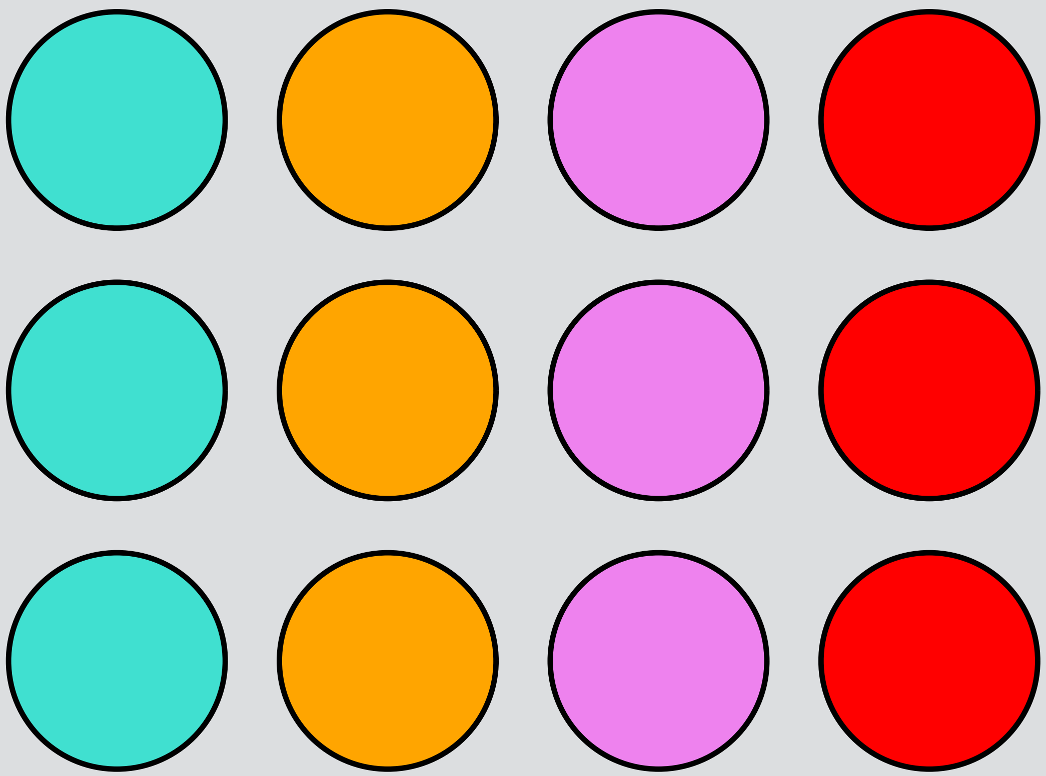
One Tile To Rule Them All: Simulating Any Tile Assembly System with a Single Universal Tile

Erik D. Demaine, Martin L. Demaine, Sándor P. Fekete,
Matthew J. Patitz, Robert T. Schweller,
Andrew Winslow, Damien Woods

Self-Assembly

Self-Assembly

Simple particles coalescing into complex superstructures.



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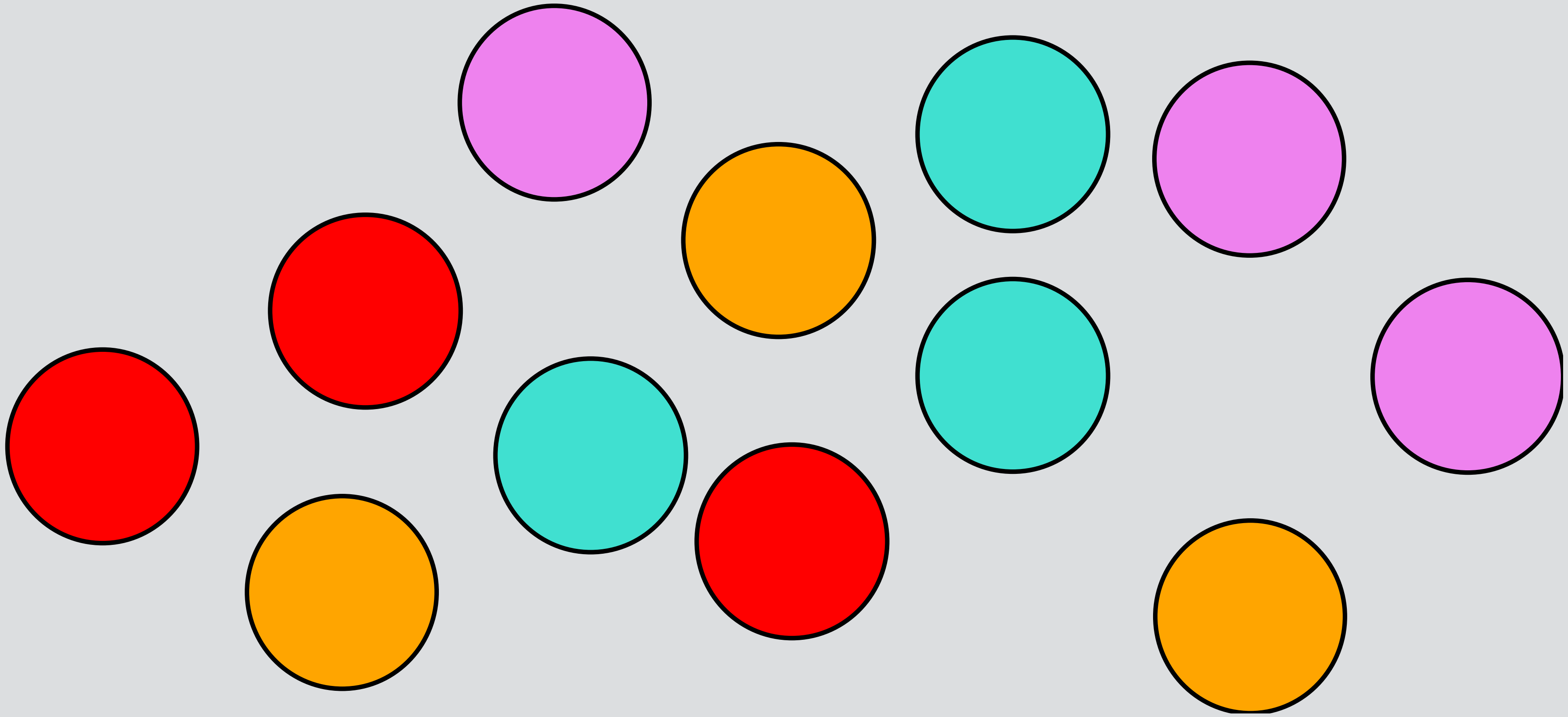

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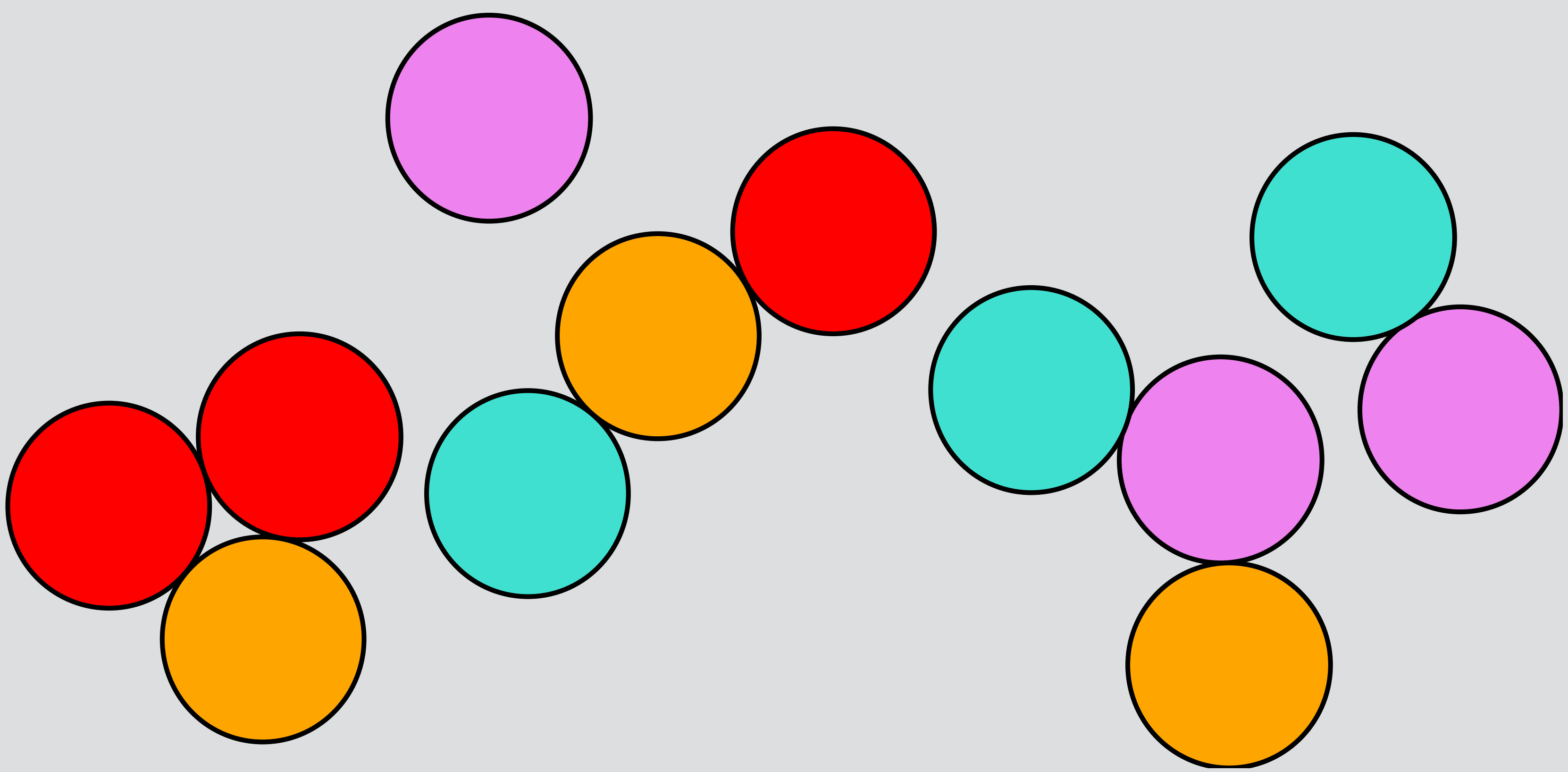

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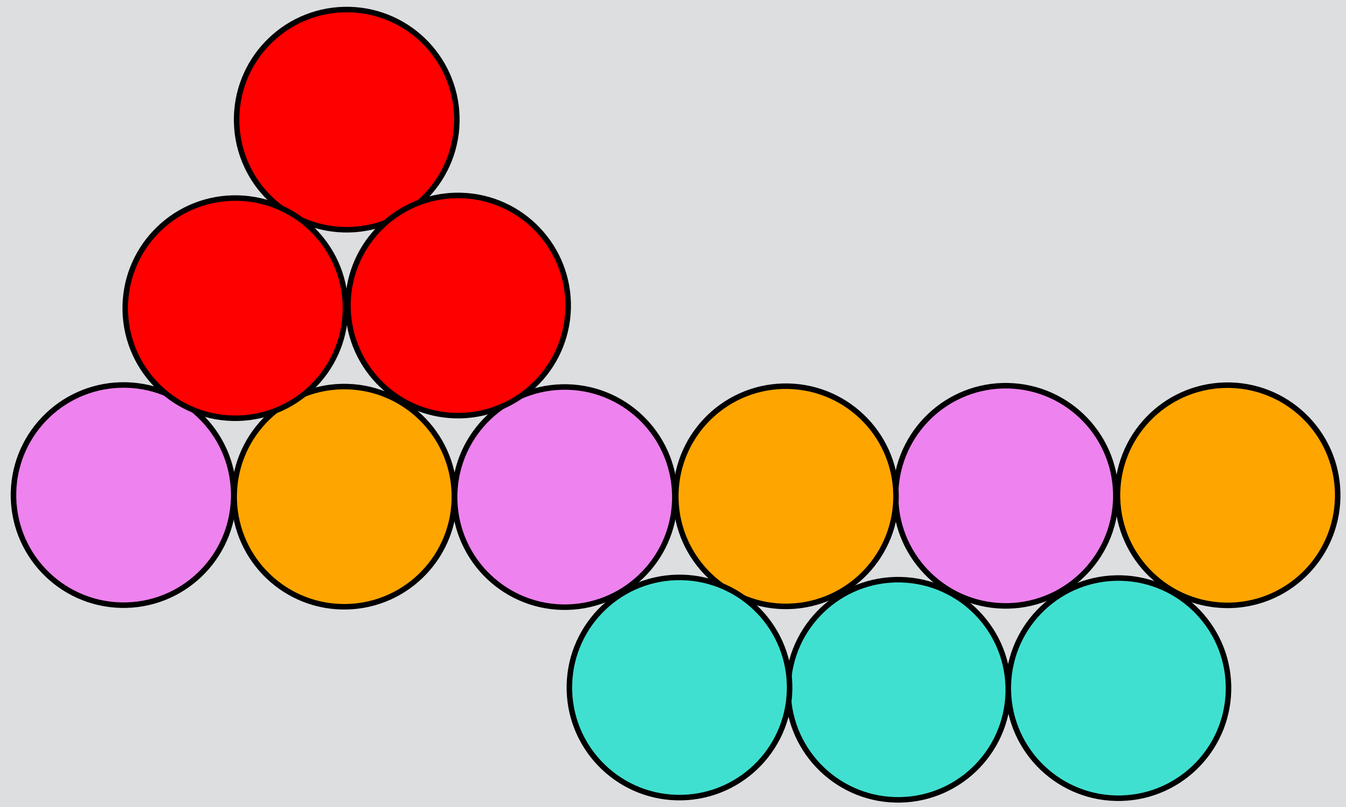
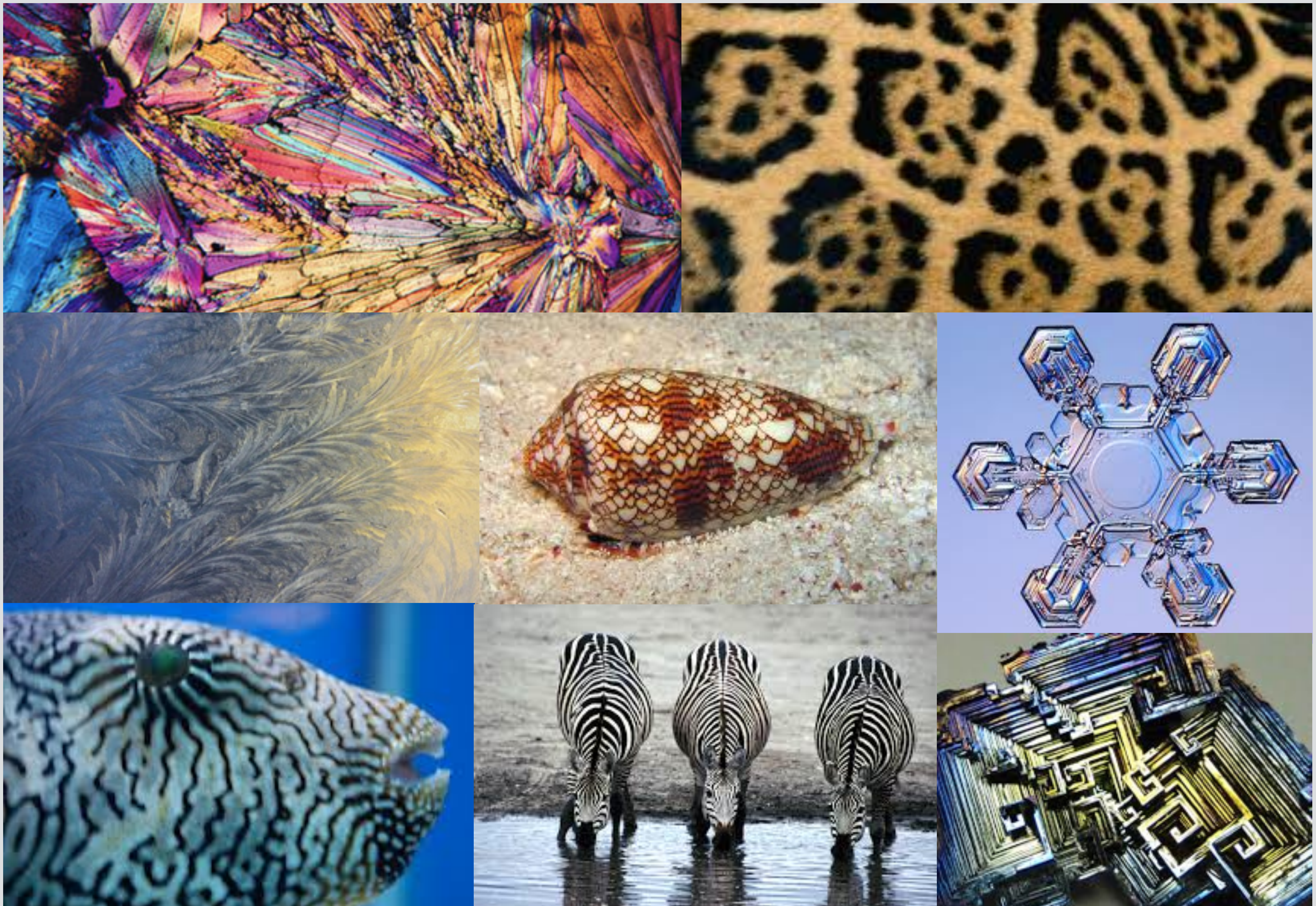
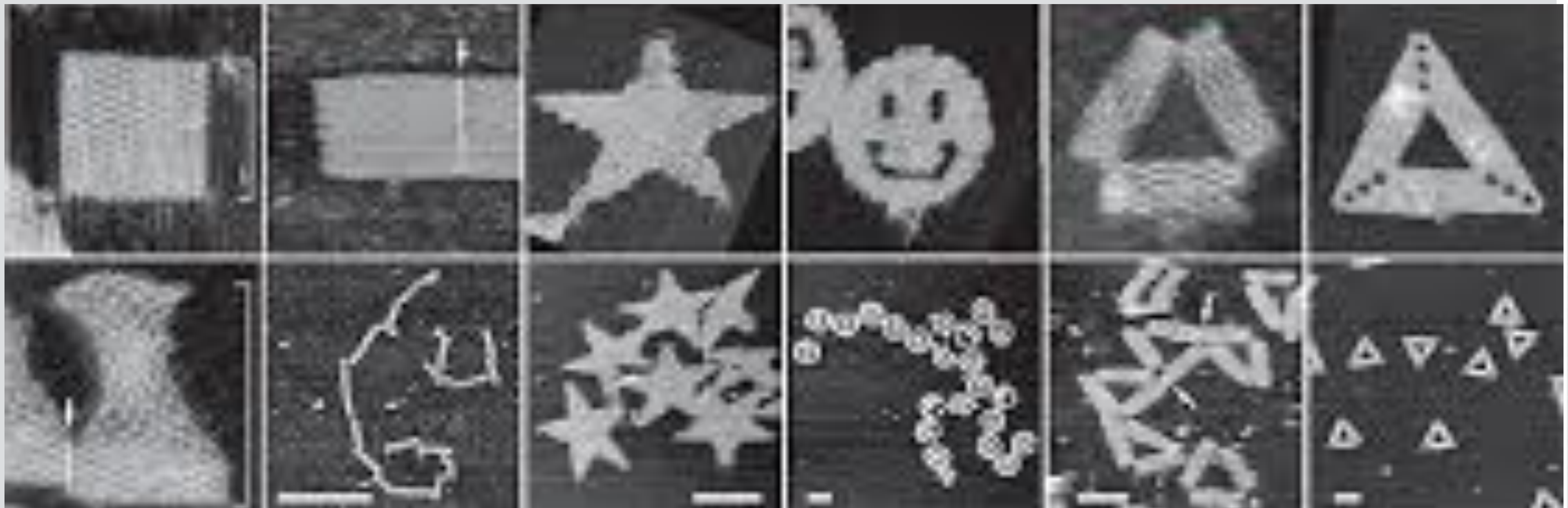
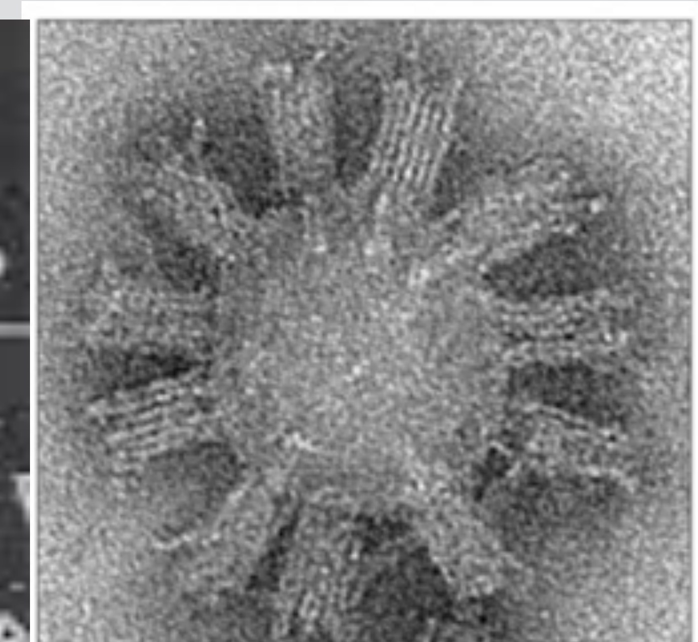


