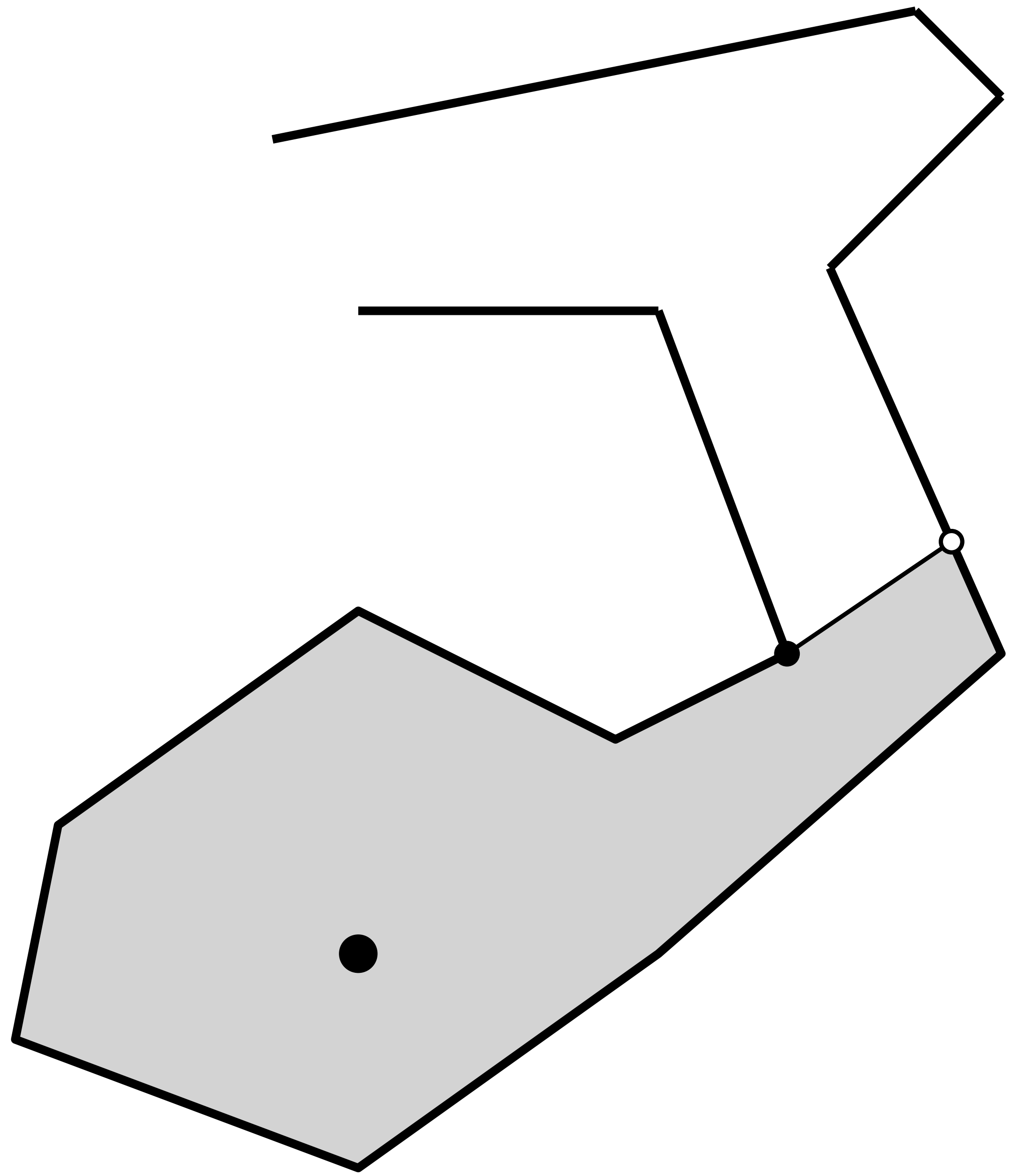
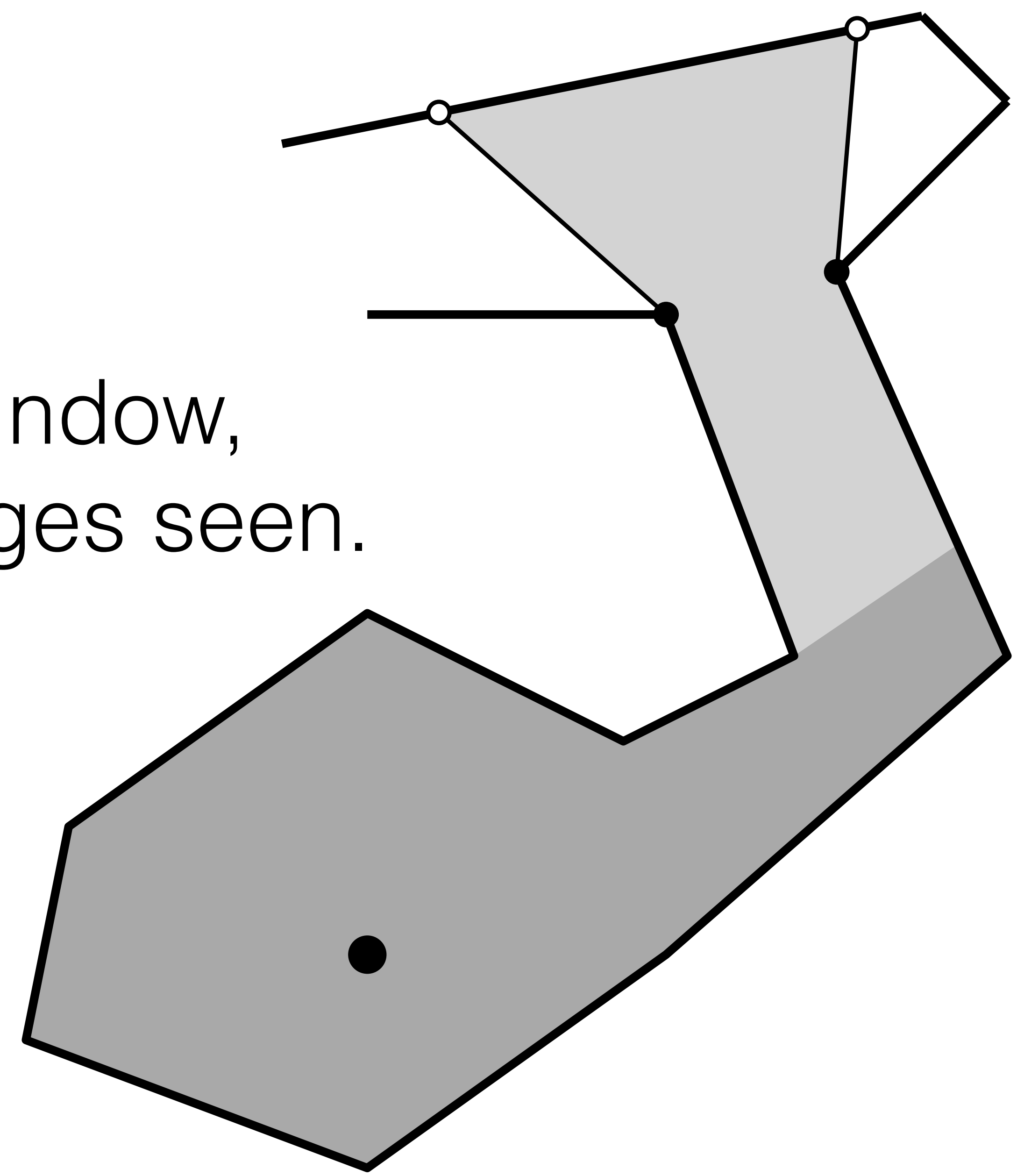


Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

If saturated window,
then two new edges seen.



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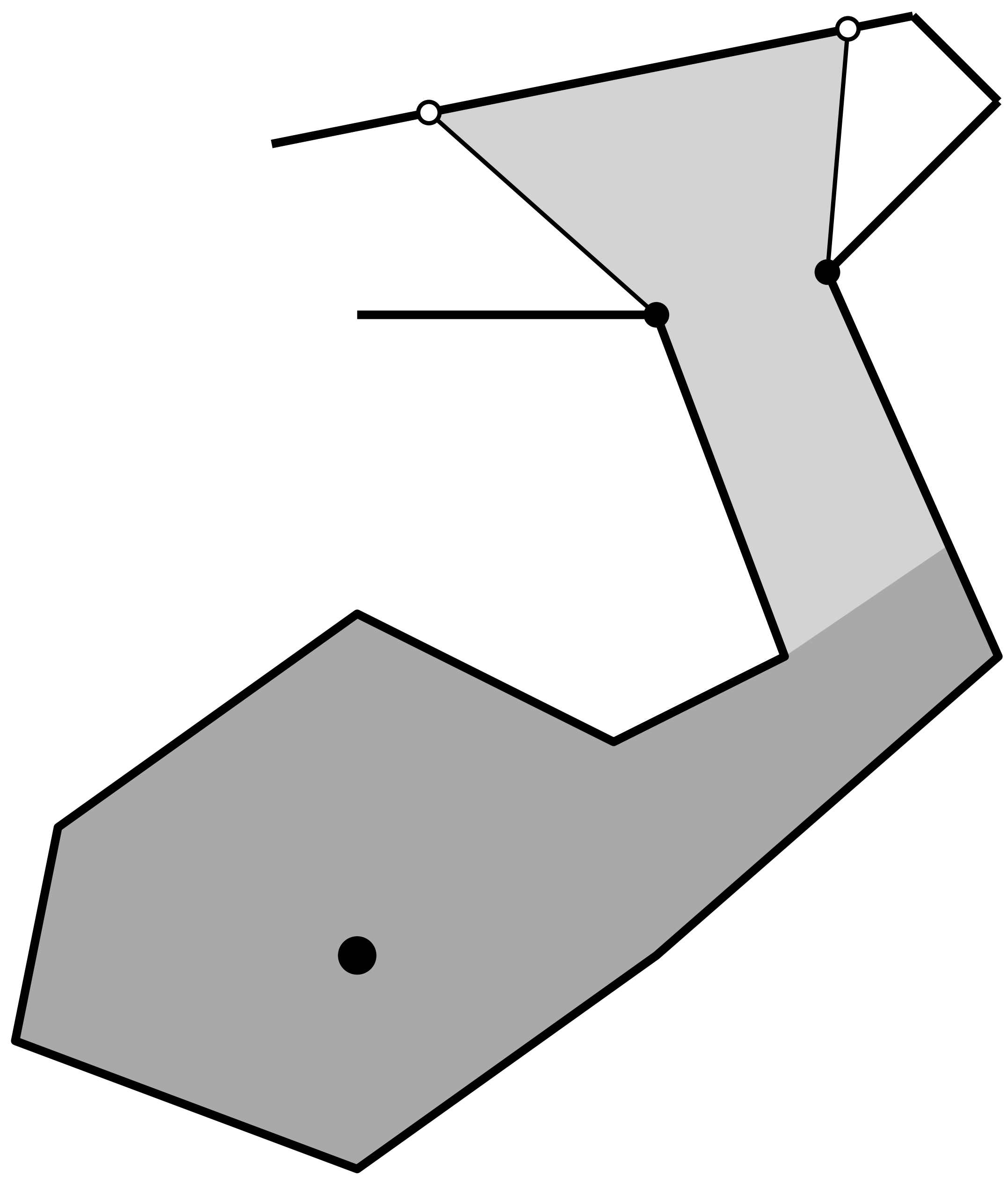