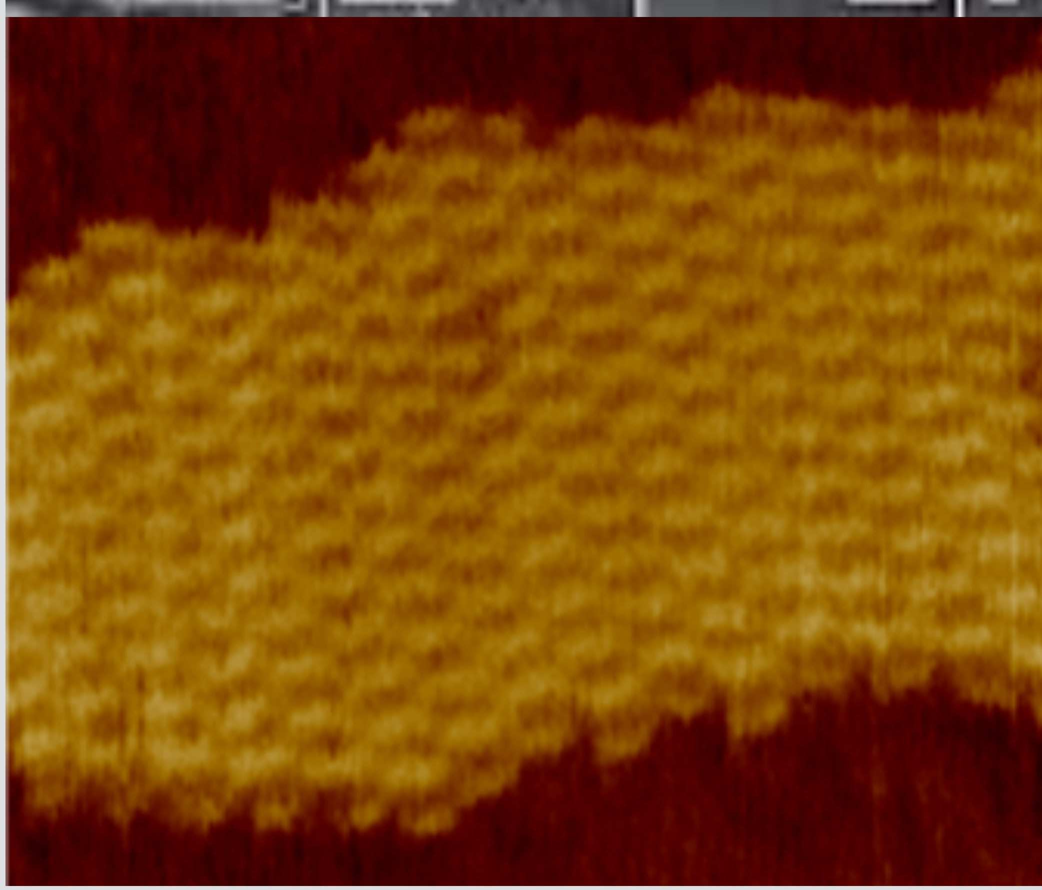
Natural self-assembly

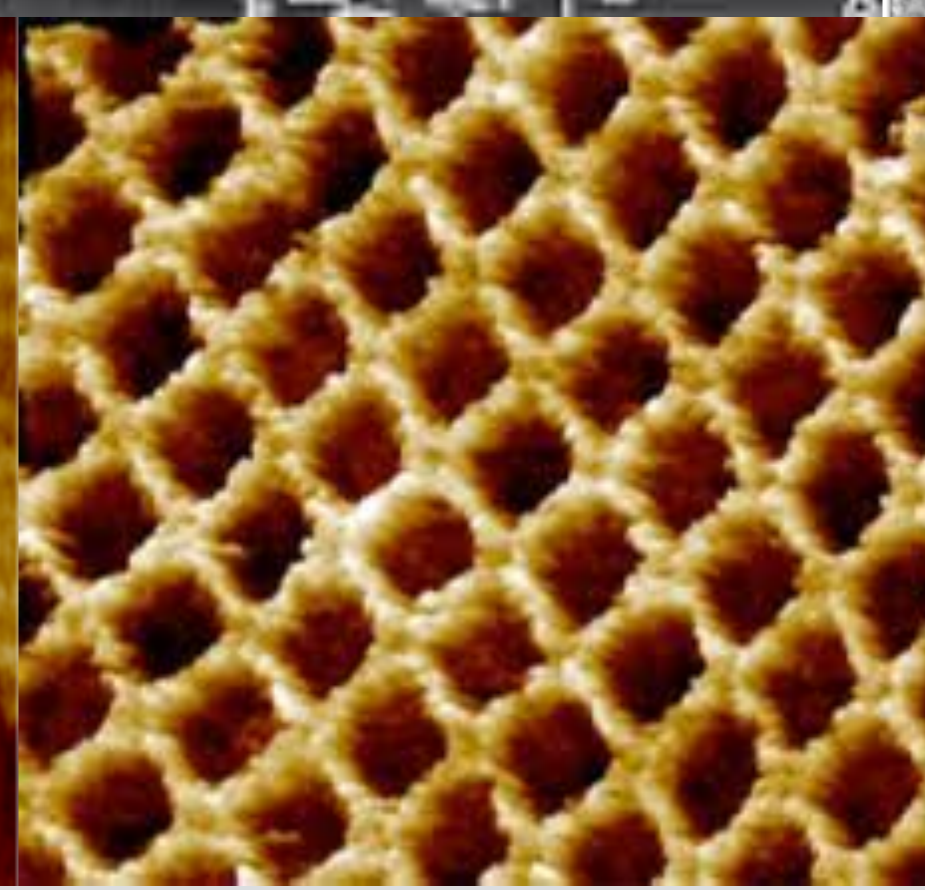


Synthetic Self-Assembly with DNA









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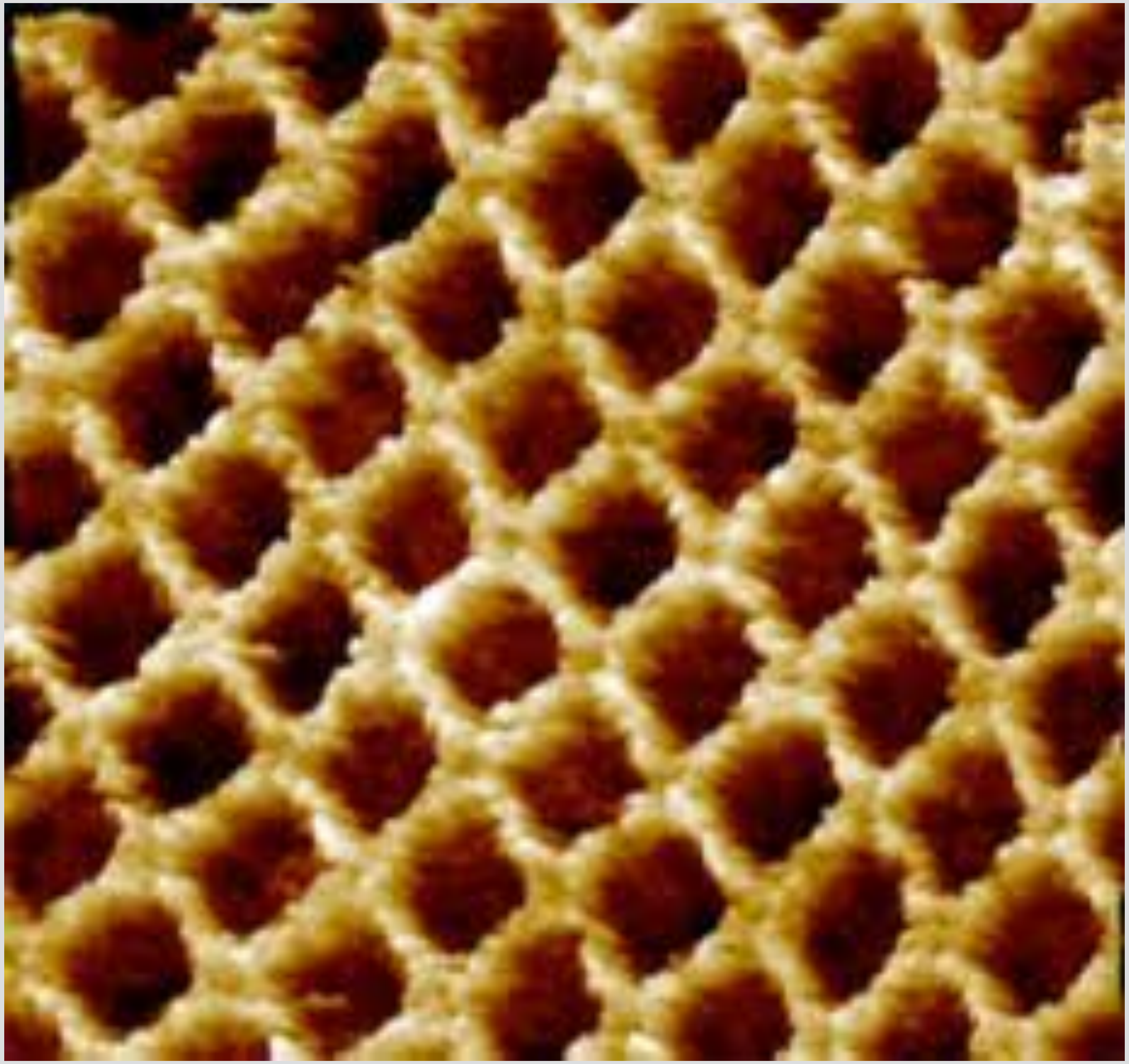

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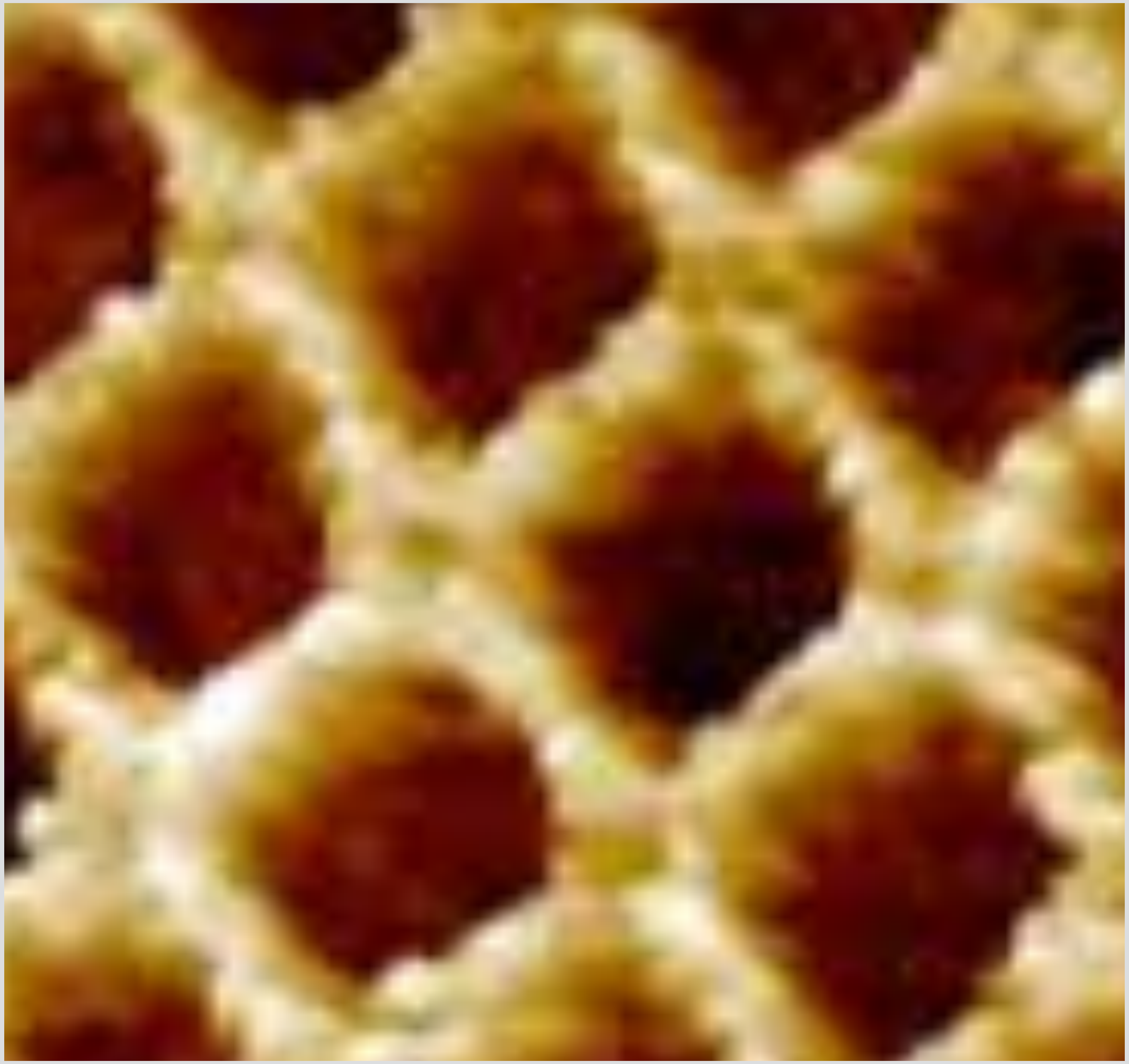

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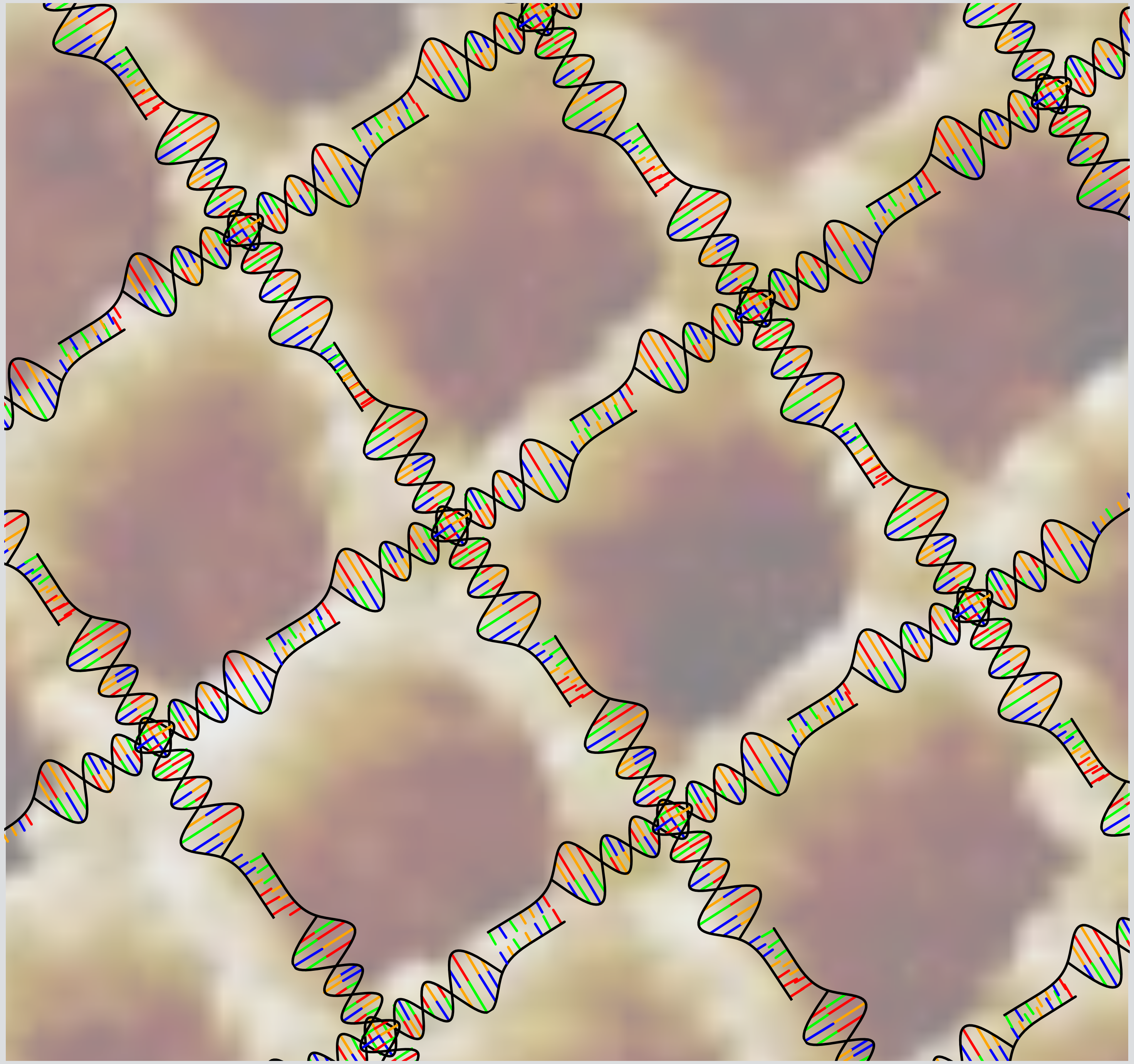
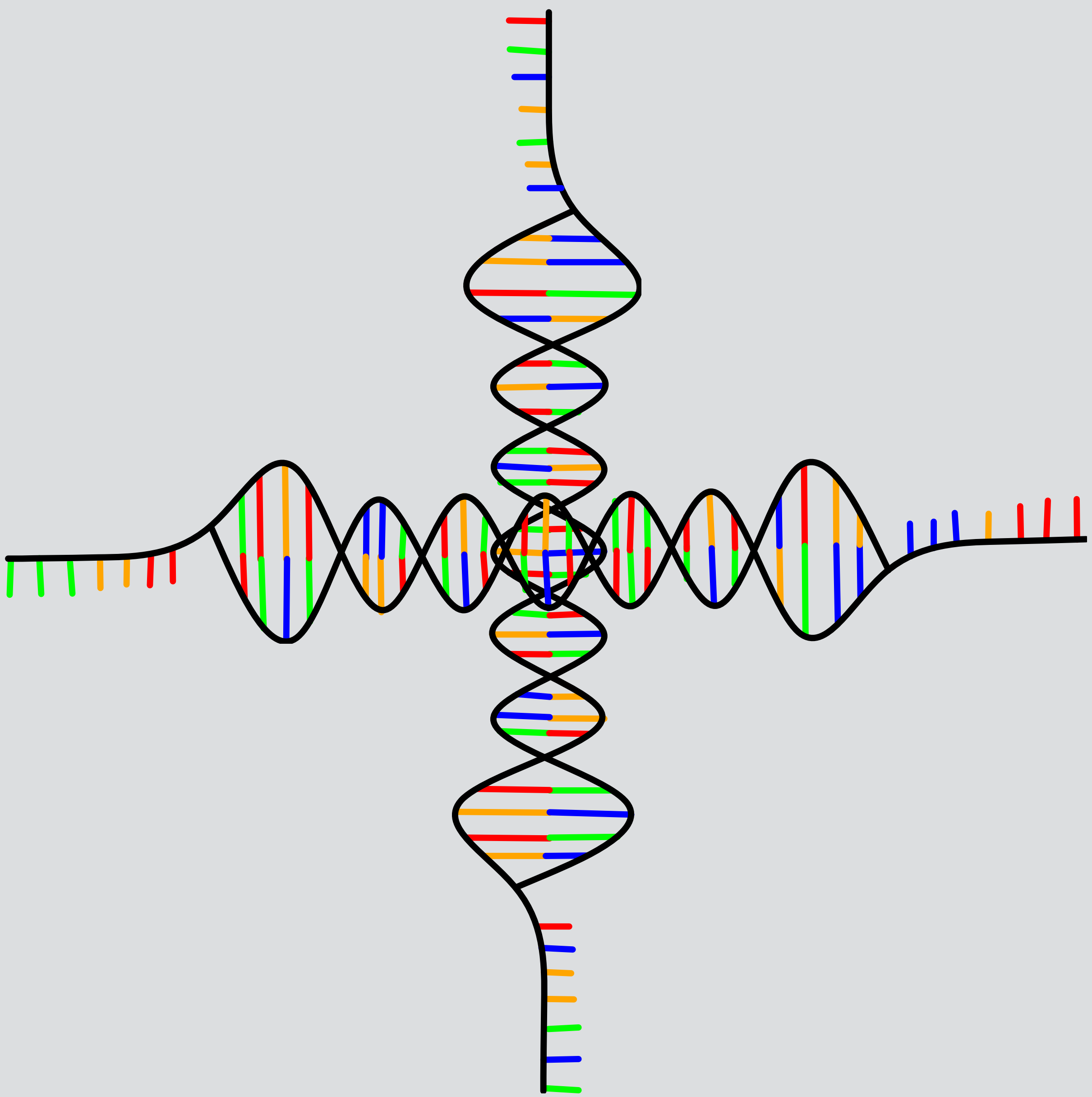


DNA Tile Self-Assembly



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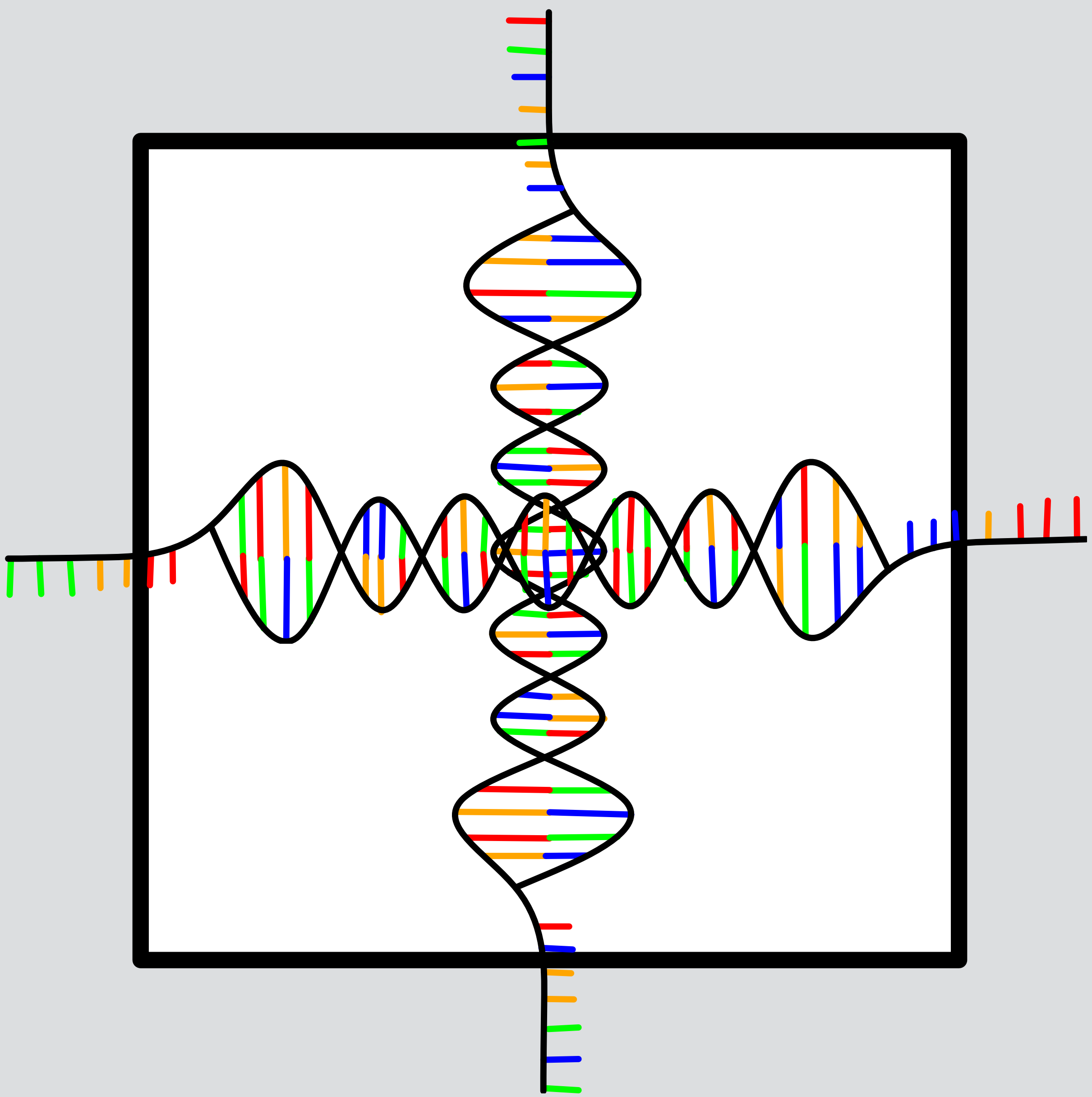
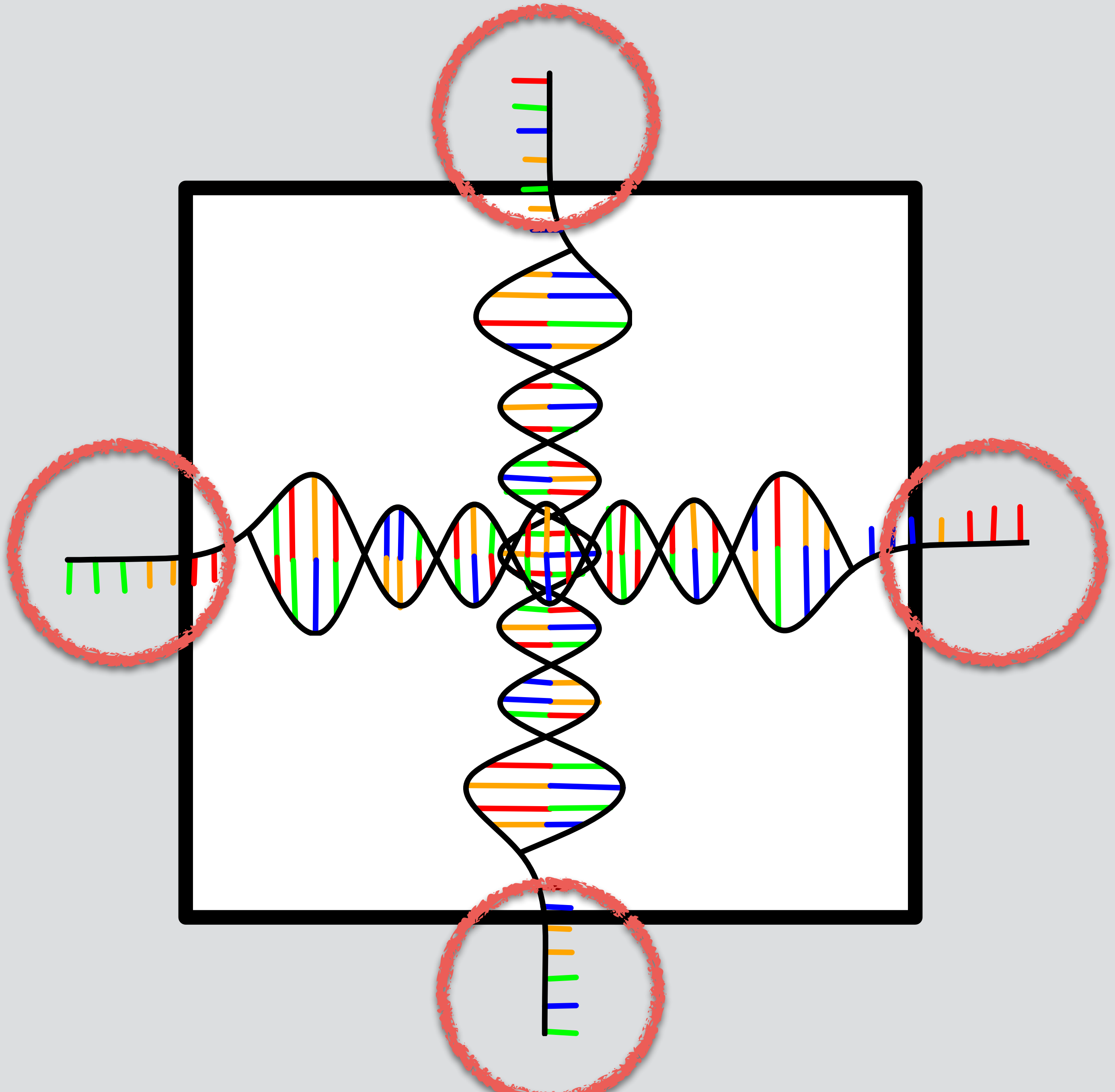
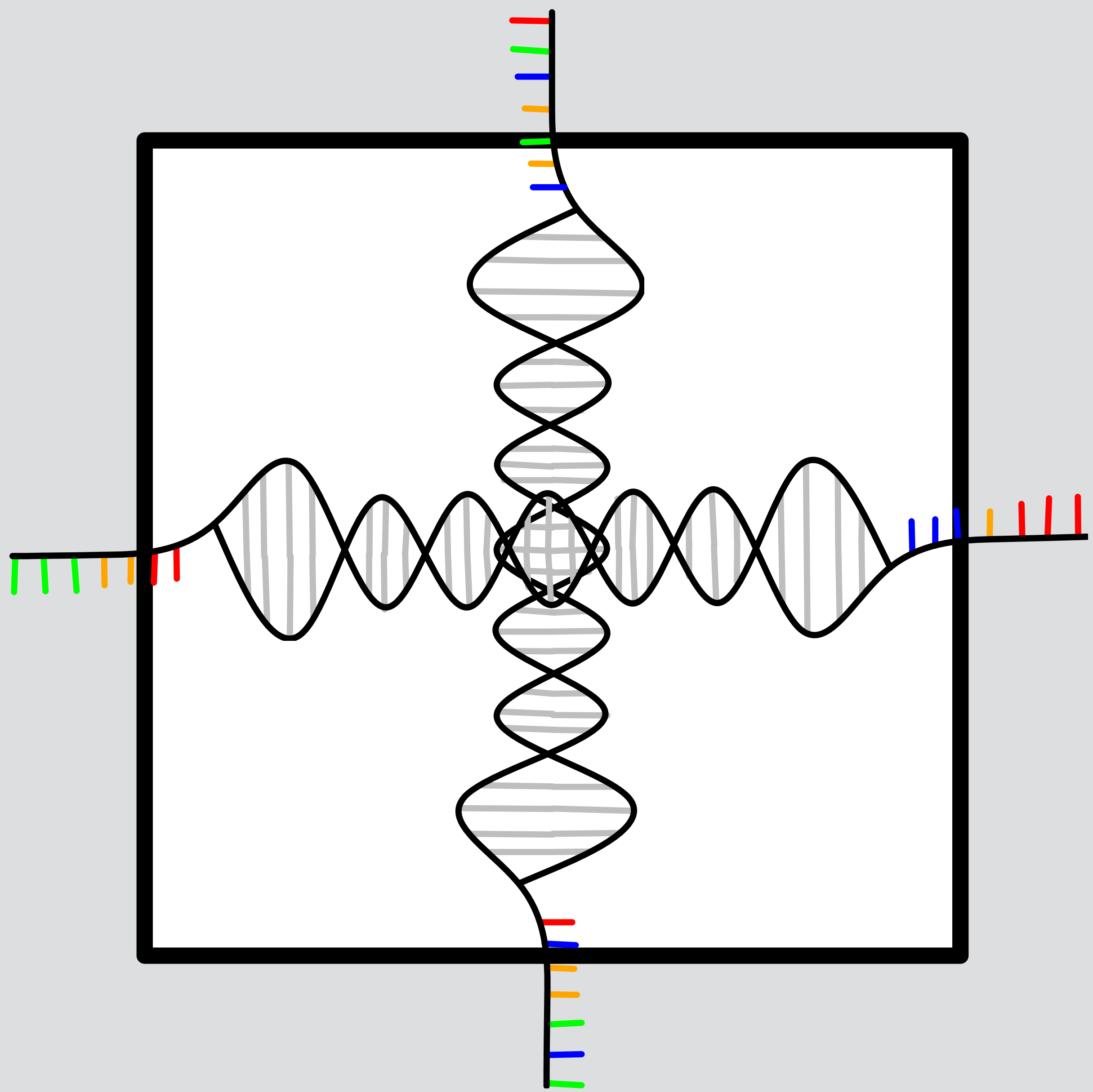


DNA Tile Self-Assembly



Sticky ends

DNA Tile Self-Assembly



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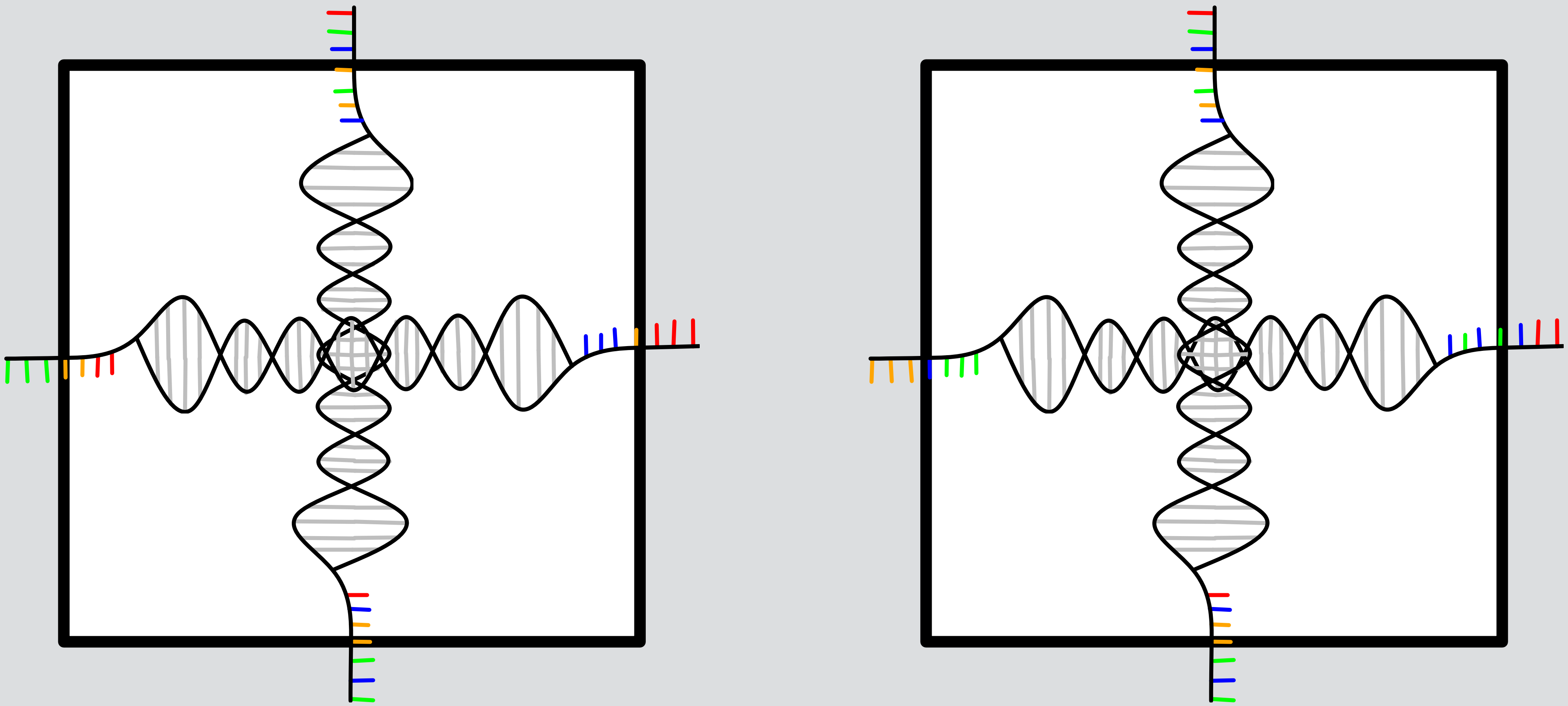

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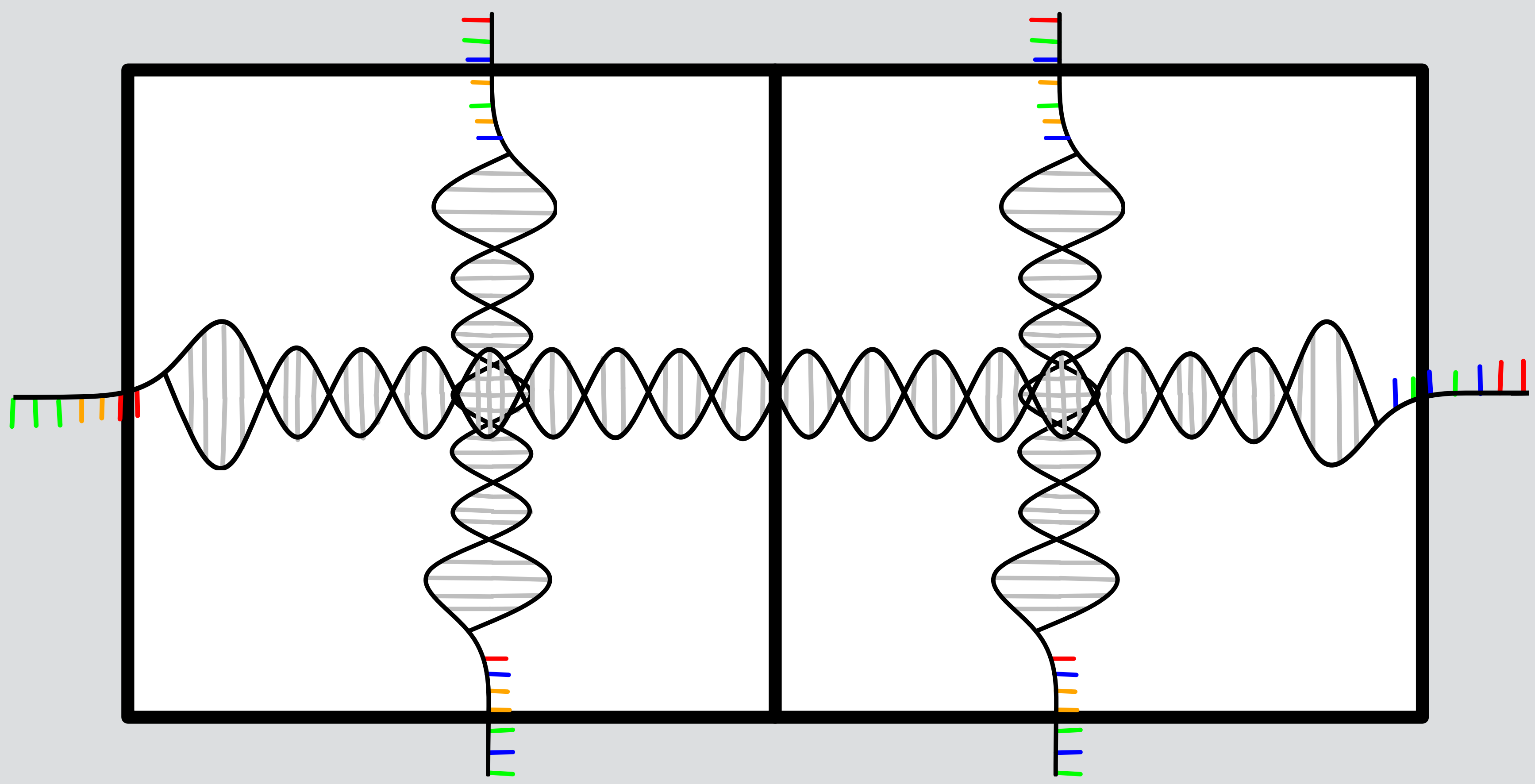

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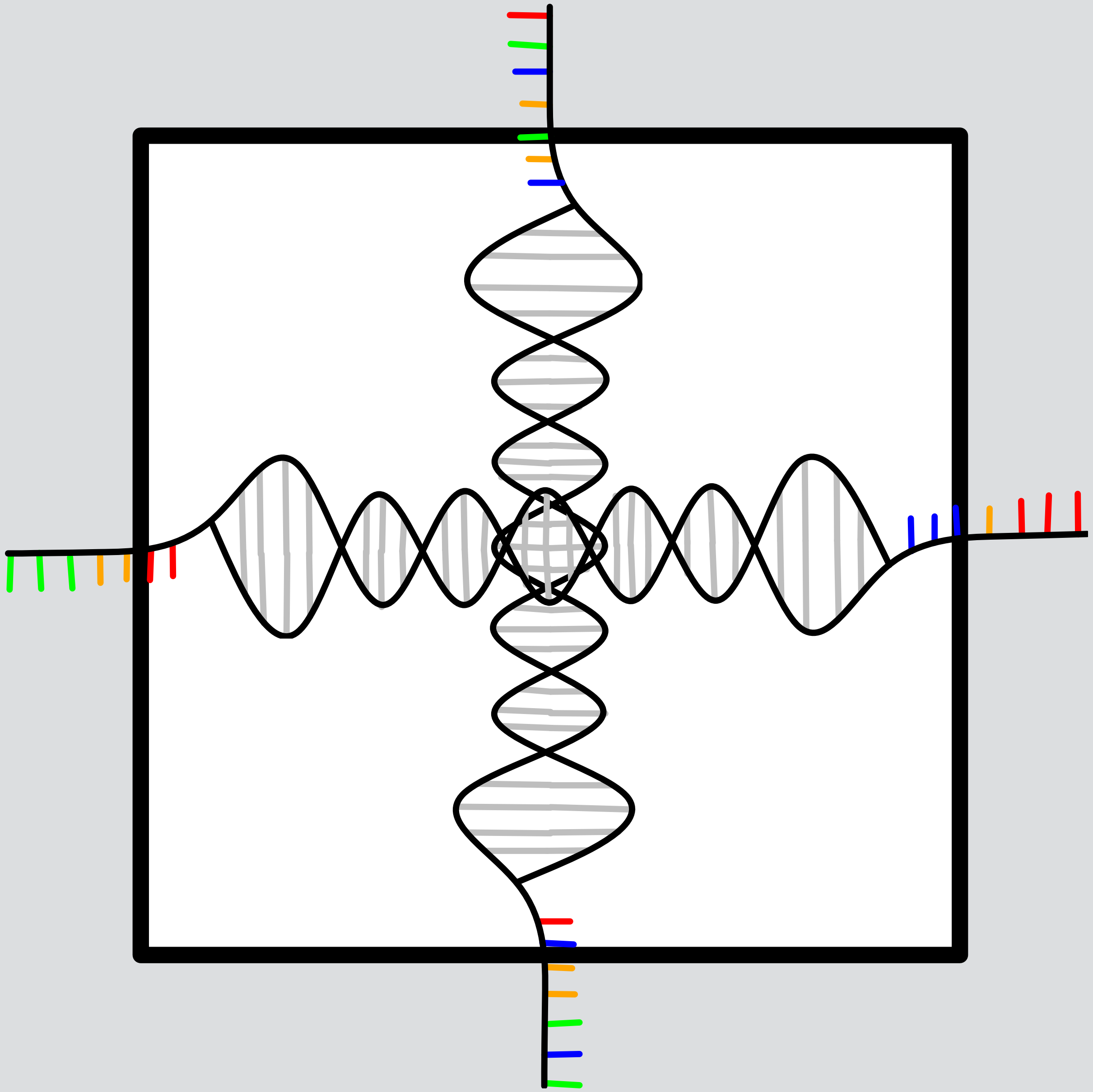
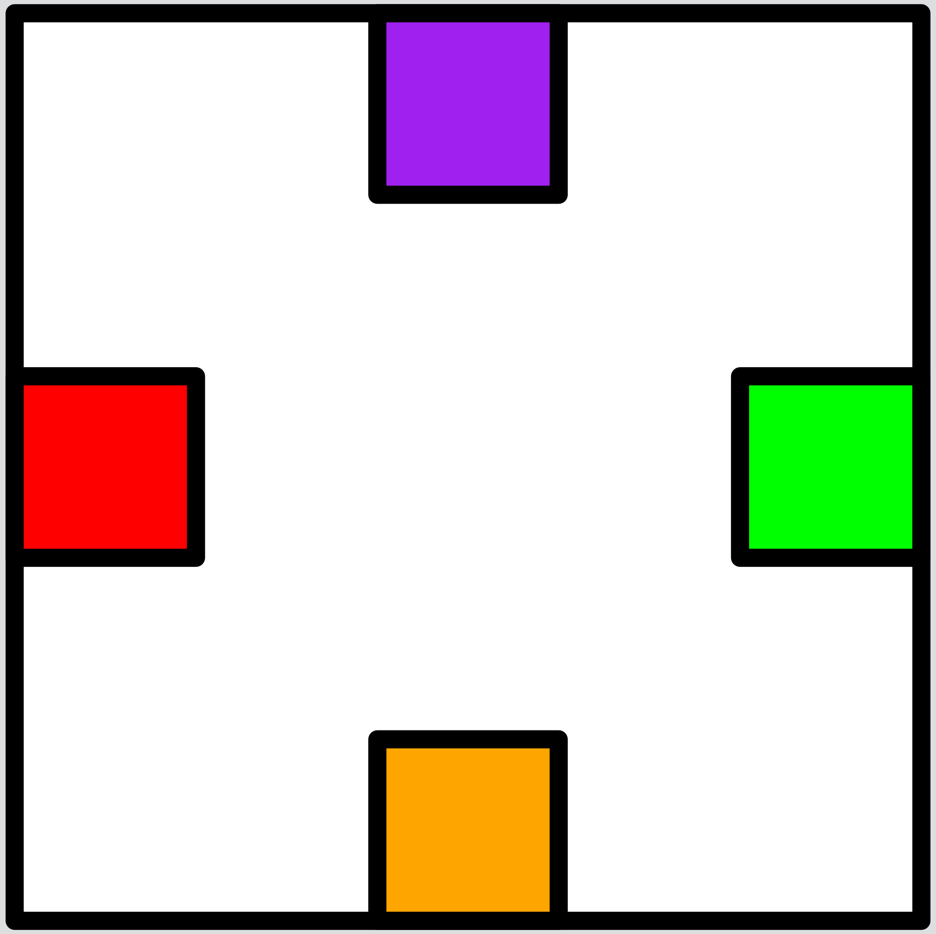


Tile Self-Assembly



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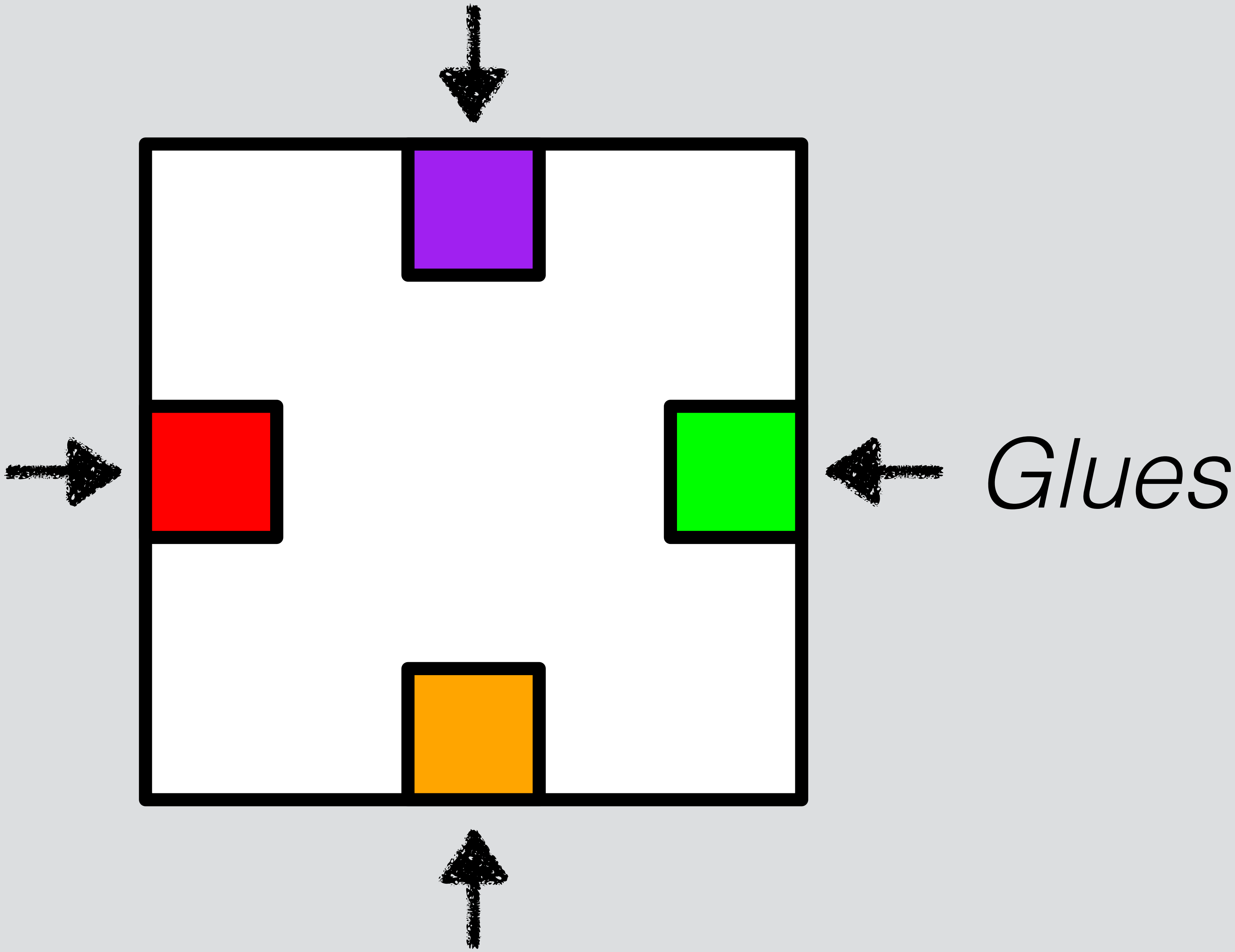