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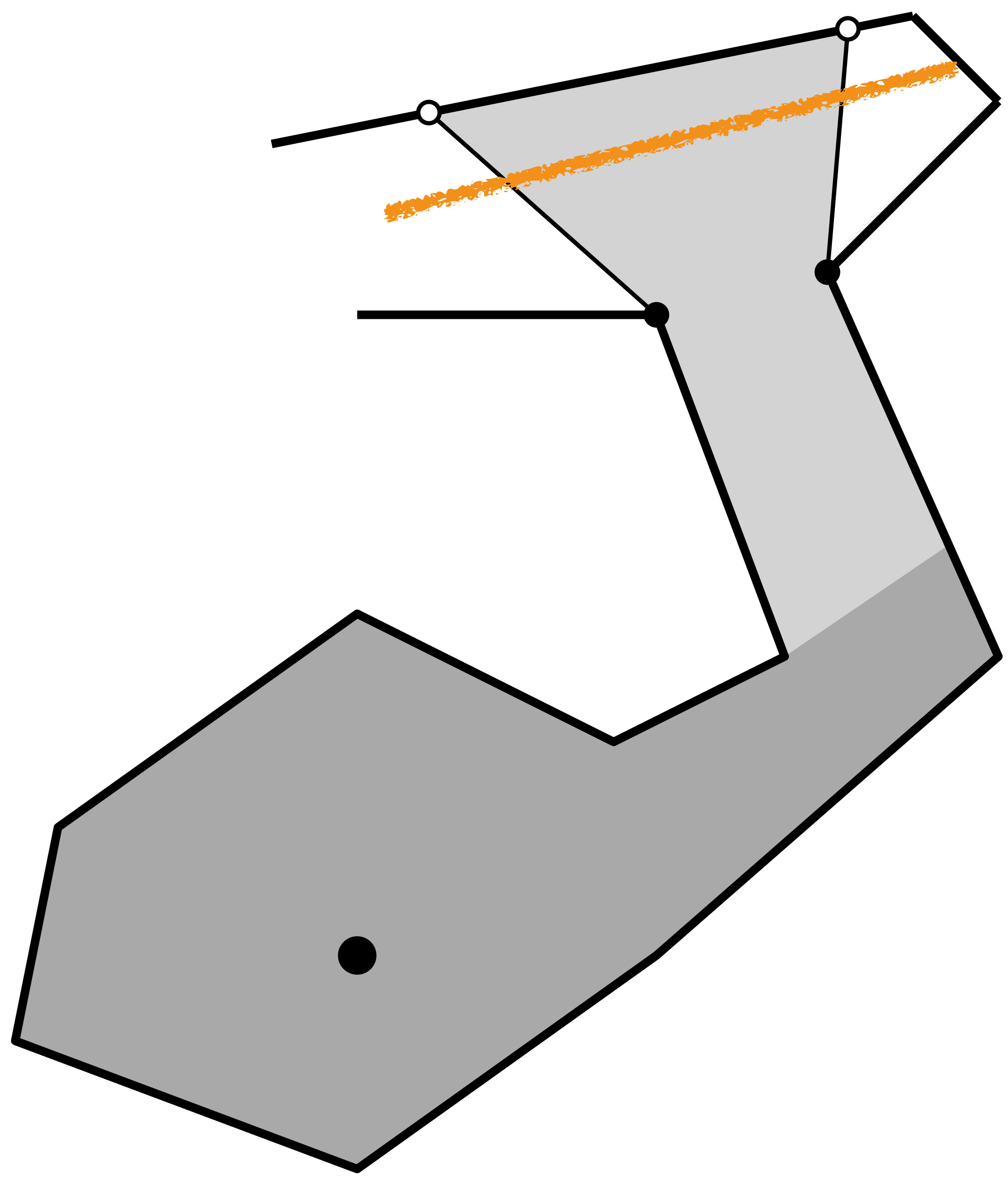