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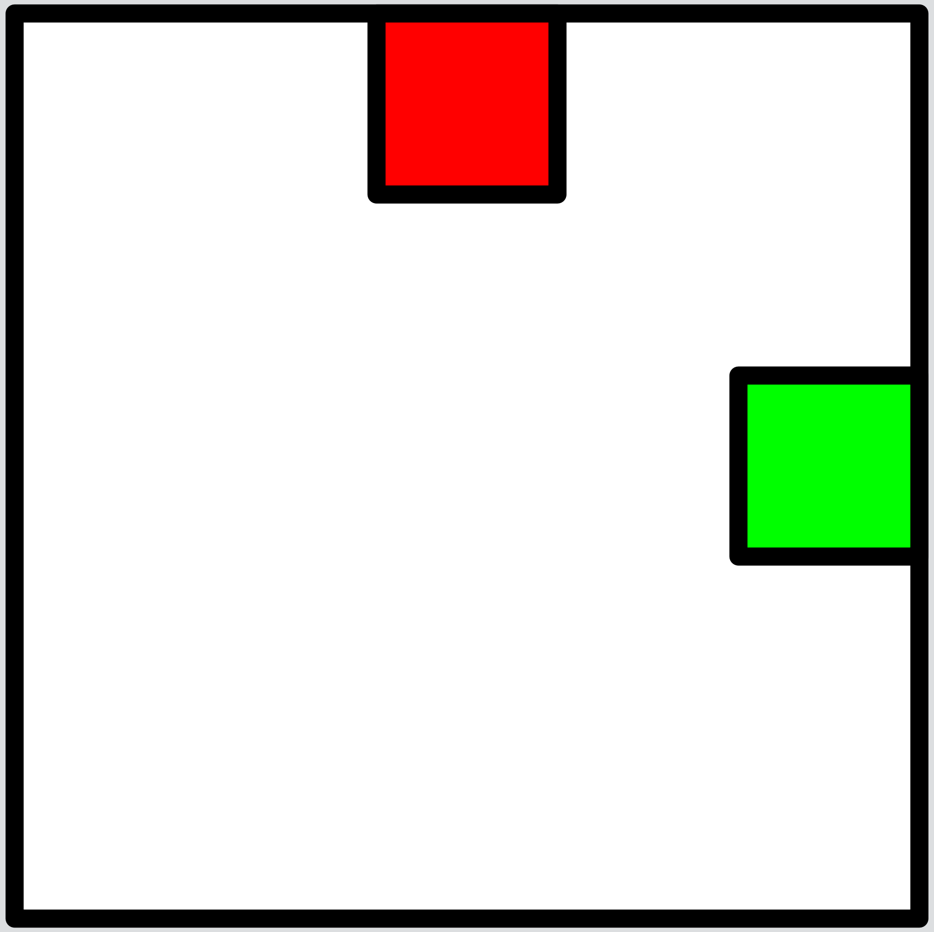

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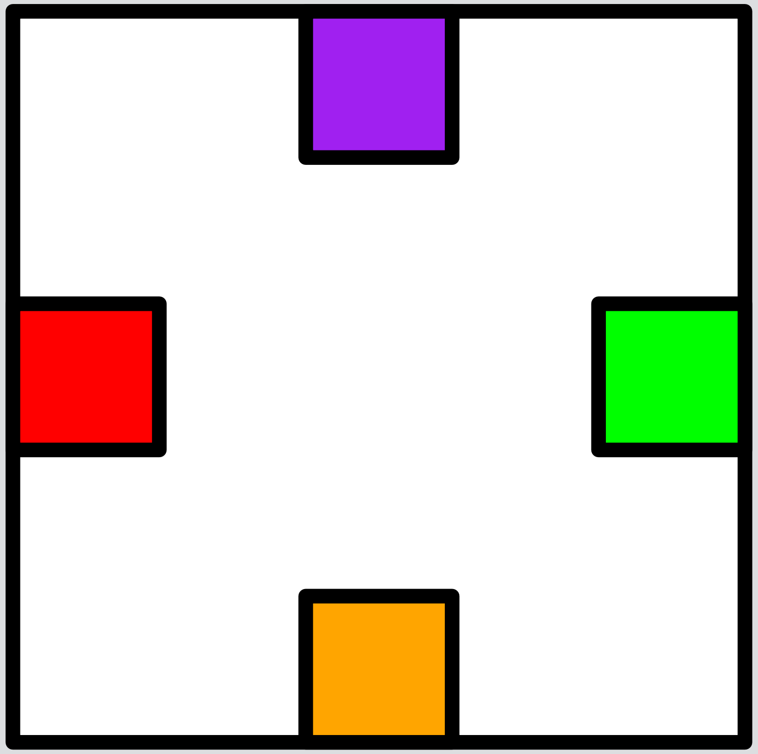

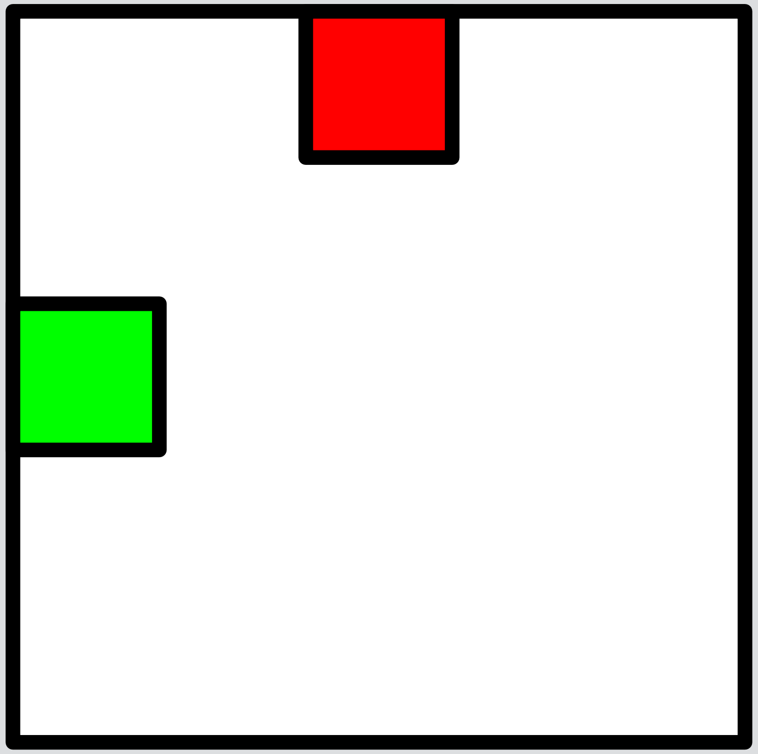

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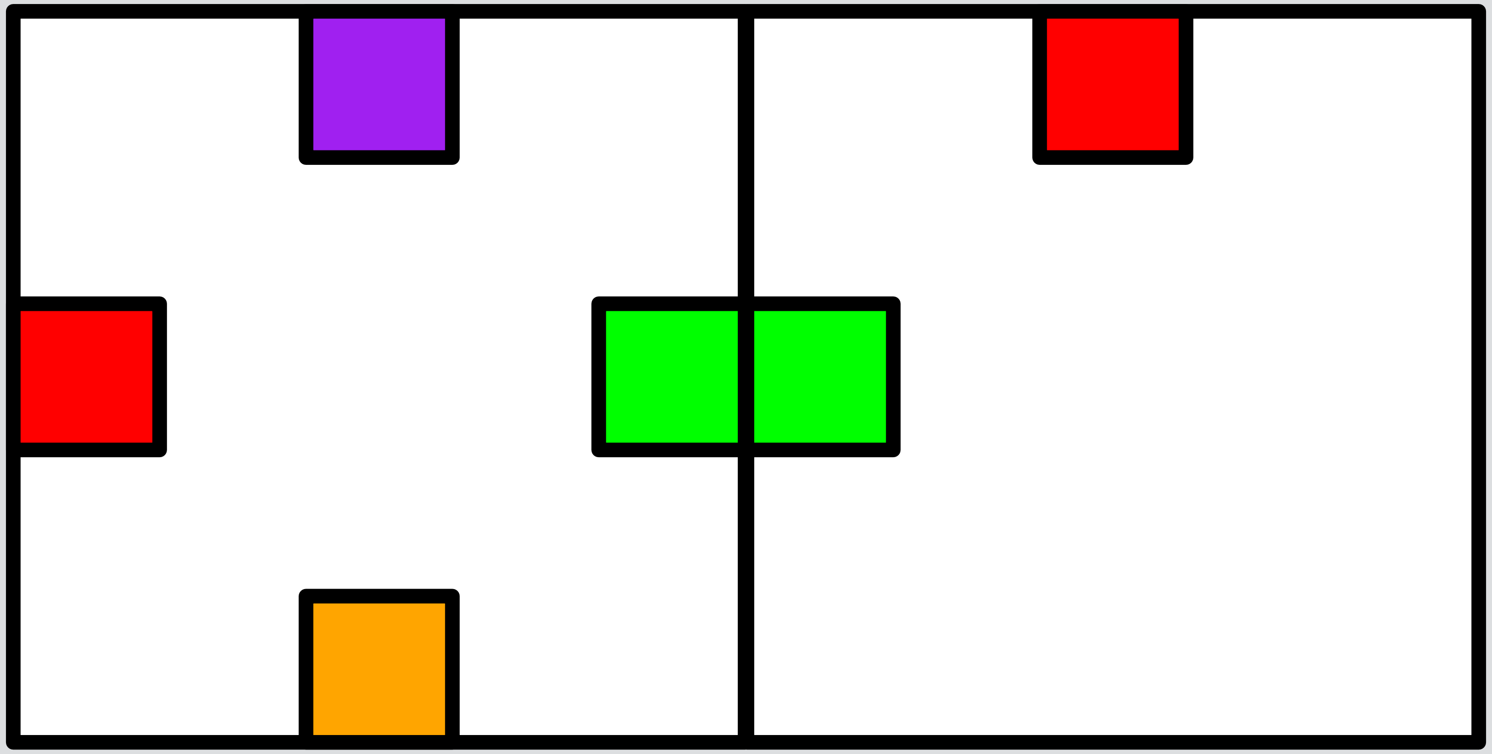

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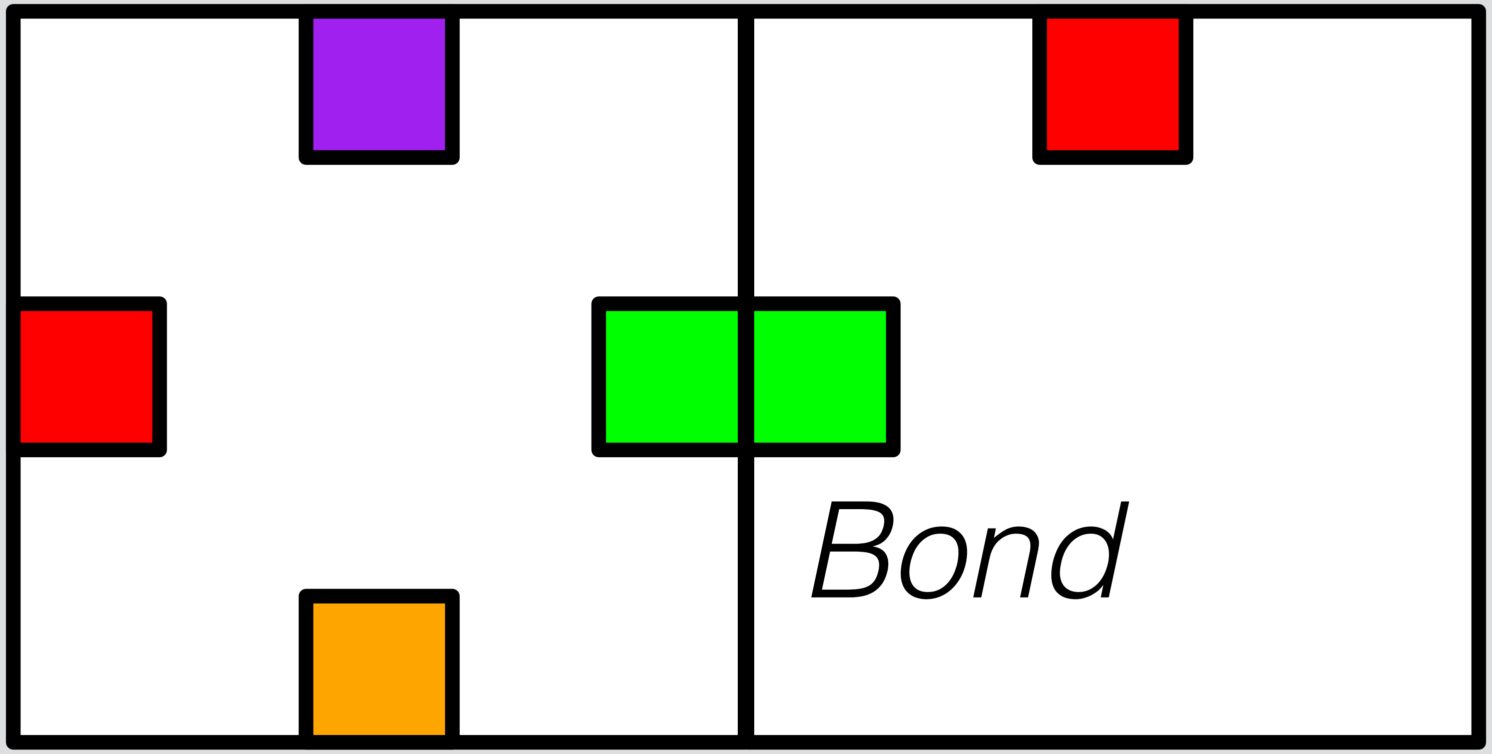

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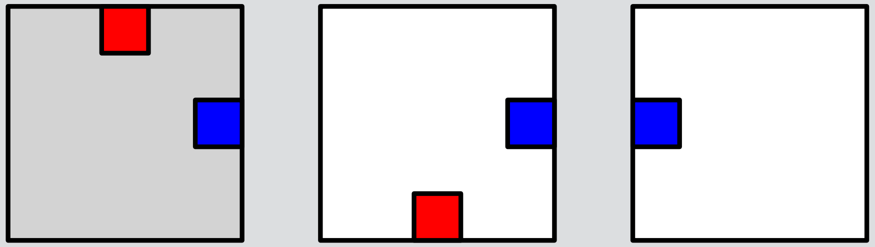

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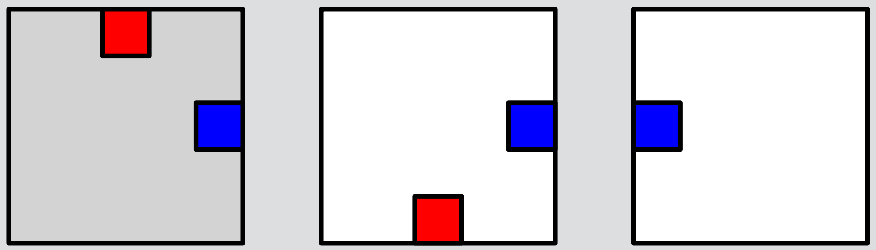
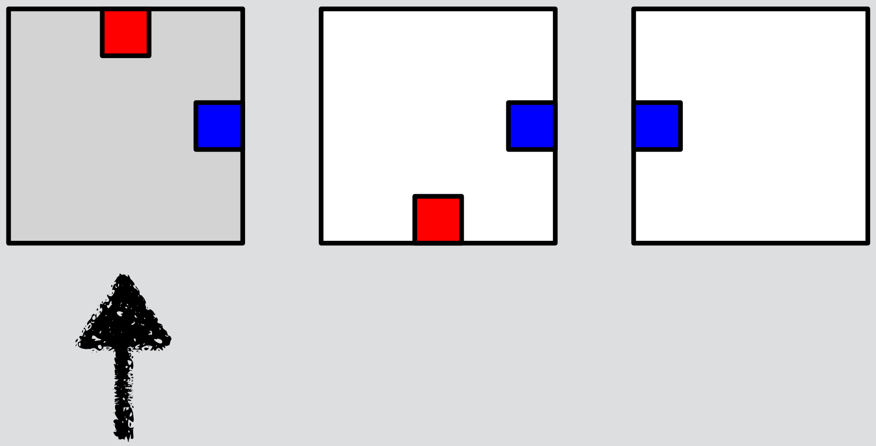