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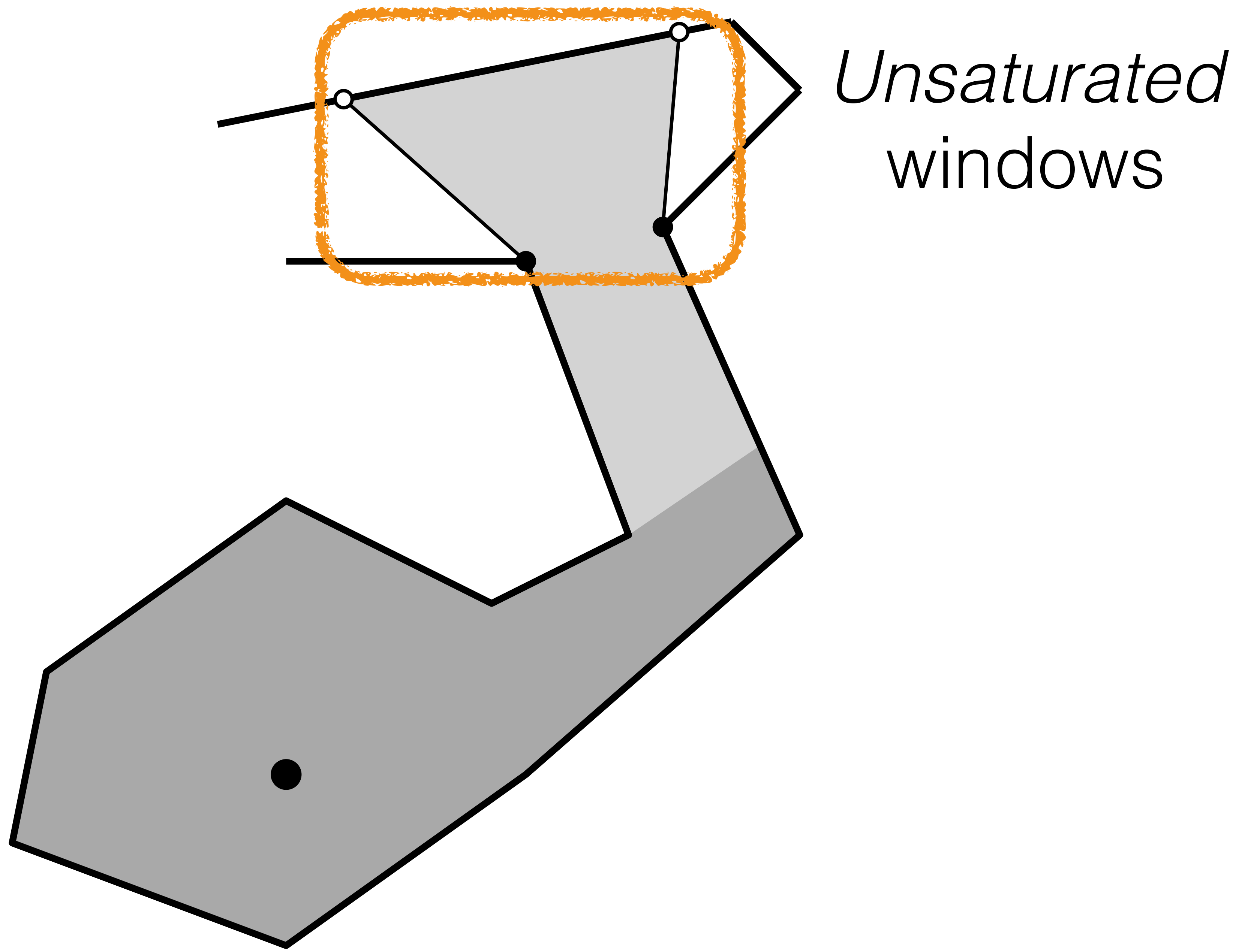
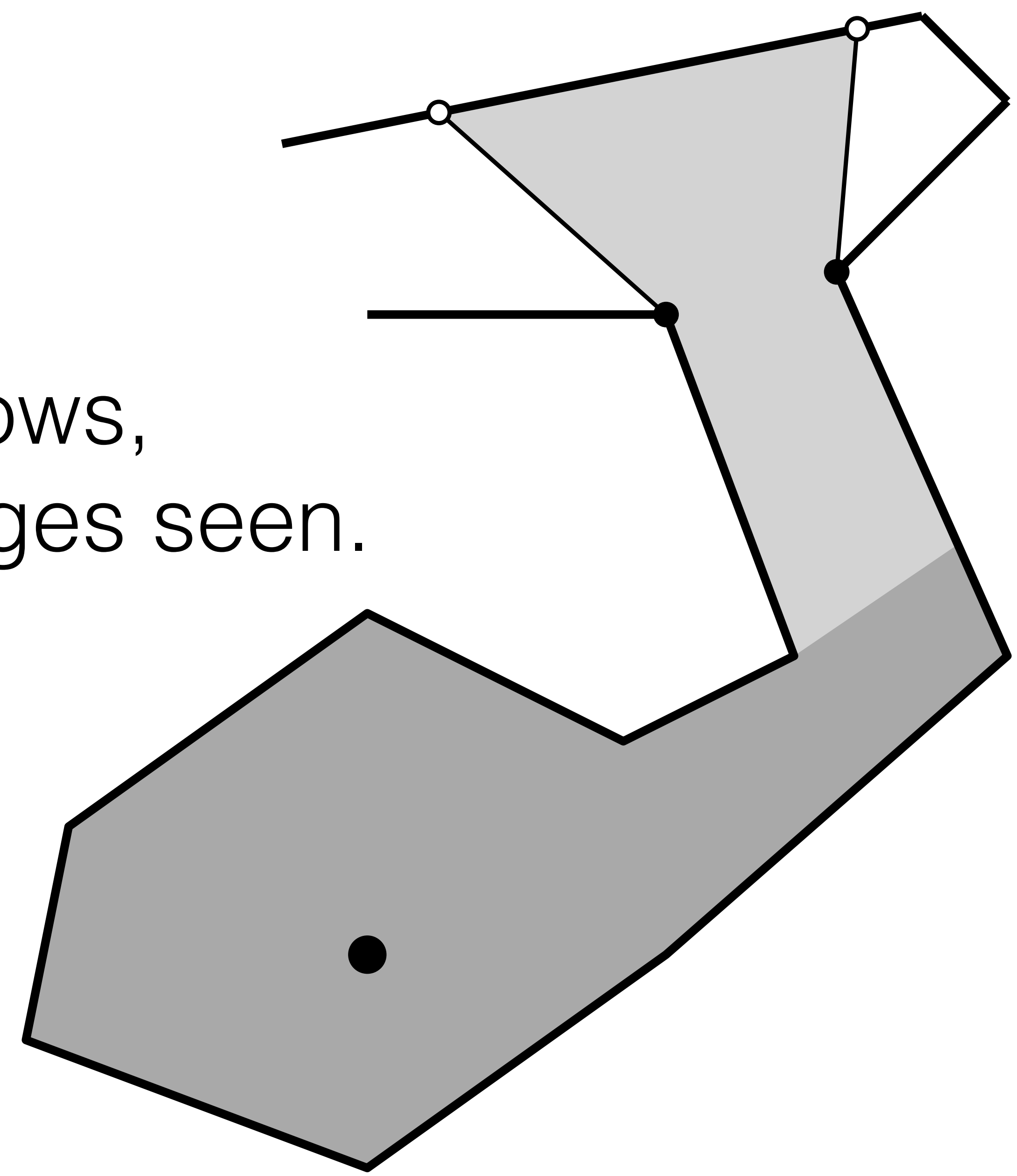


Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

If two windows,
then two new edges seen.



Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

Can show that either a saturated window or multiple windows always exist.