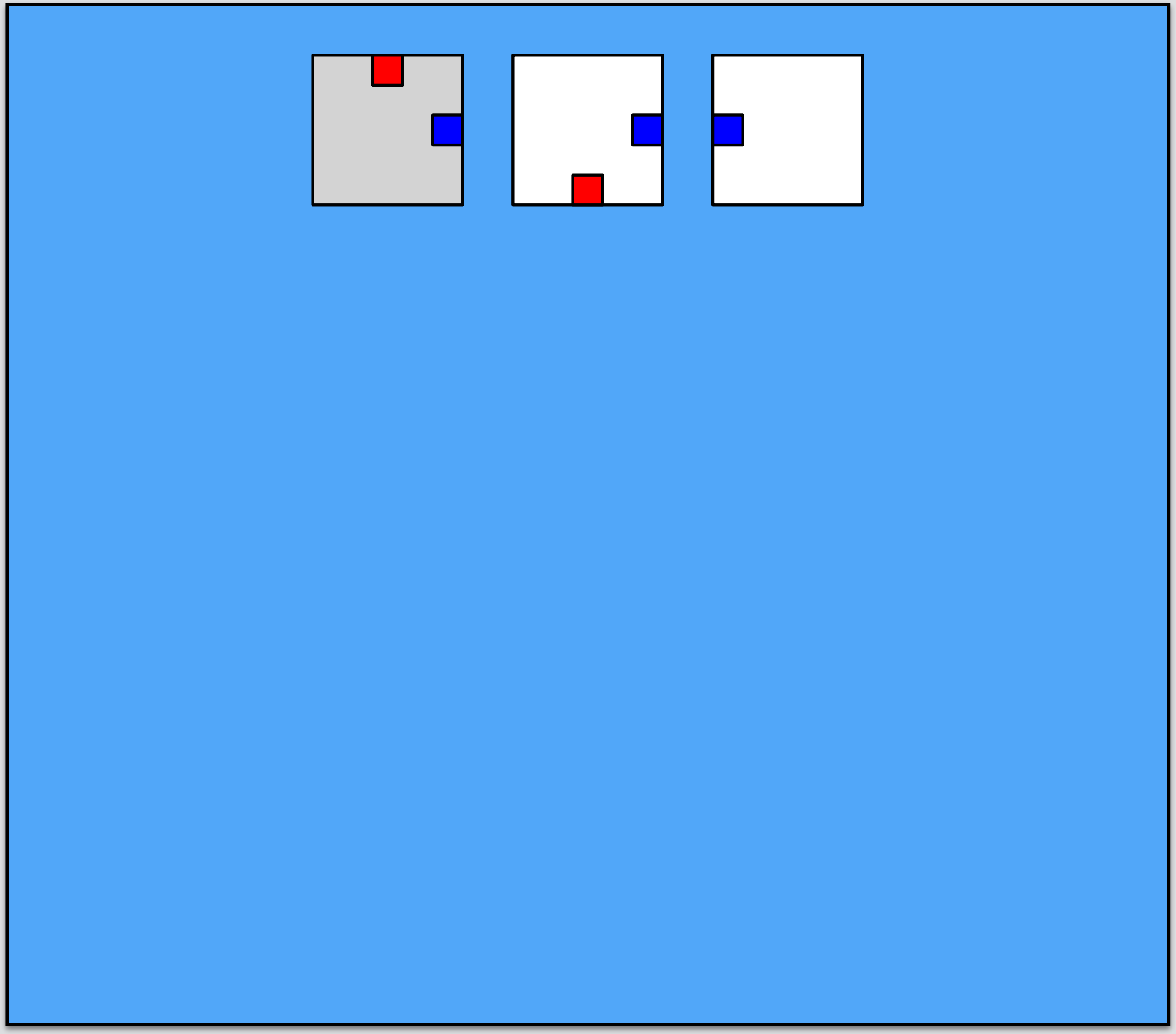
Tile set

Tile Self-Assembly

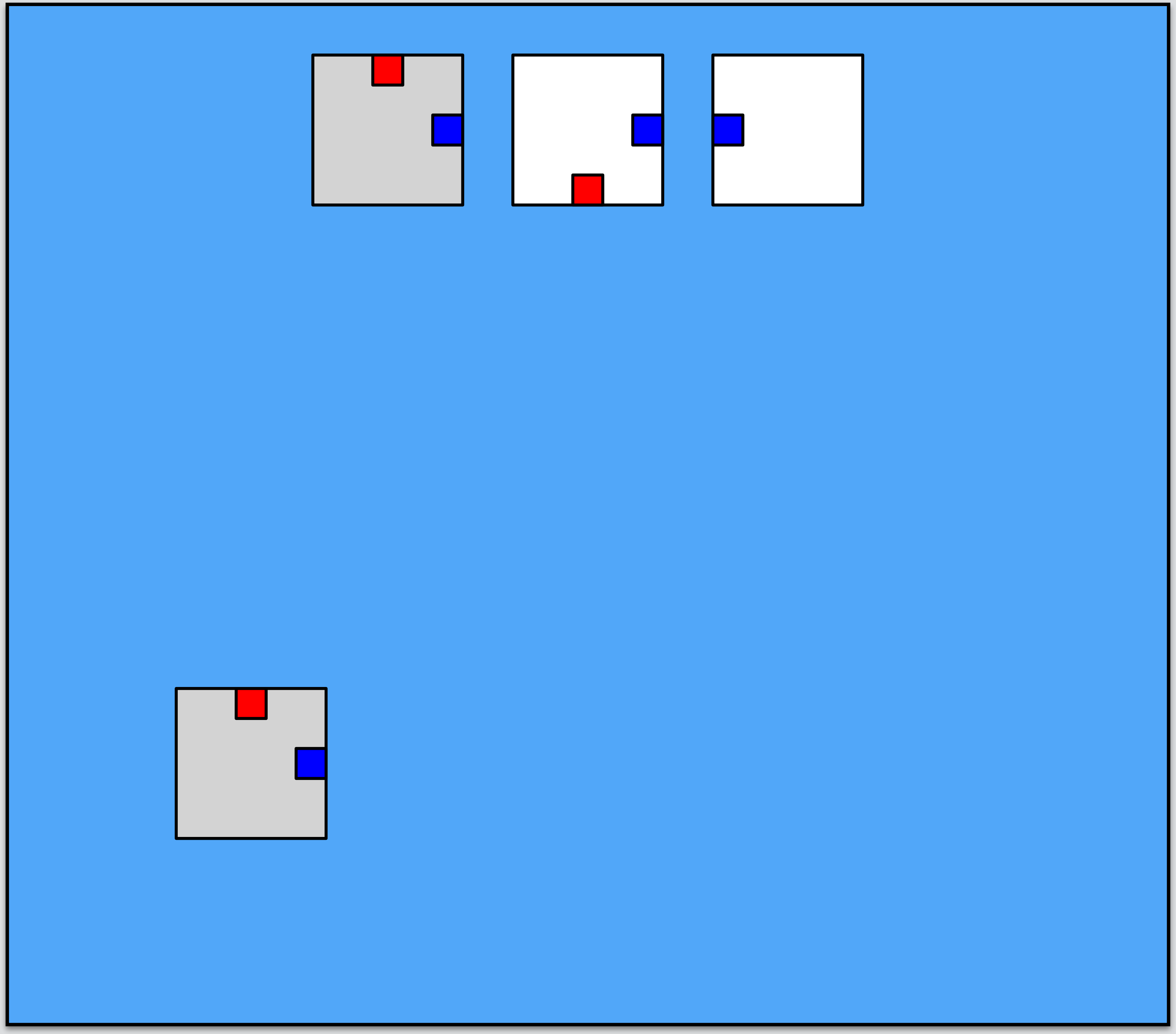


Seed tile

Tile Self-Assembly



Tile Self-Assembly



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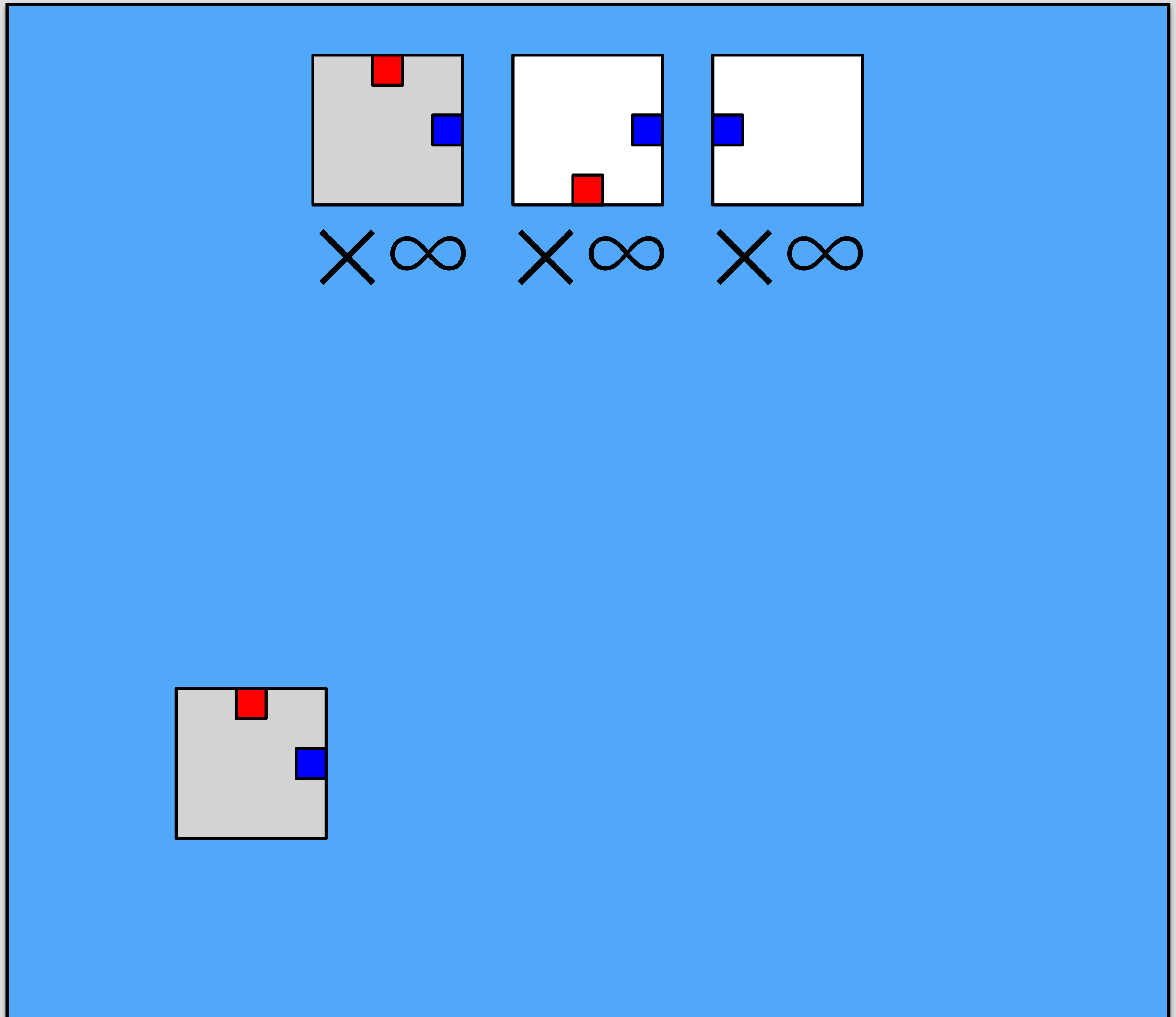

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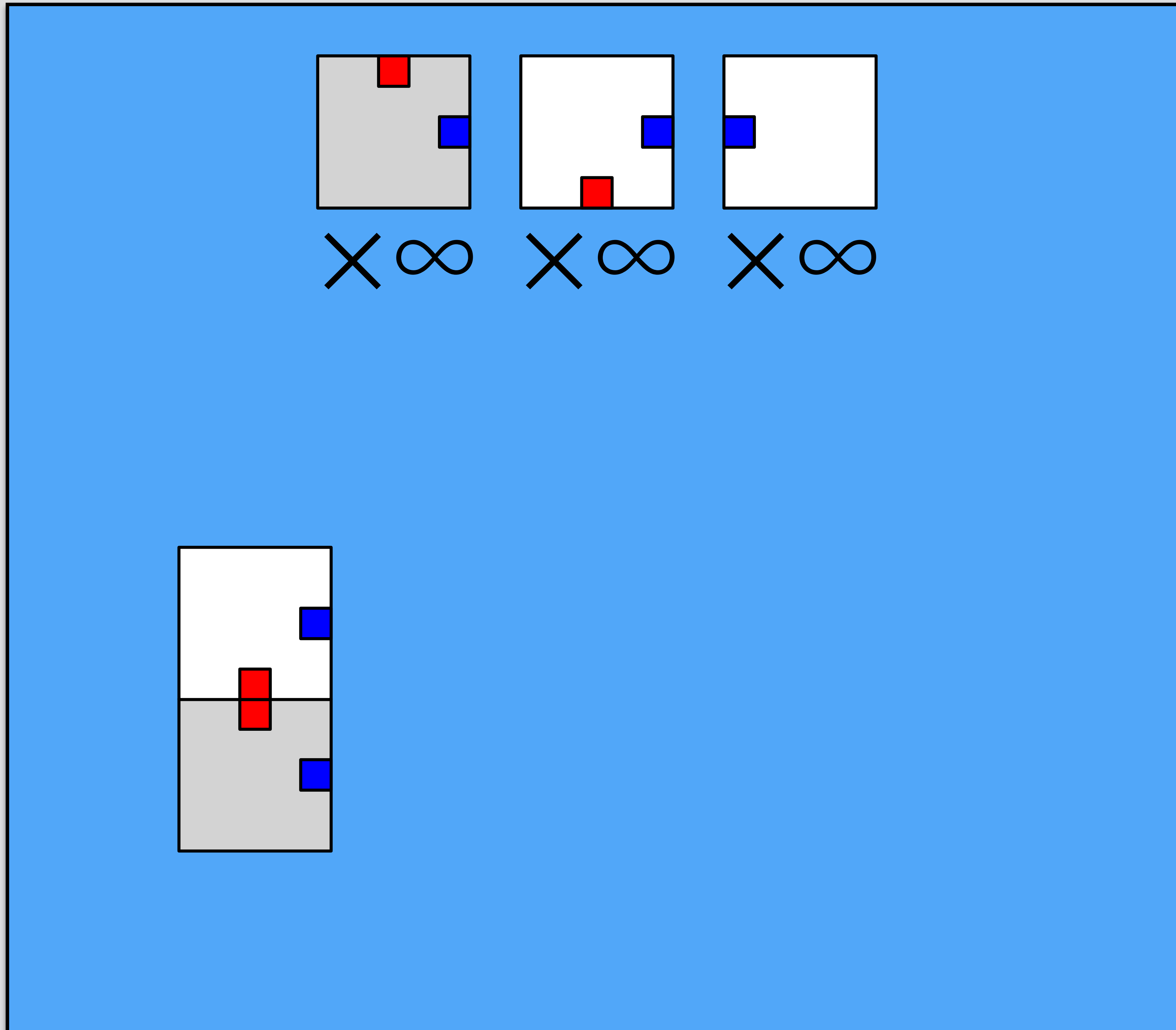

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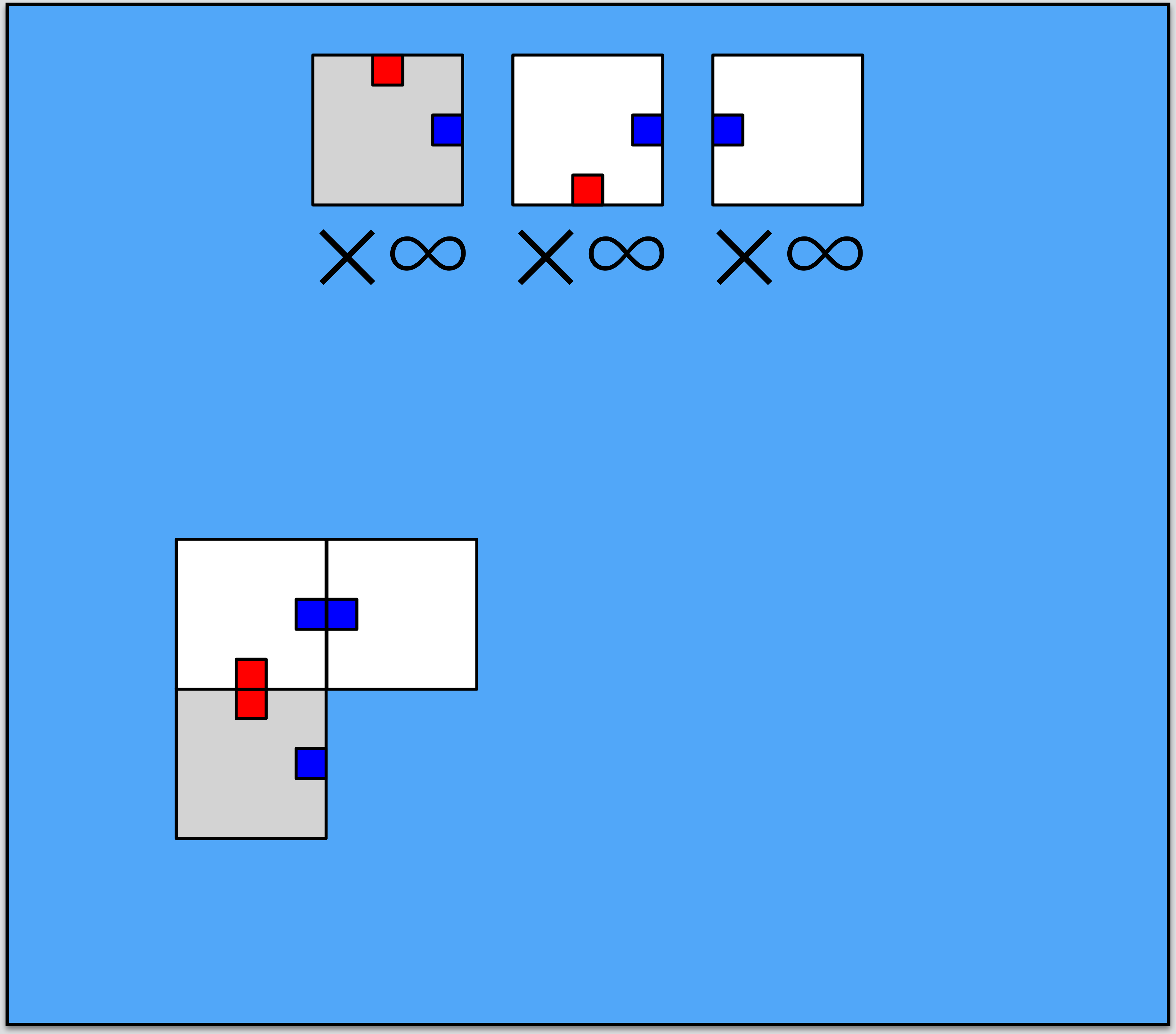

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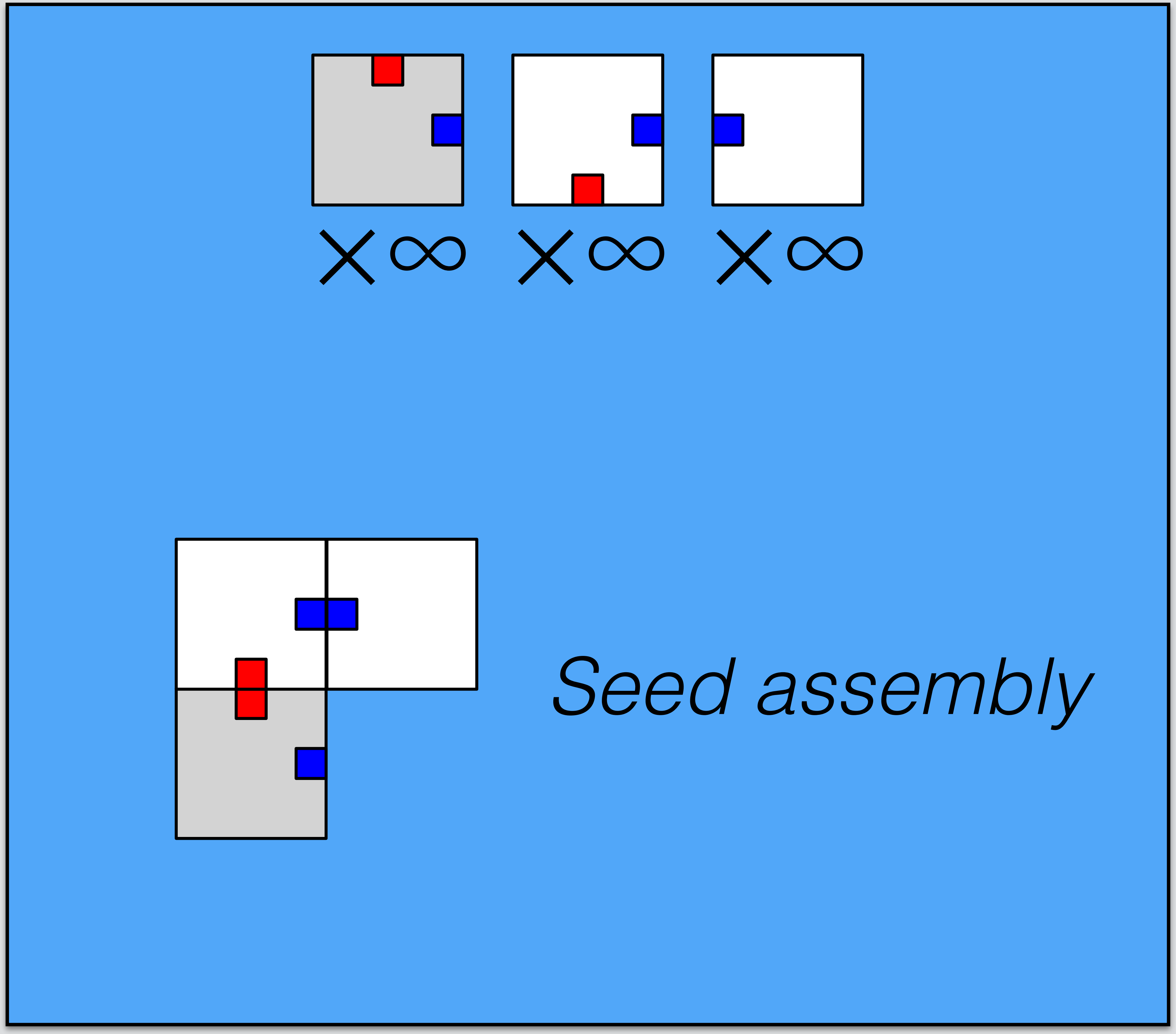

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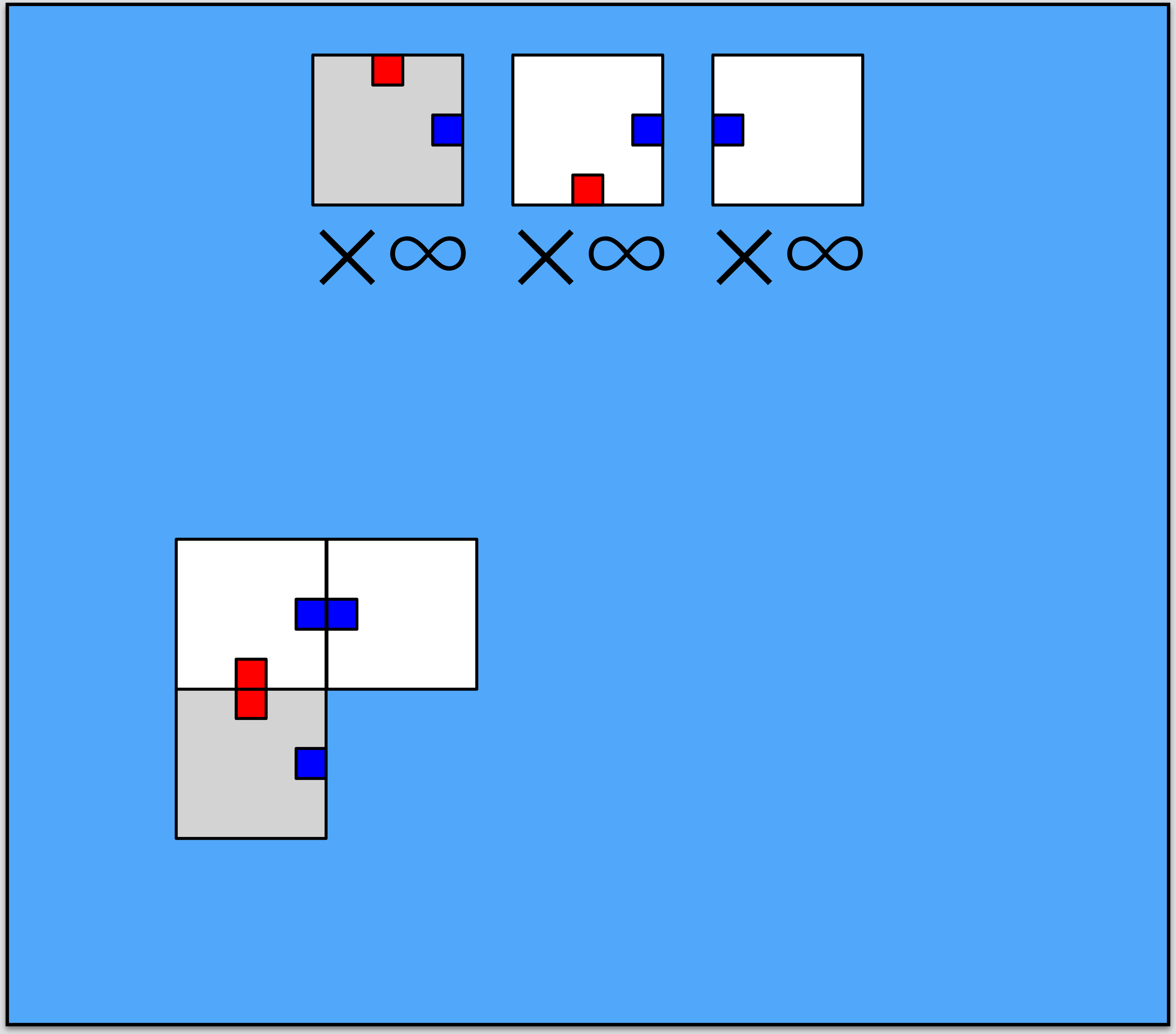

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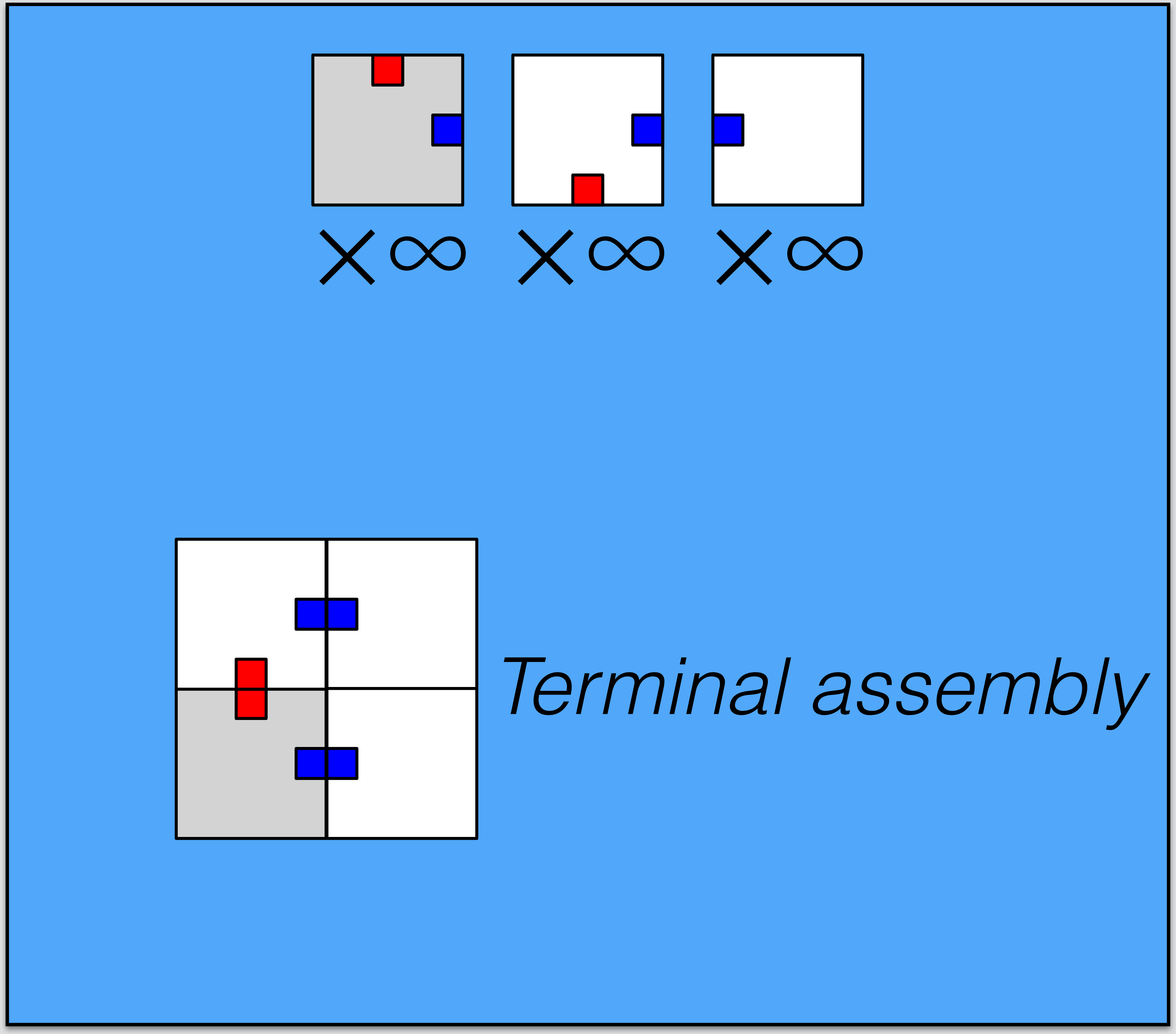
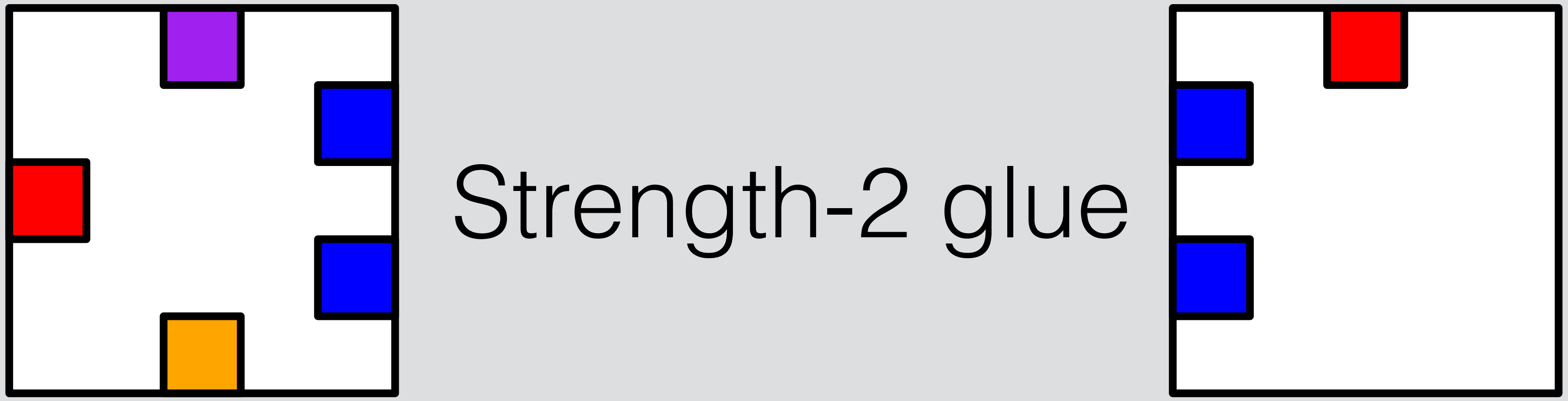


Tile Self-Assembly

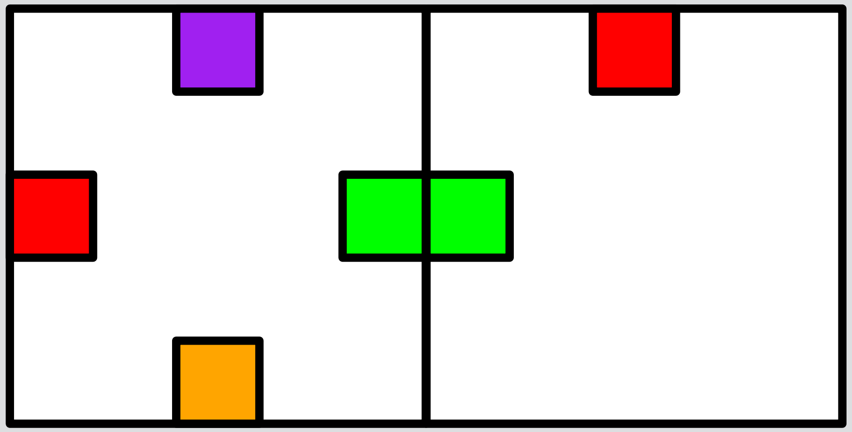
- Unit square *tiles* that cannot rotate.
- Up to four *glues*, one per side.
- Tiles attach edgewise to form *bonds*.
- Tiles attach to a growing *seed assembly*.

Glues have strength

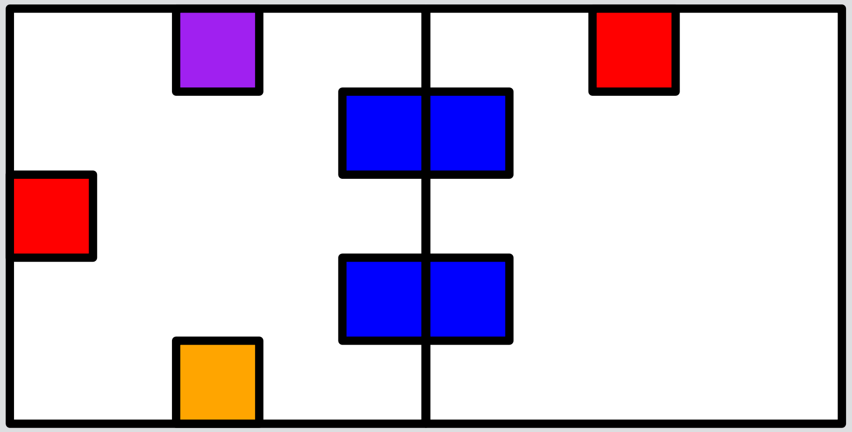


Bonds have strength

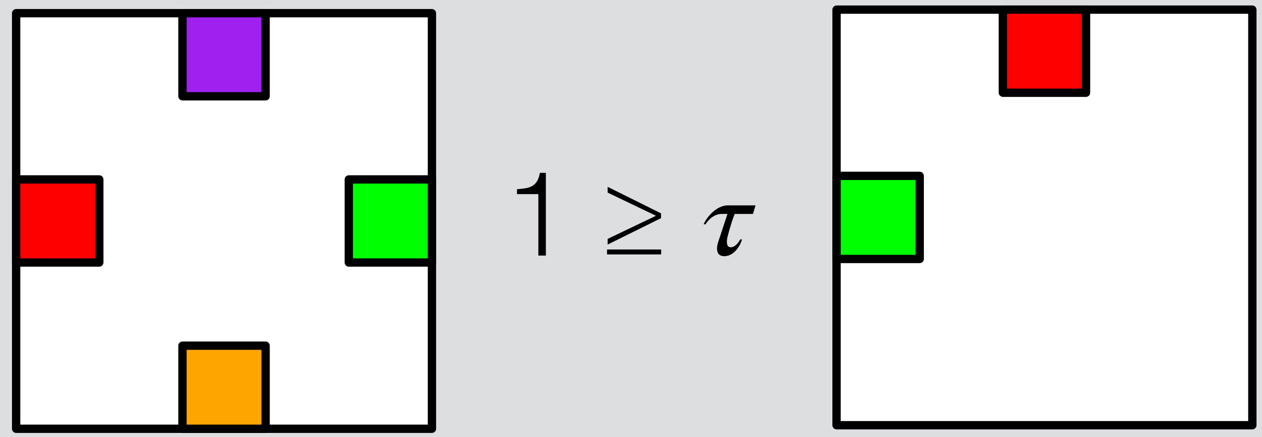
Strength-1 bond

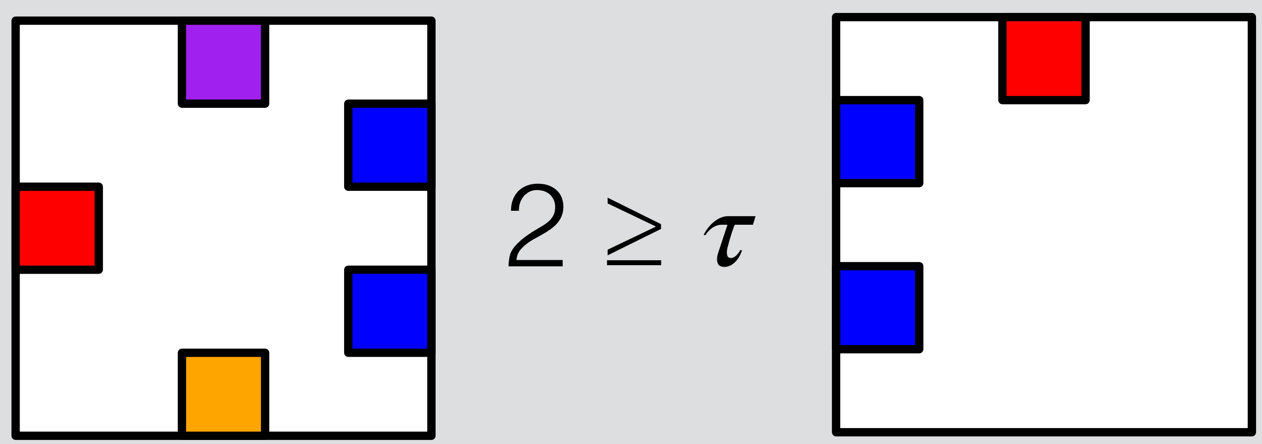


Strength-2 bond

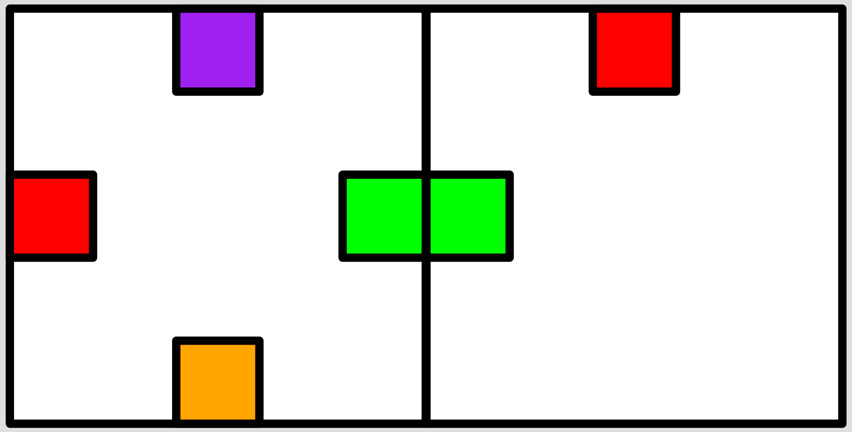


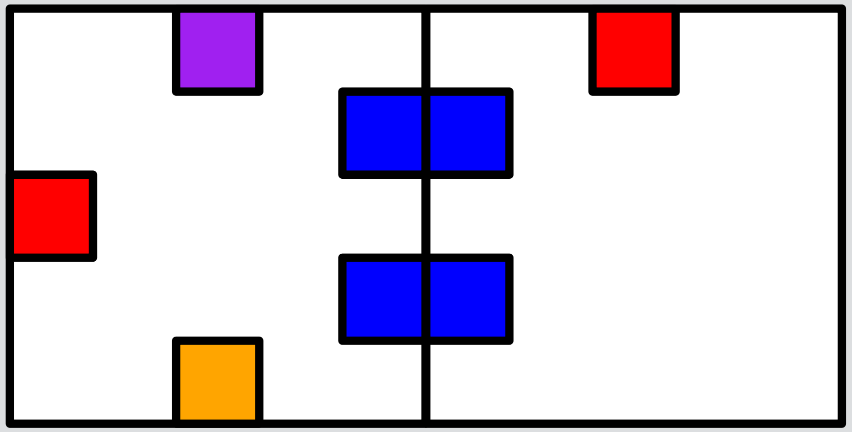
At temperature $\tau = 1$



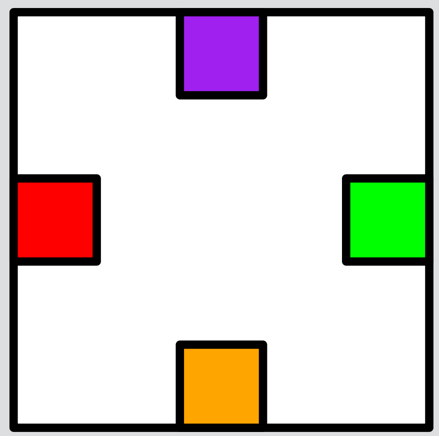


At temperature $\tau = 1$

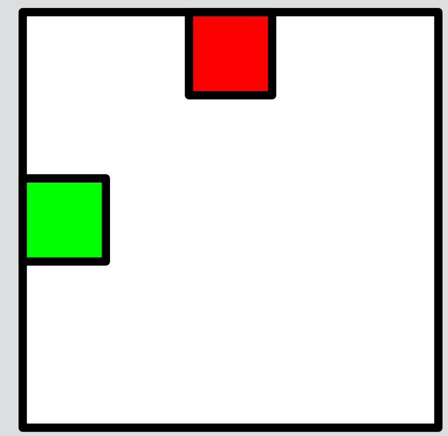


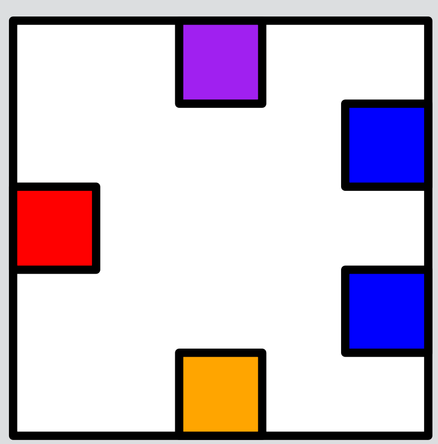


At temperature $\tau = 2$

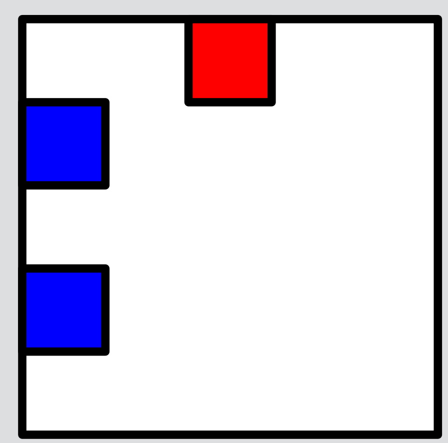


$1 \not\geq \tau$

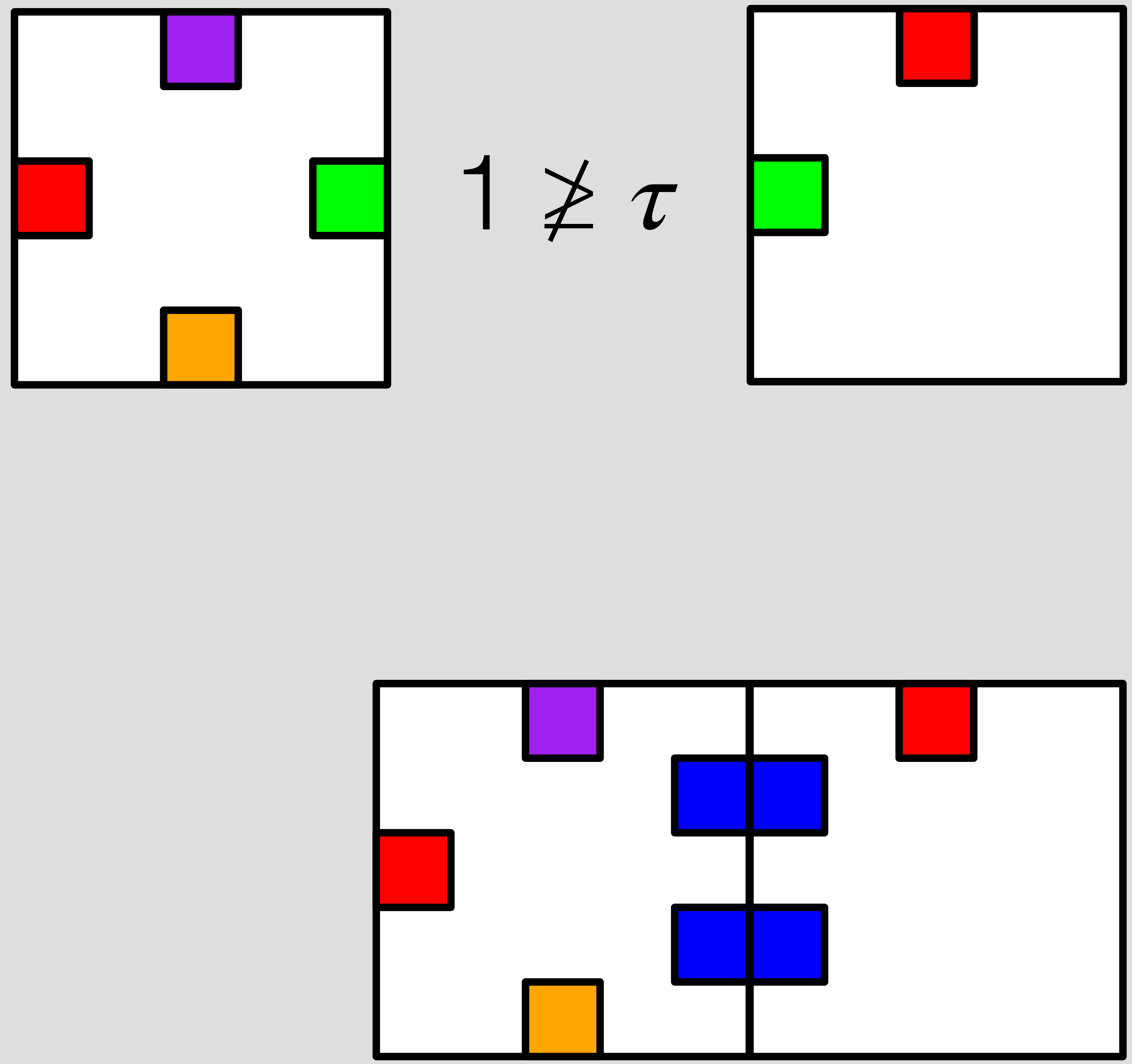




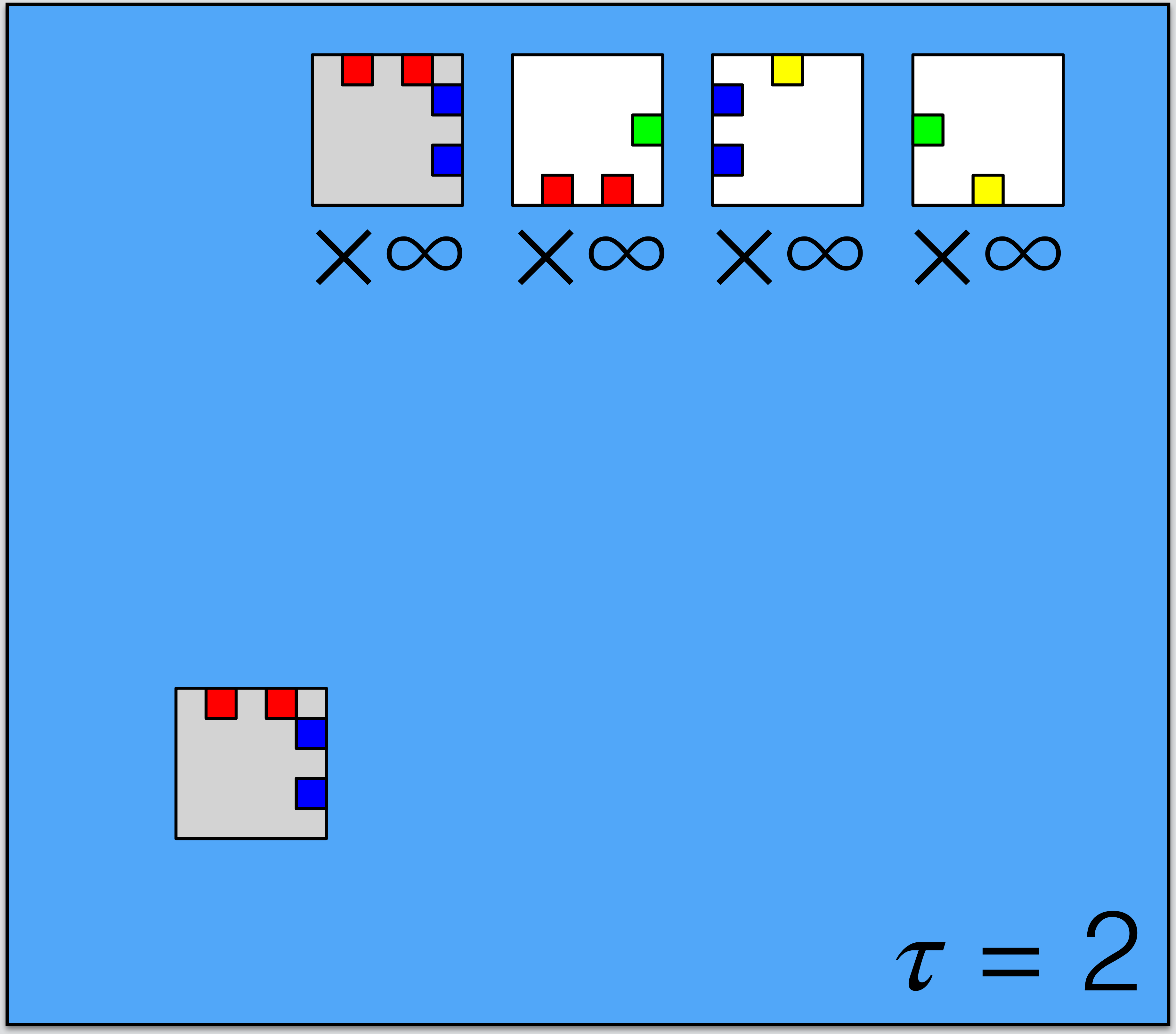
$2 \geq \tau$



At temperature $\tau = 2$



Tile Self-Assembly



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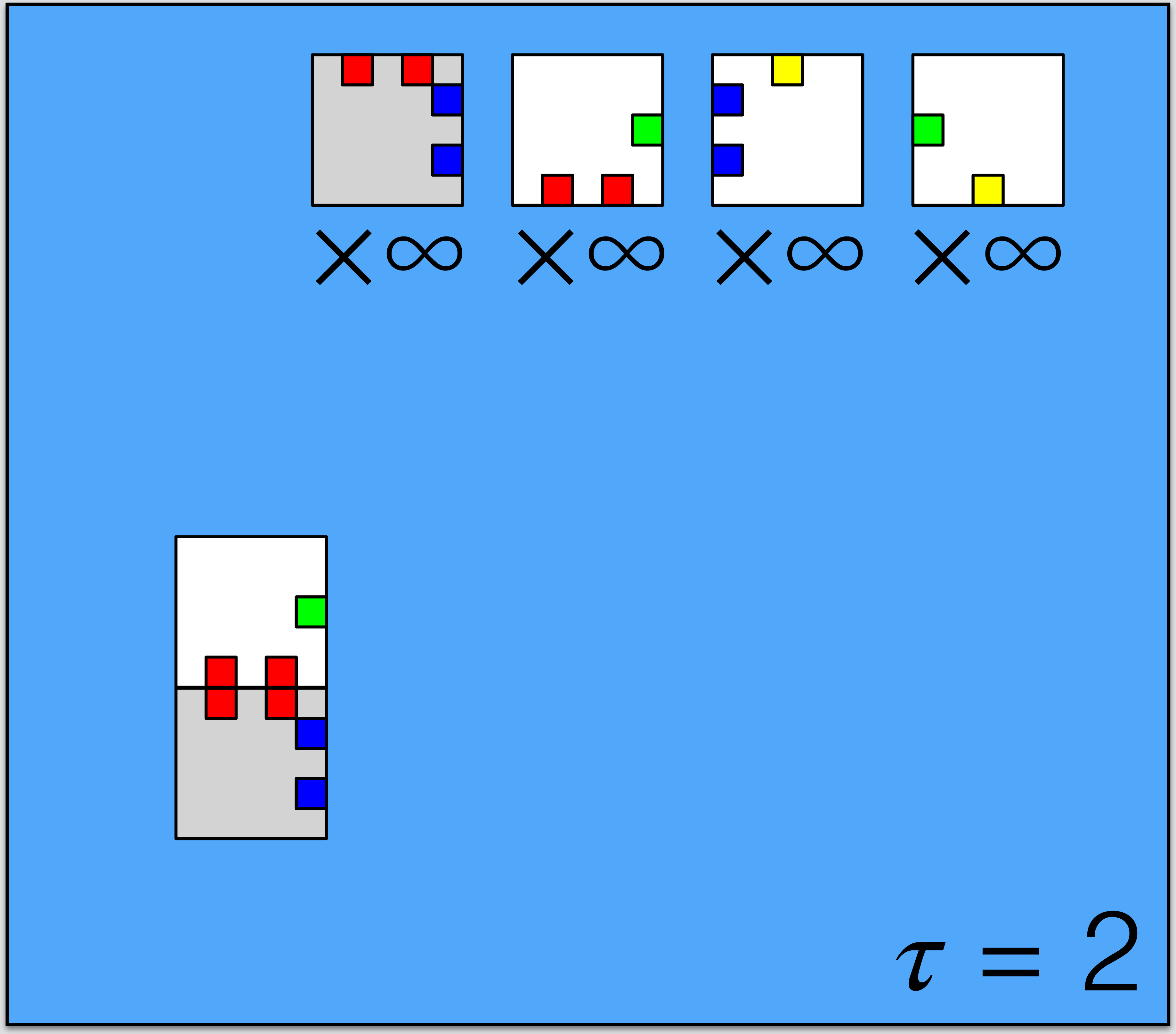