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

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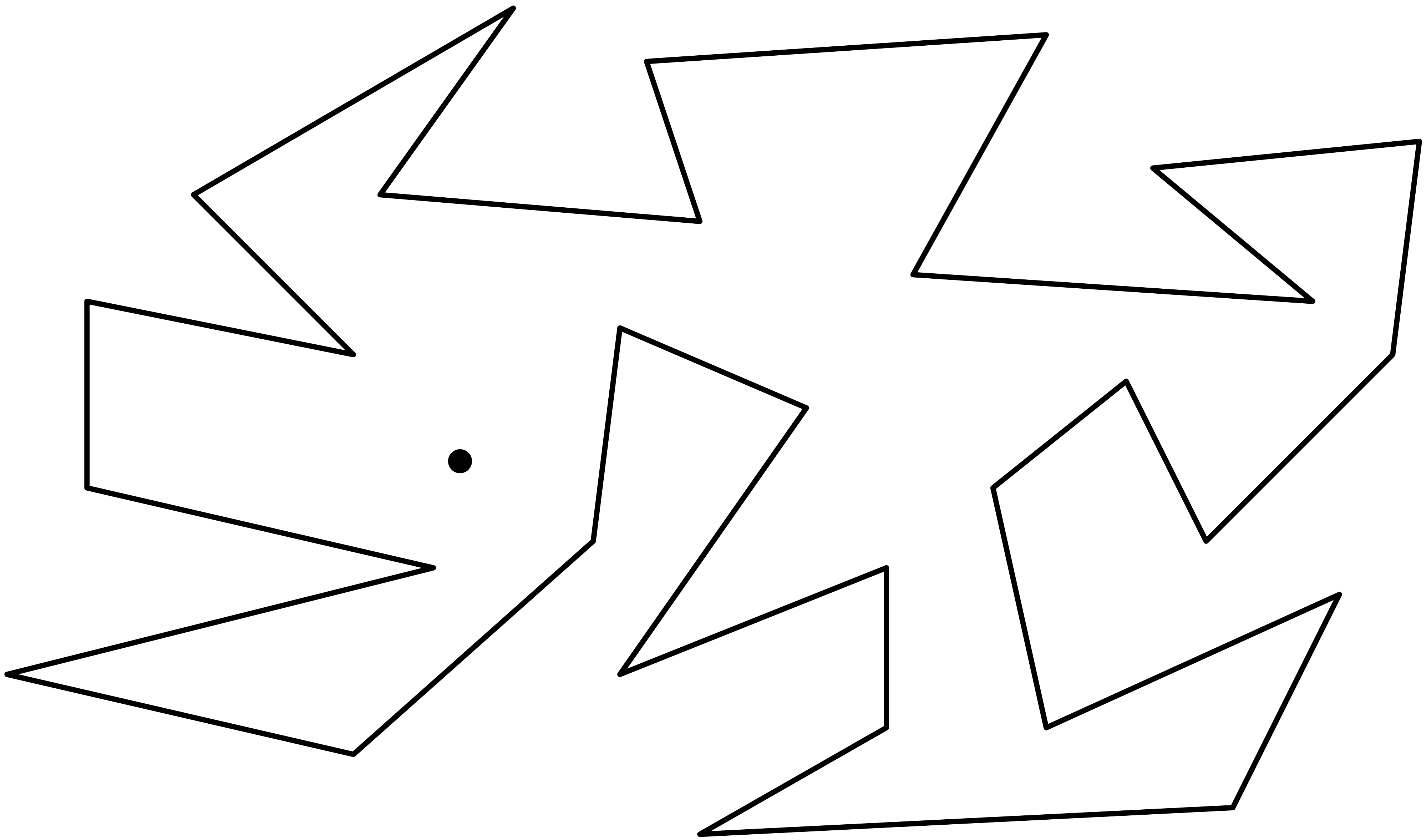

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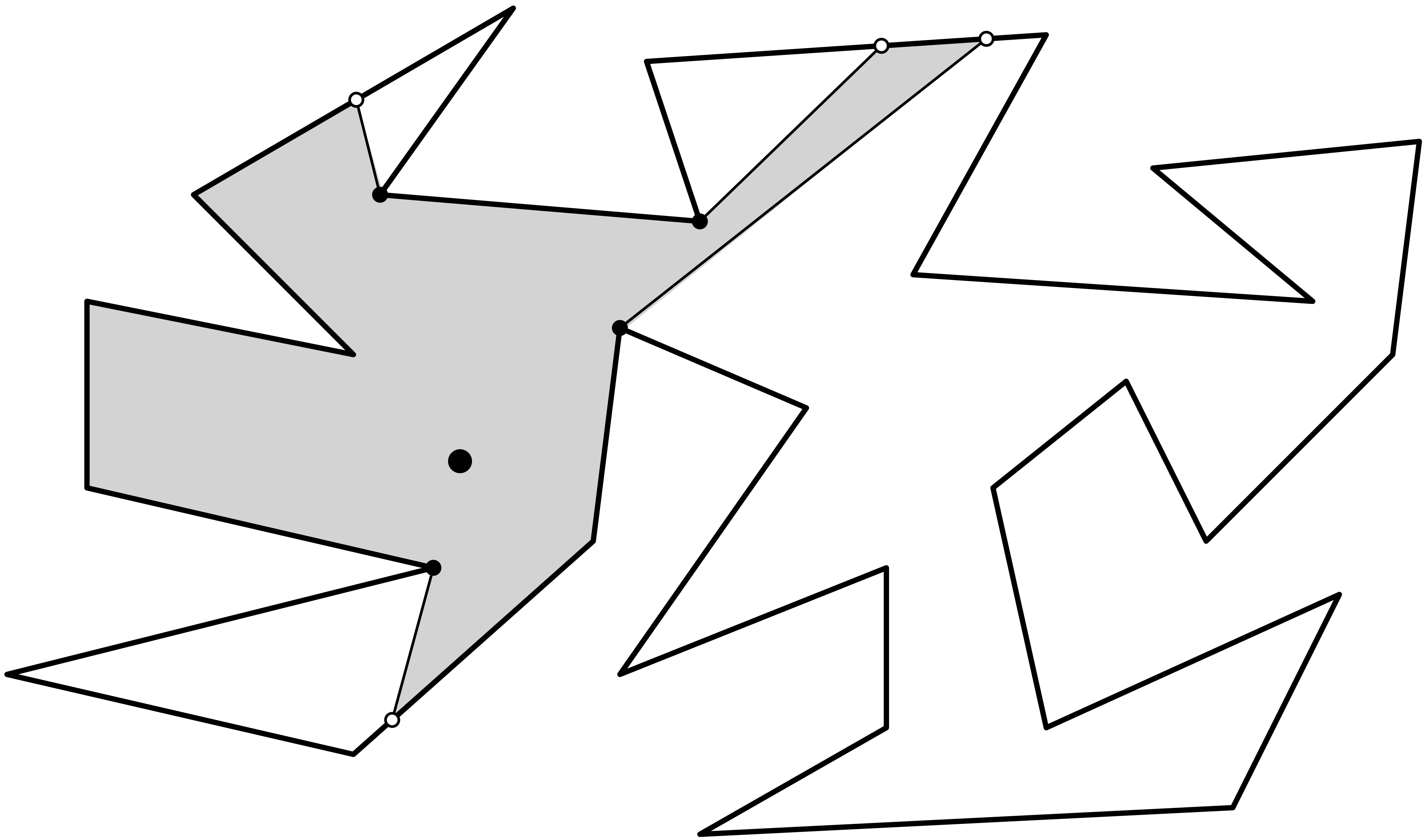
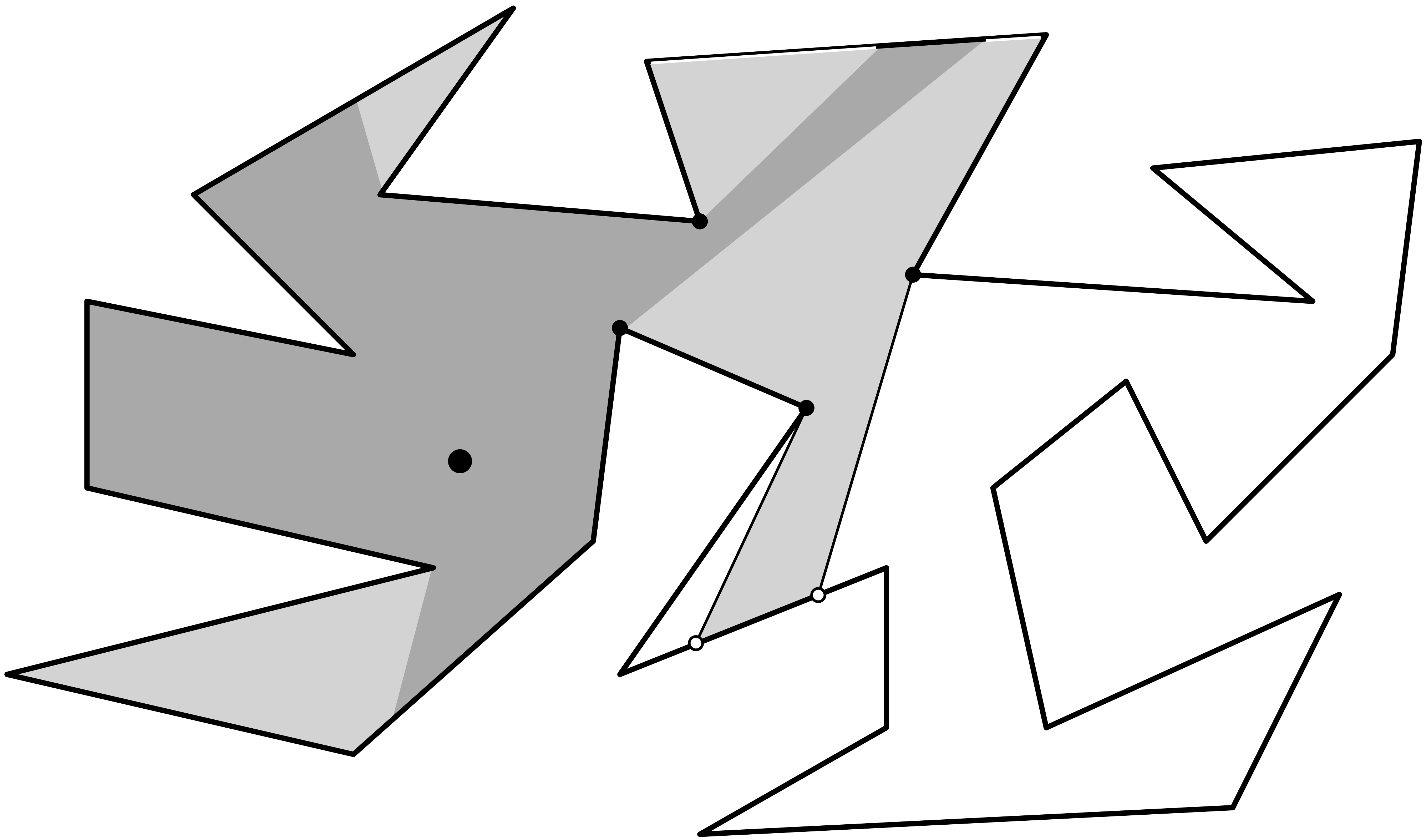