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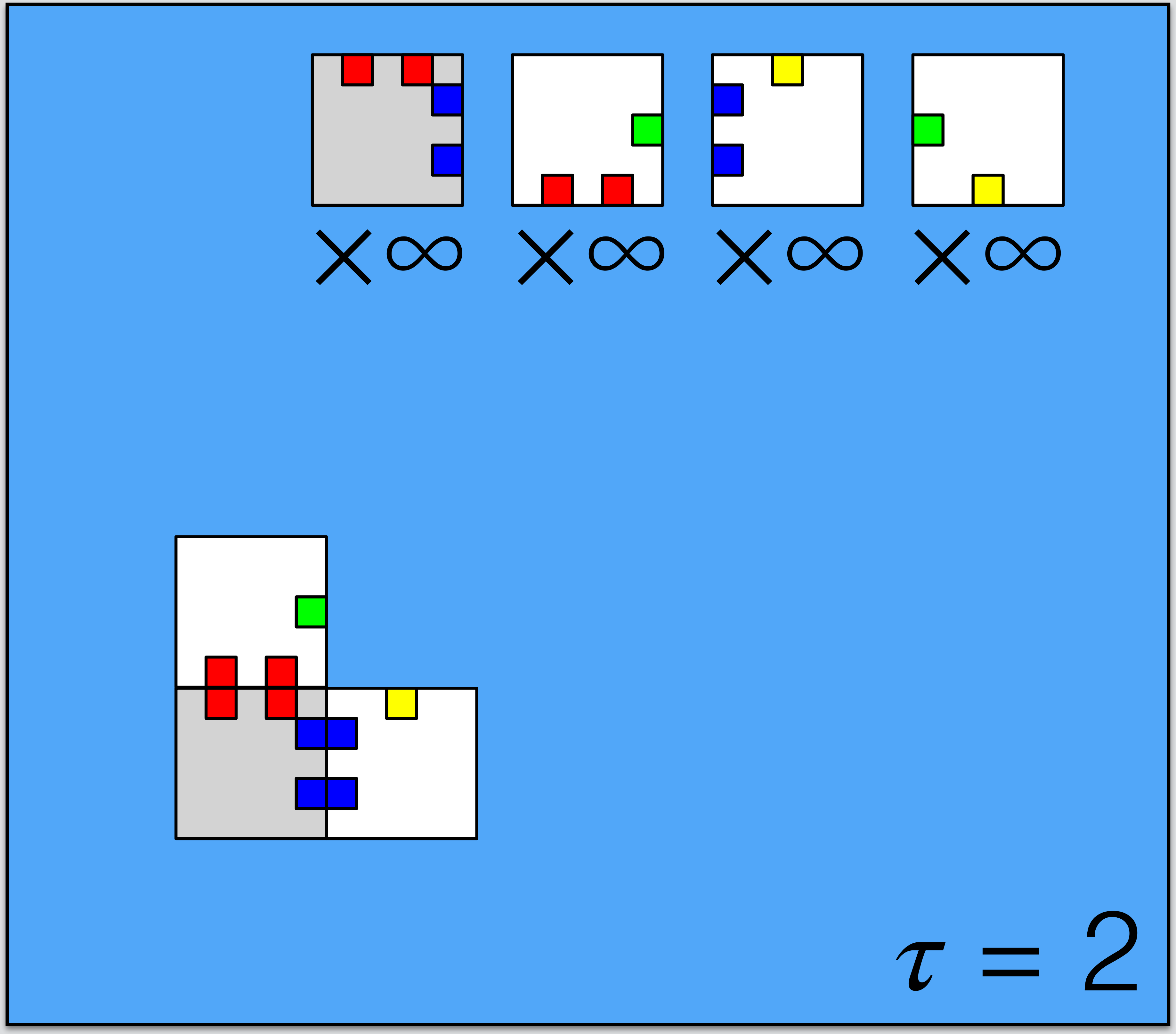

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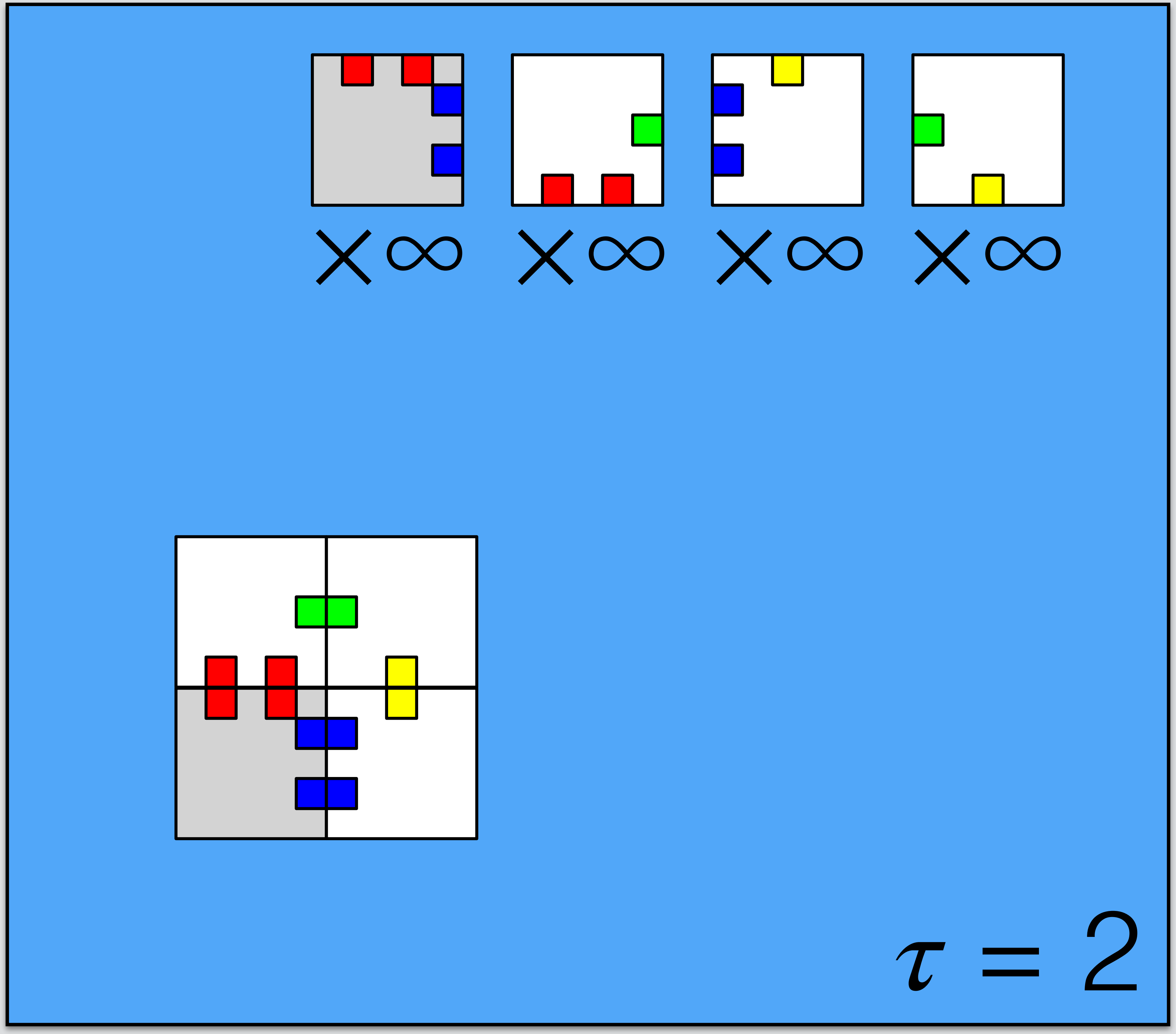


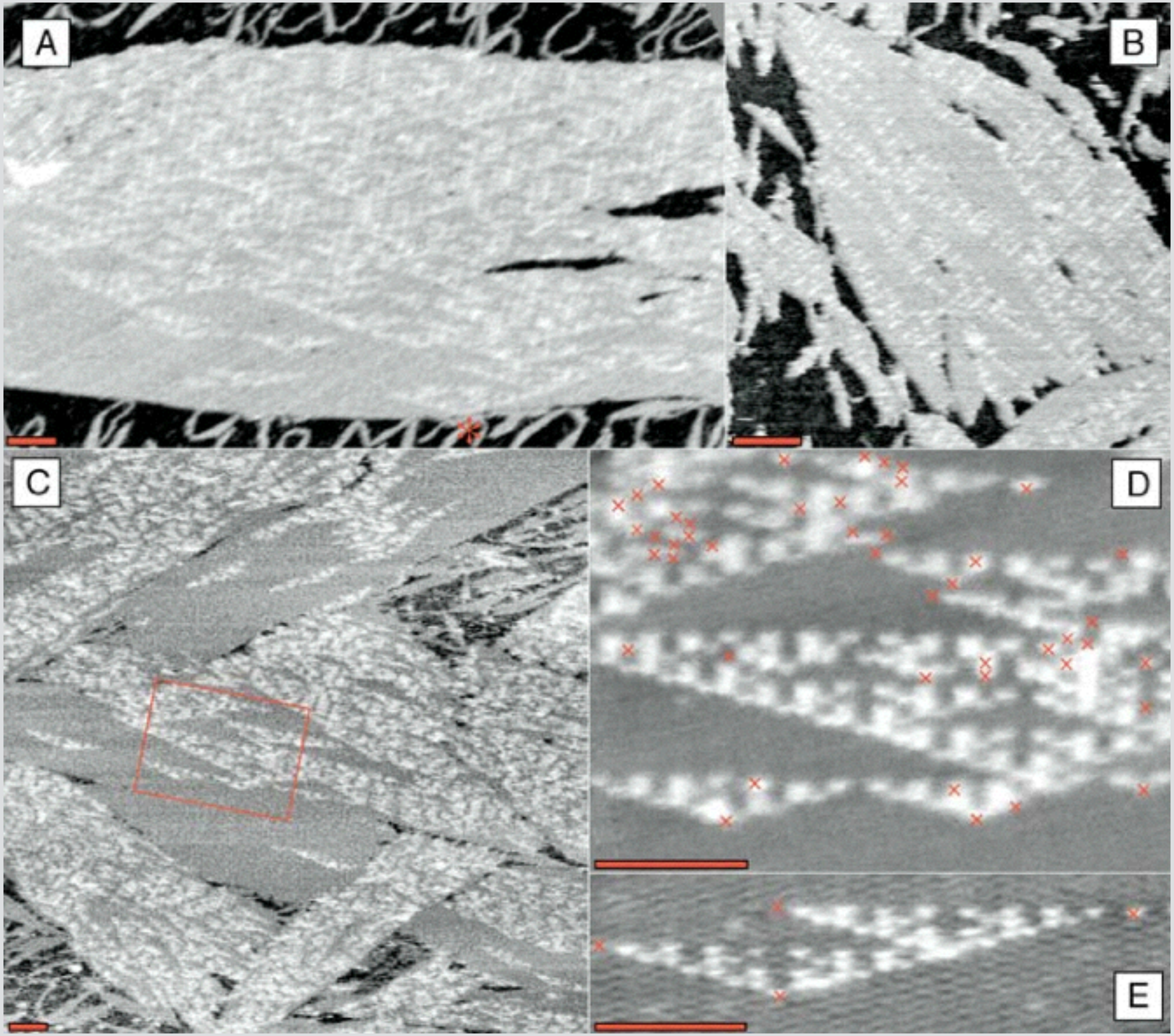
Tile Self-Assembly

- Glues and bonds have *strength*.
- System has *temperature* τ.
- Tile can attach to seed assembly if the total bond strength is at least τ.

abstract Tile Assembly Model

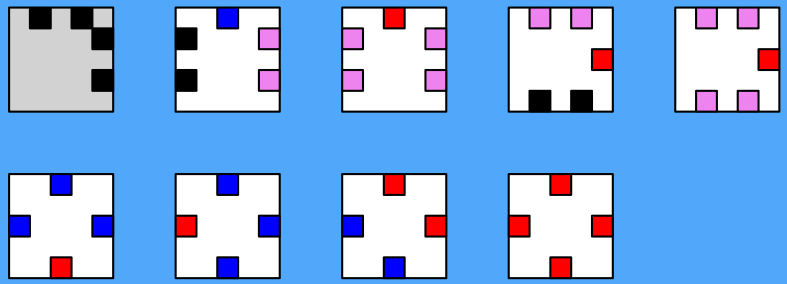
- Introduced by Erik Winfree in mid-1990s.
- Implemented in DNA at the same time.
- Based on two previous works:
    - DNA lattices of Ned Seeman in 1980s.
    - Wang tilings of Hao Wang in 1960s.

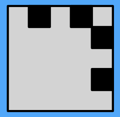
Image from [Papadakis, Rothemund, Winfree 2004]:



Scale bars = 100 nm

$\tau = 2$





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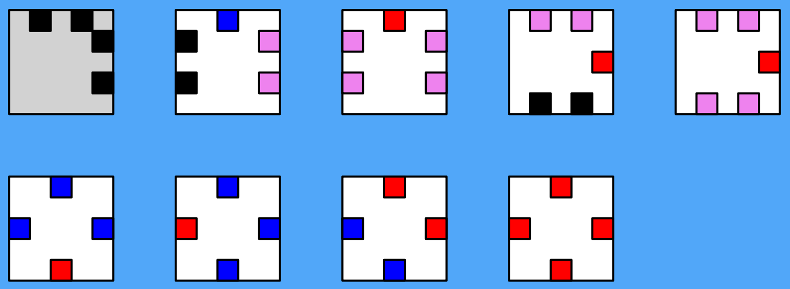

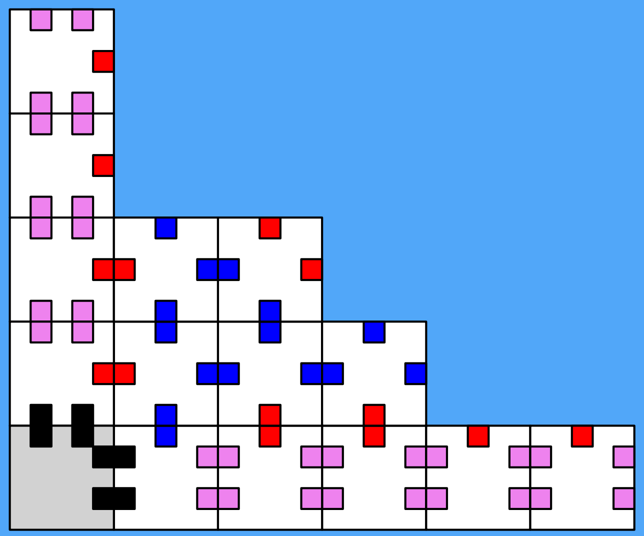

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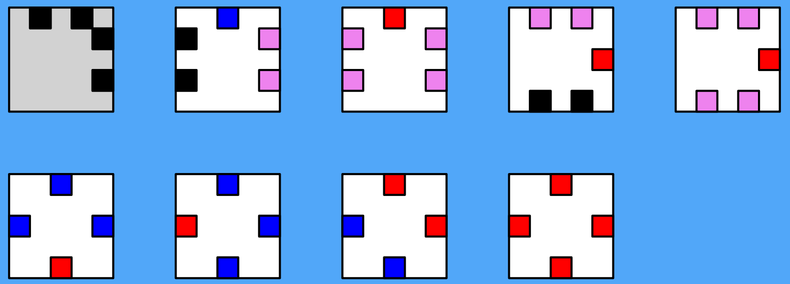

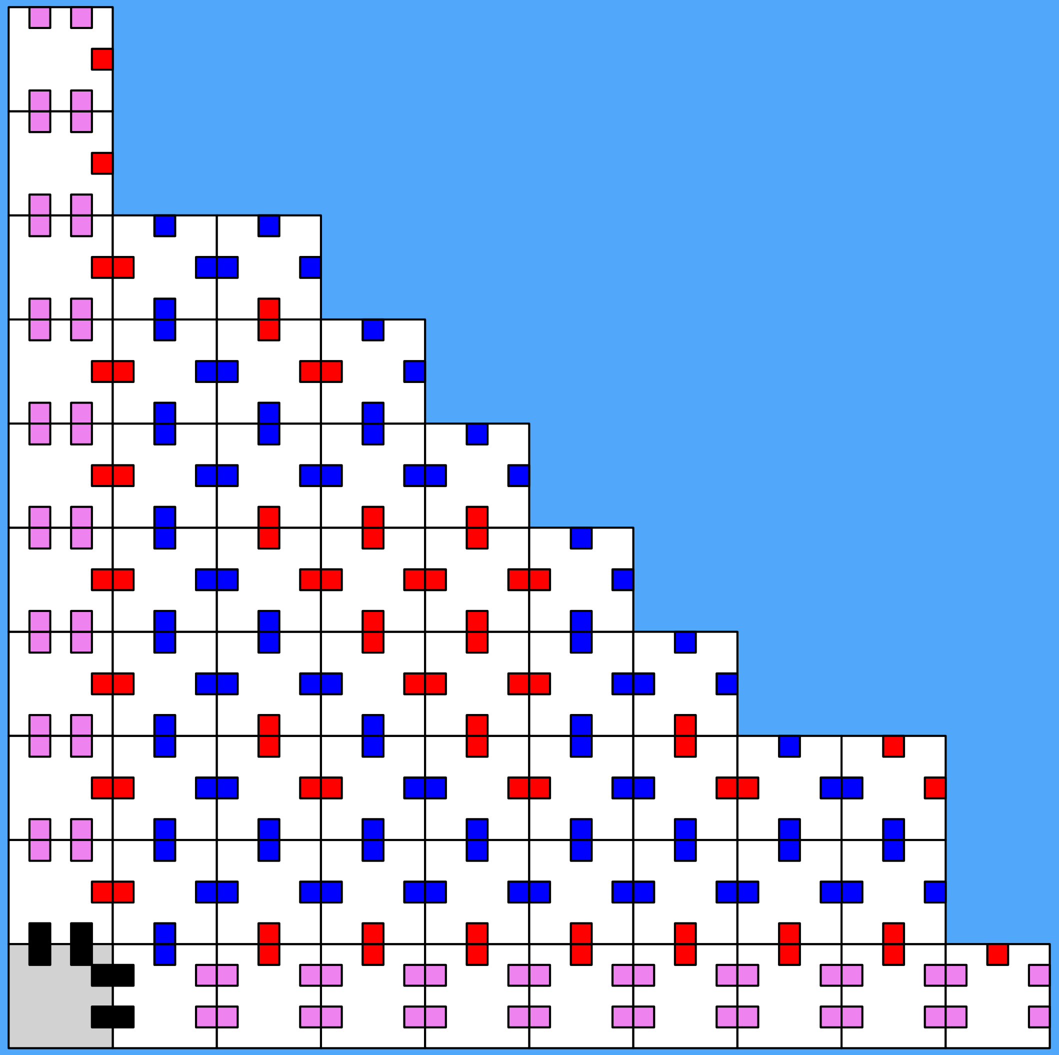

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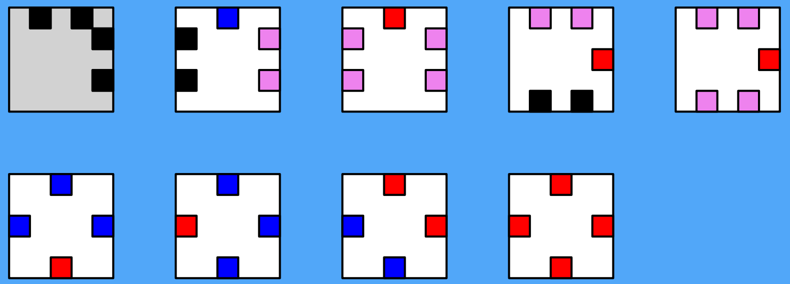