Region R_0

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

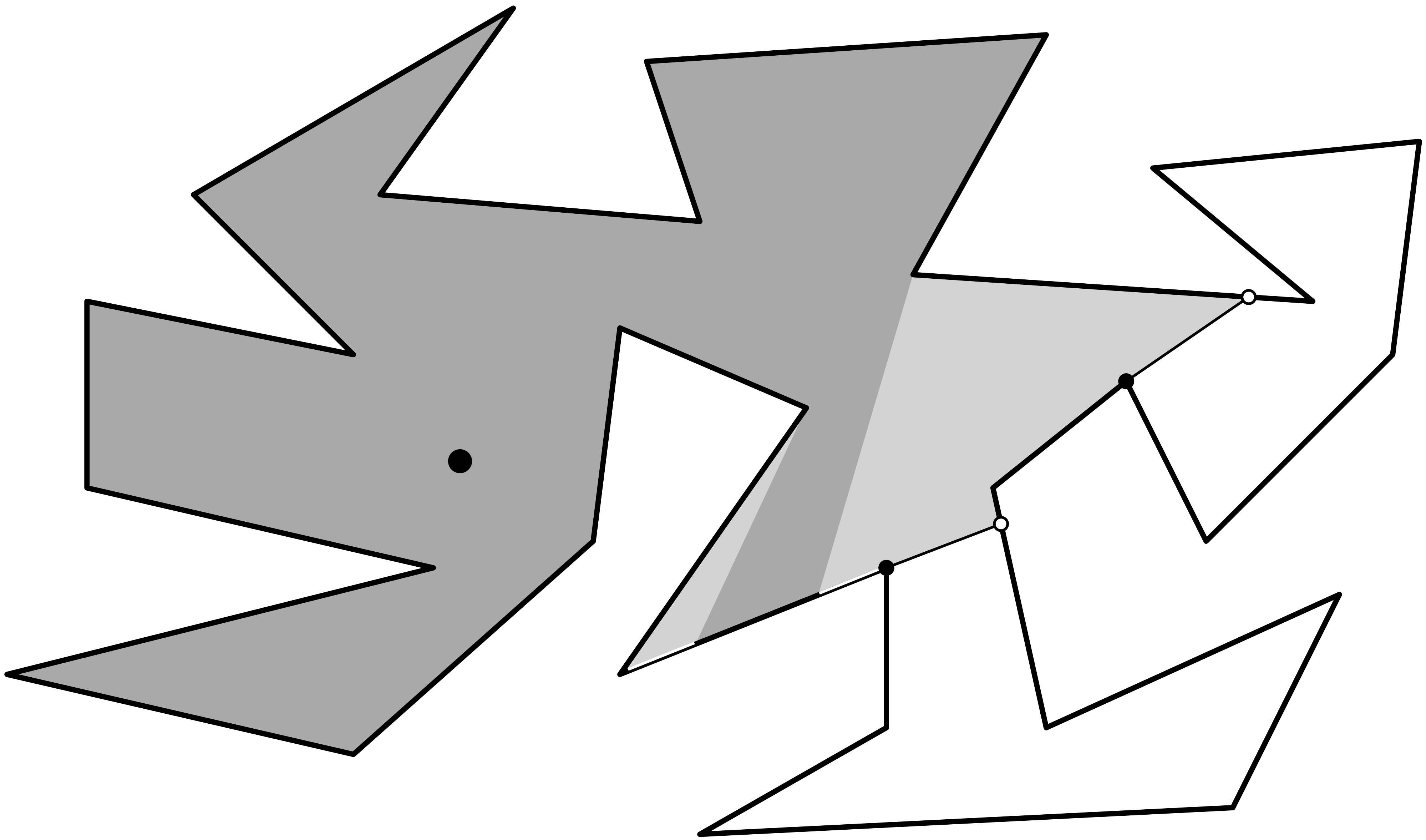


Region R_1

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

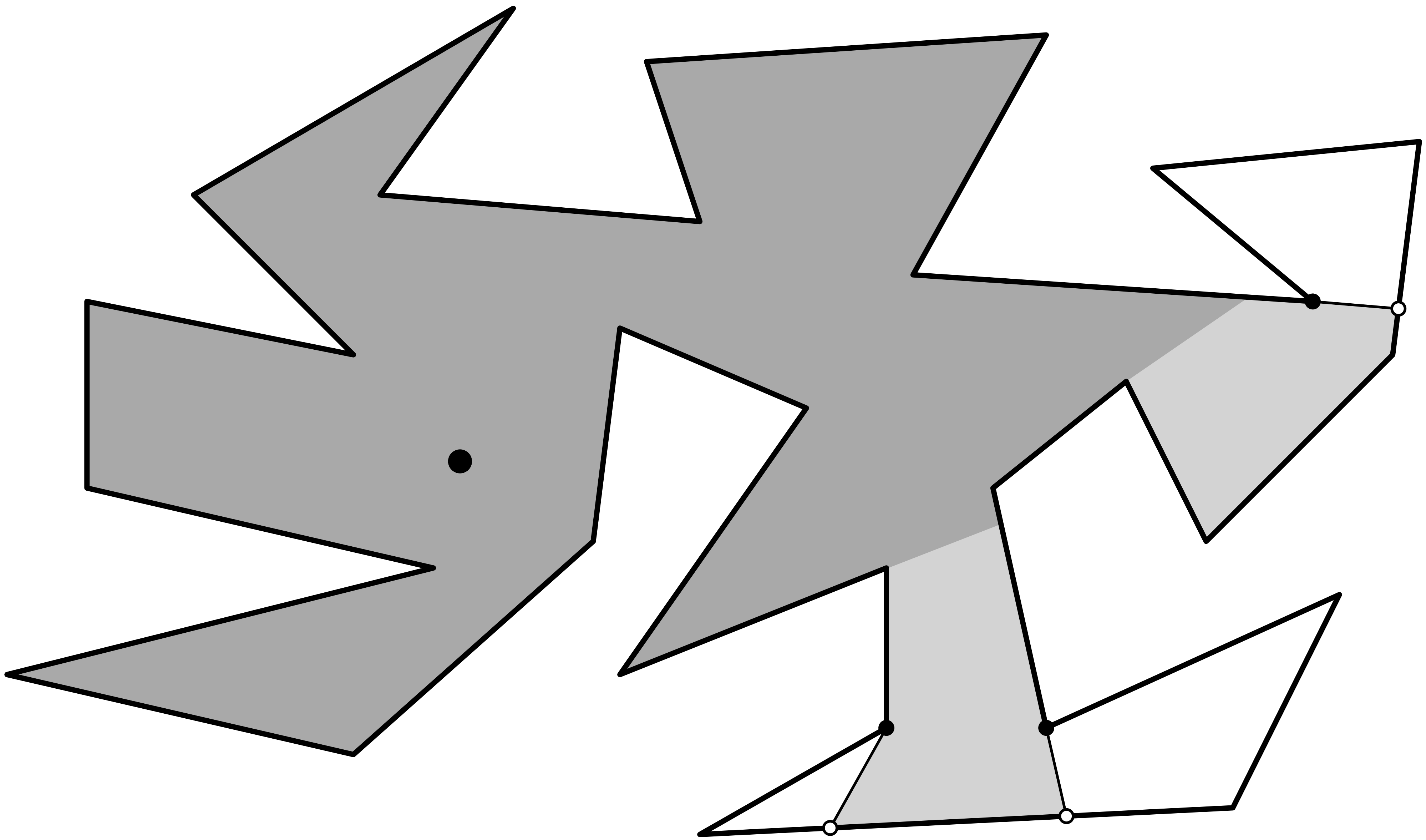


Region R_2

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.

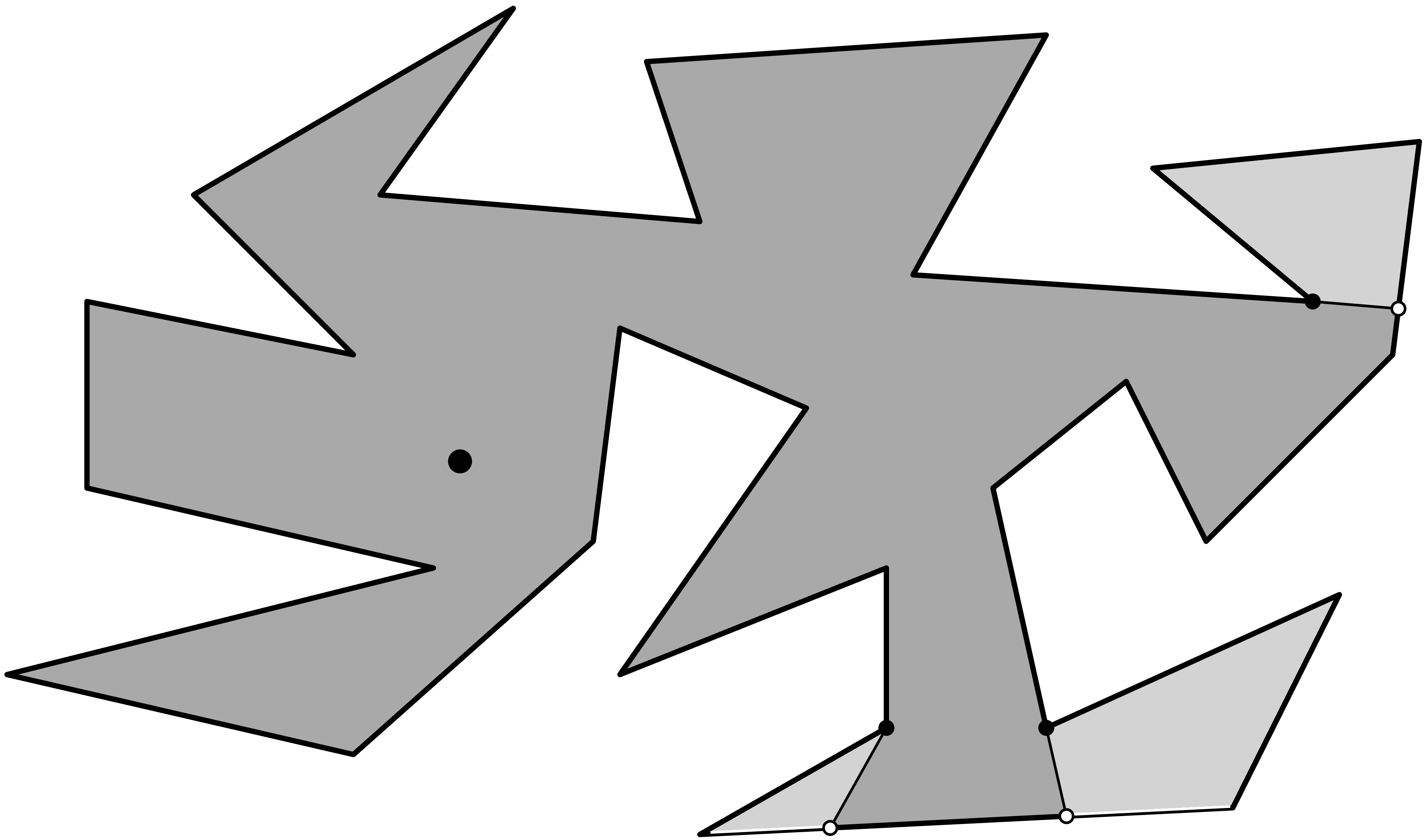


Region R_3

Proof Idea

Use a simpler subset of diffuse reflection visibility region.

Count total # new edges seen after each step.



Region R_4

Proof Idea

- Diameter proof: at every step, two new edges are seen, so $n/2$ steps until all edges (and interior) seen.
- Radius proof: there always exists a location with:
  - Saturated windows to regions of $\leq n/2$ sides.
  - And unsaturated windows to multiple regions of $\leq n/2$ *total* sides.
  - So enough progress (4 sides per reflection) is always made.
  - Many more details than this…

Conclusion

- We prove that every simple n-gon has a point that illuminates the entire polygon after at most (n-2)/4 diffuse reflections.

- We also give O(n*log(n)) algorithm to compute such a point.

- Proof uses a subset of the actual visibility region plus *saturation* of this region's boundary.

    - Among a number of other things.

Open Problems

- Extending our results to polygons with holes.
  - Also open for diameter.
- An algorithm to compute the diffuse reflection radius of a given polygon.
  - Also open for diameter.
- A $o(n^{10})$ algorithm for shortest diffuse reflection path between a pair of points.

Thank you.

Diffuse Reflection Radius in a Simple Polygon

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