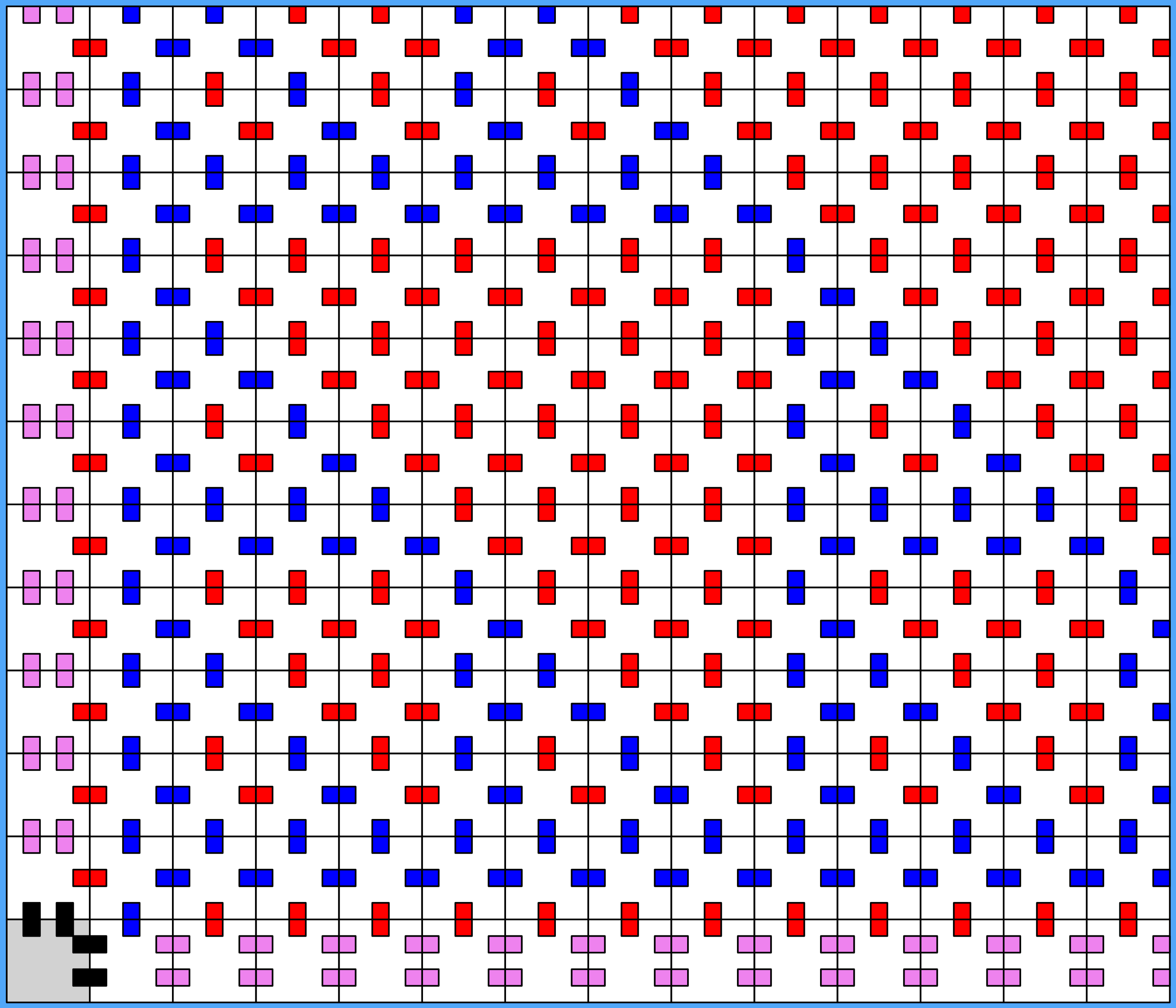

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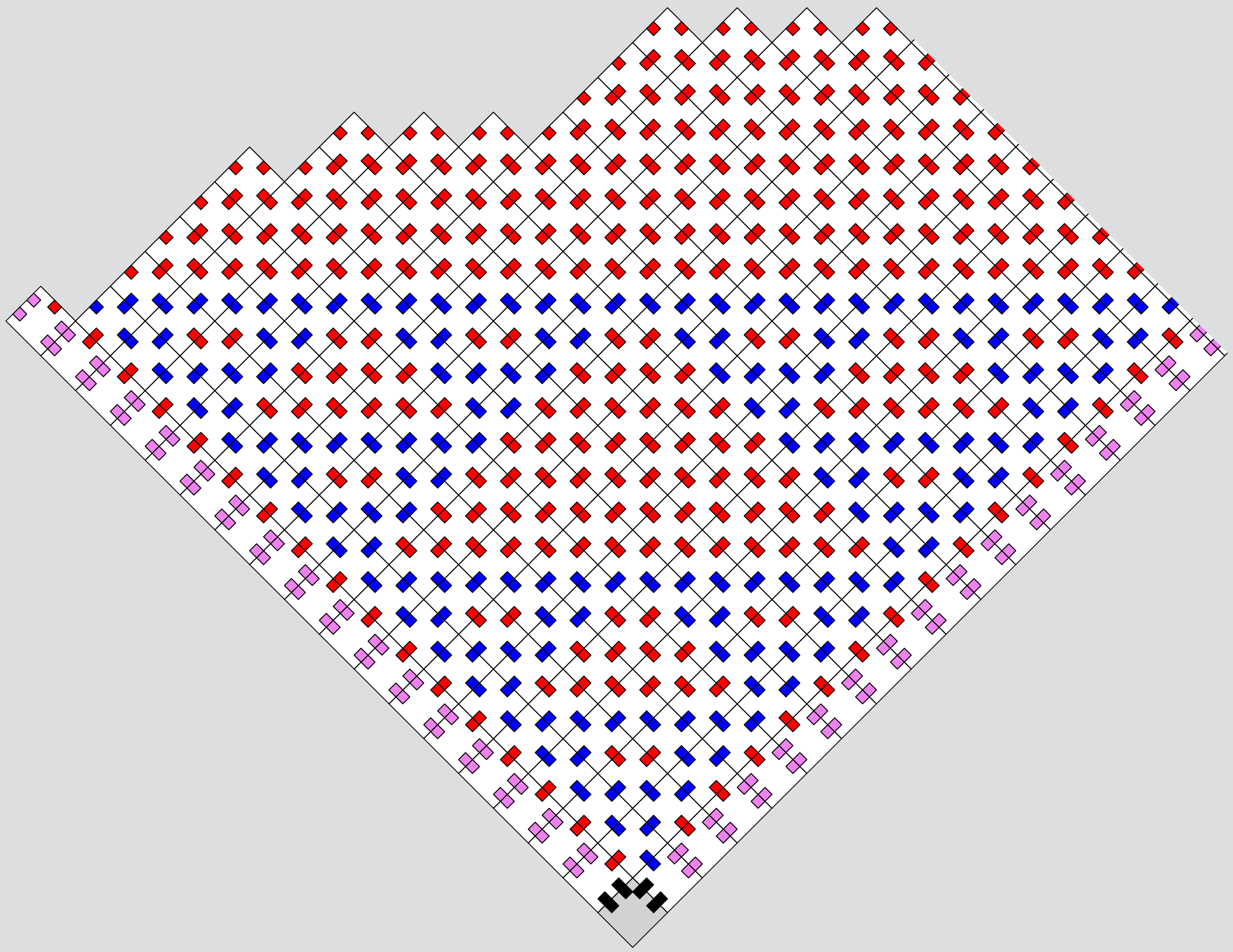
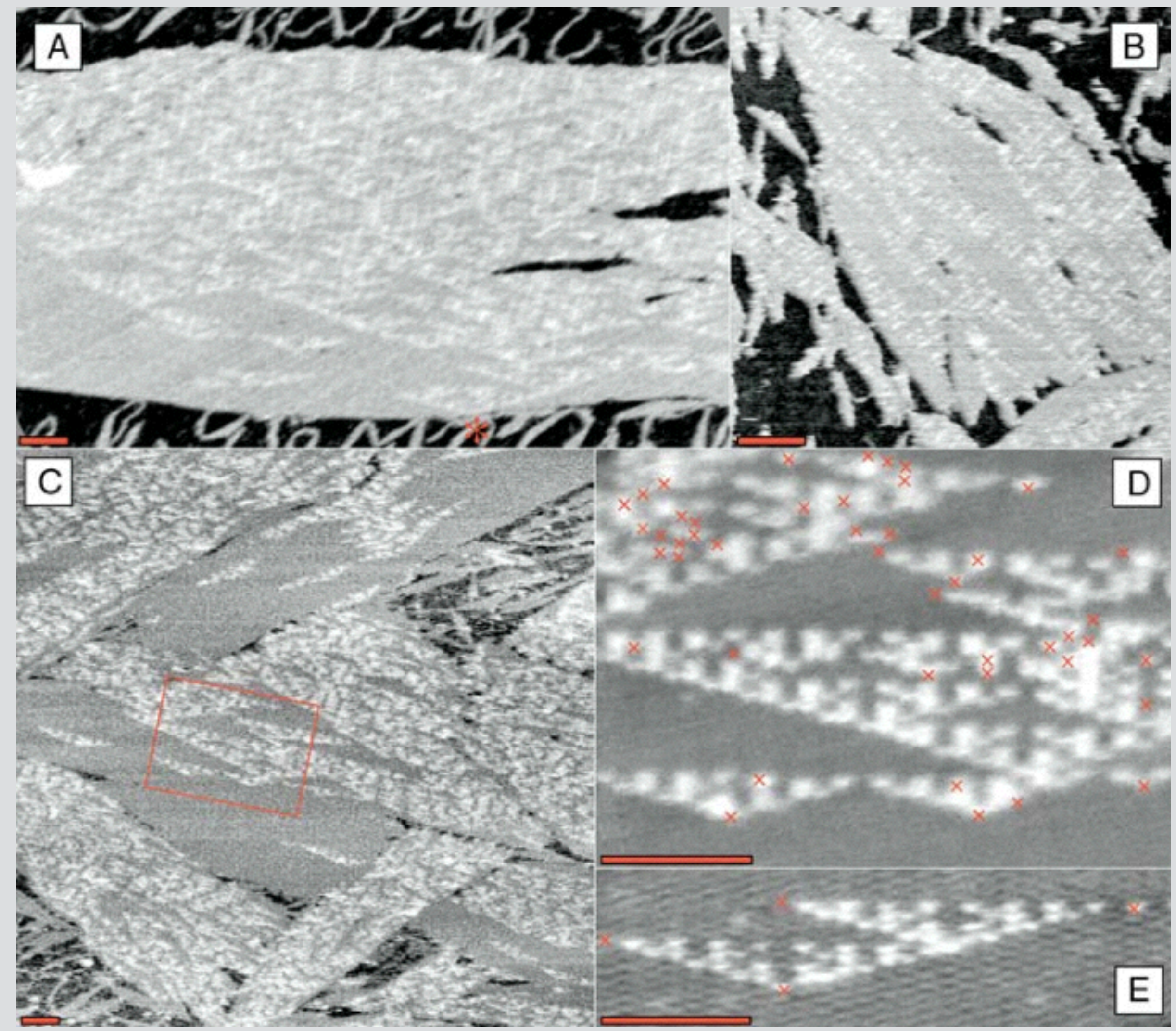
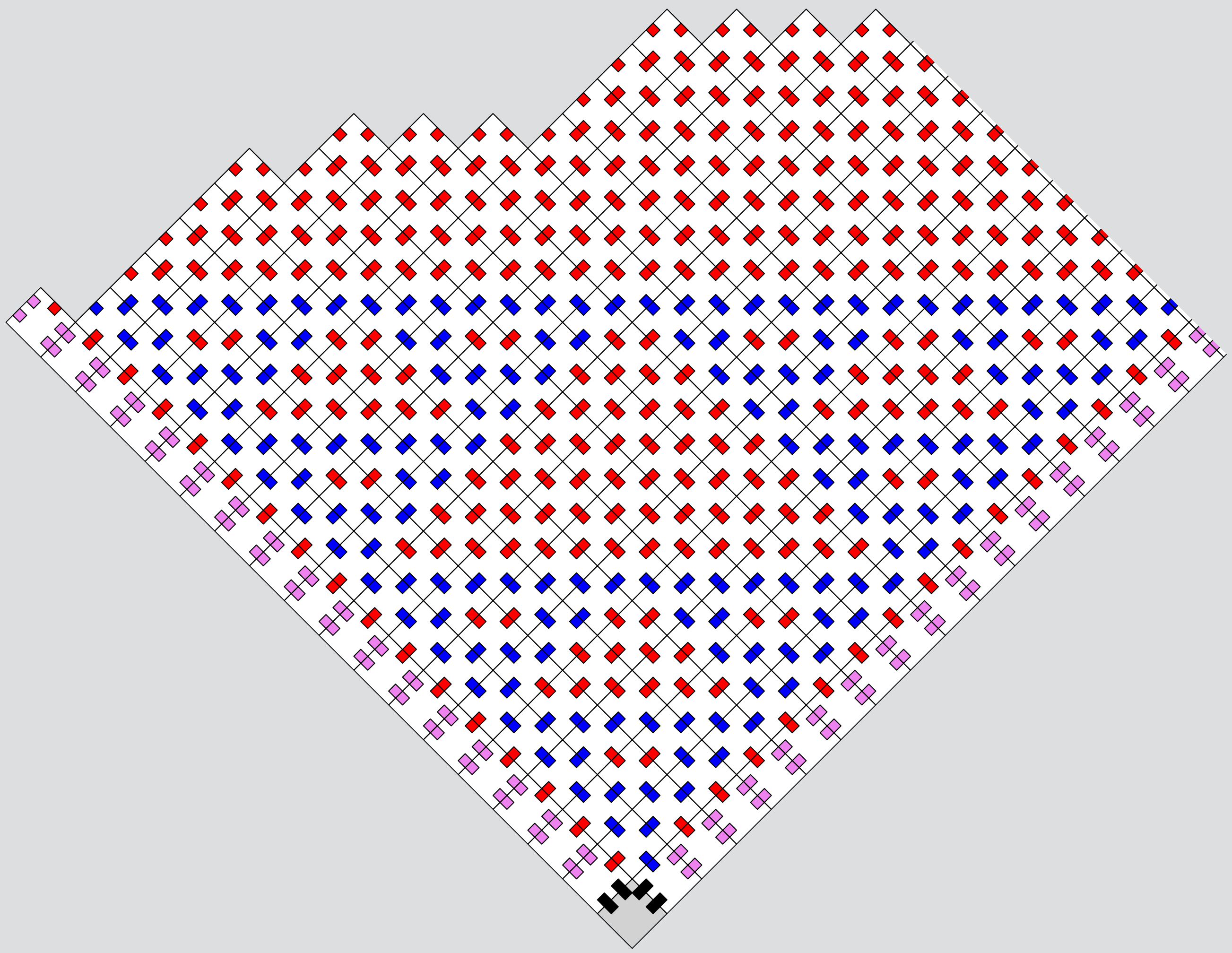
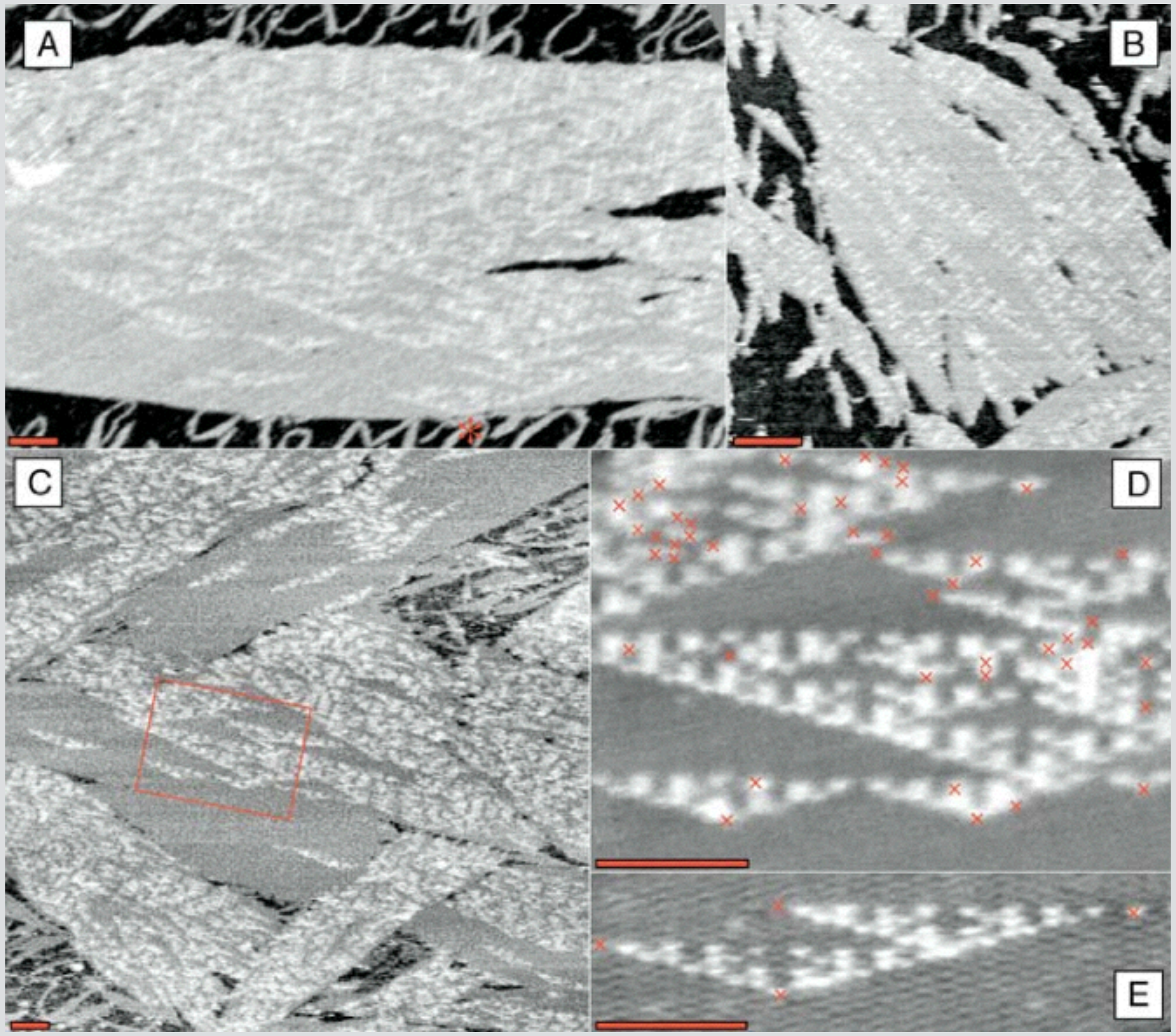



A
B
C
D
E

Theoretical model



Implementation



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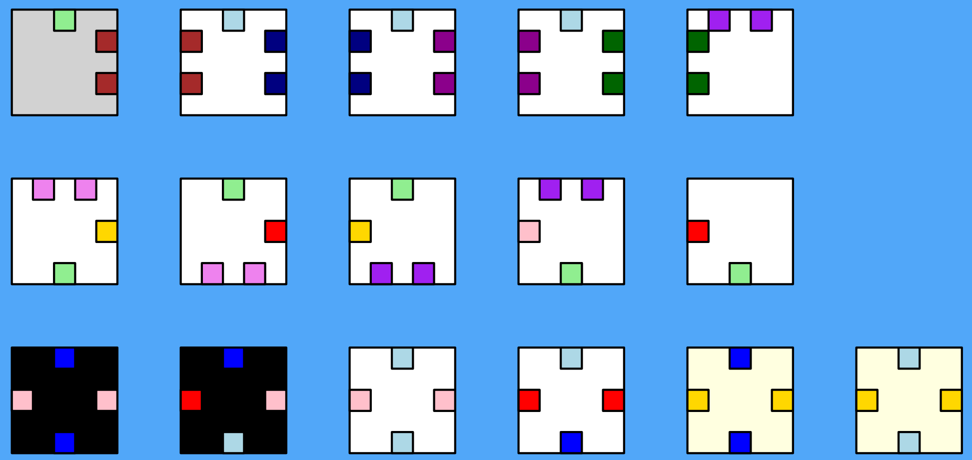

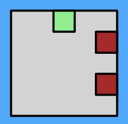

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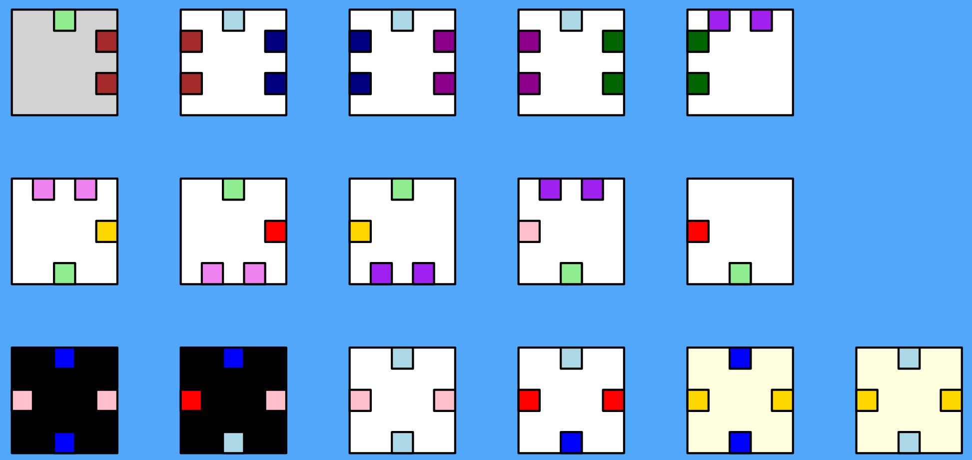

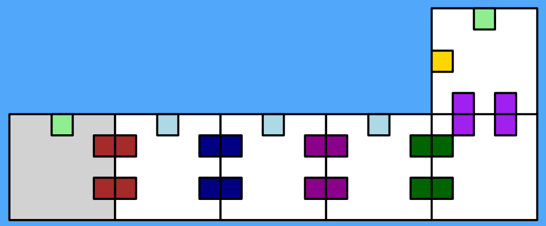

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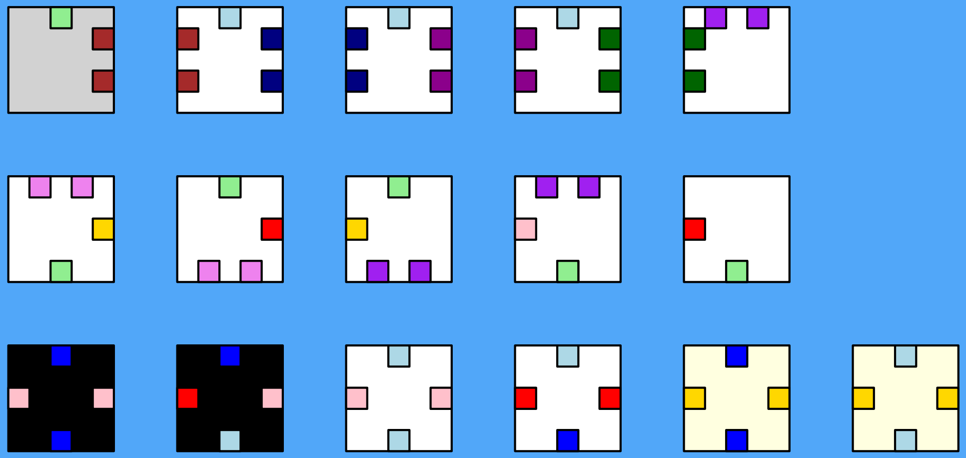

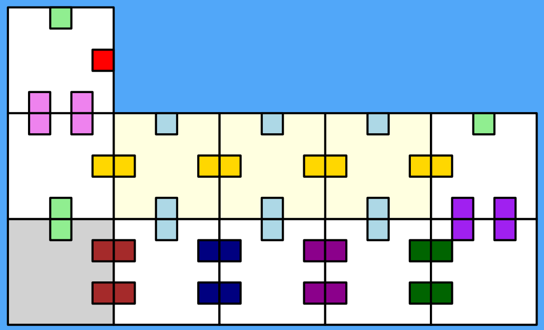

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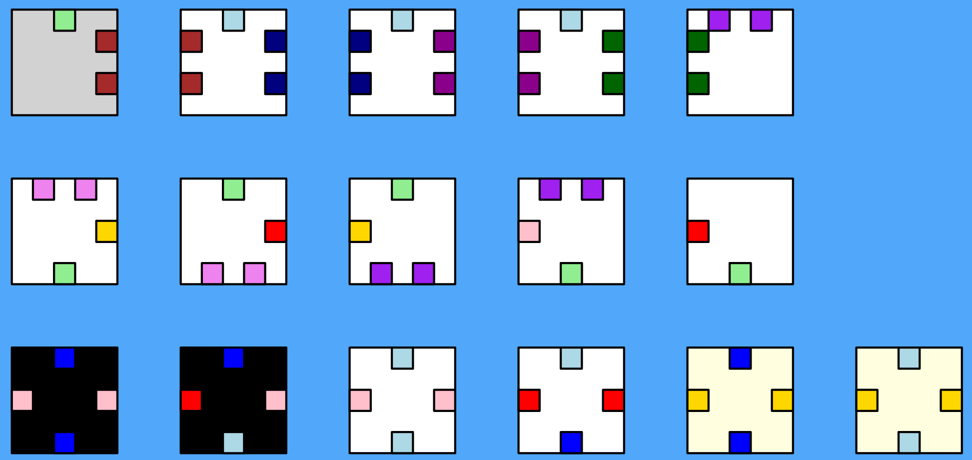

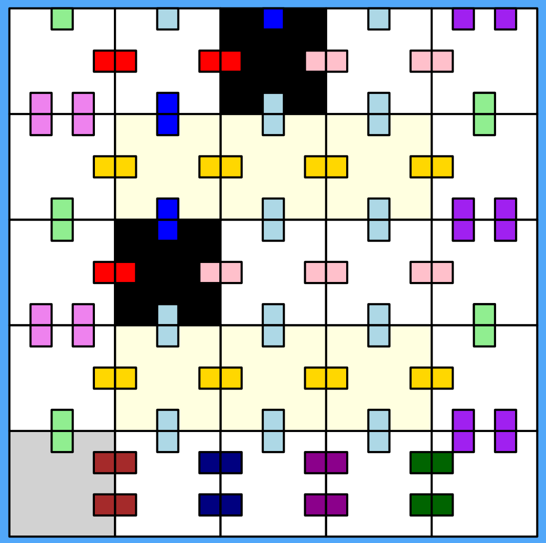

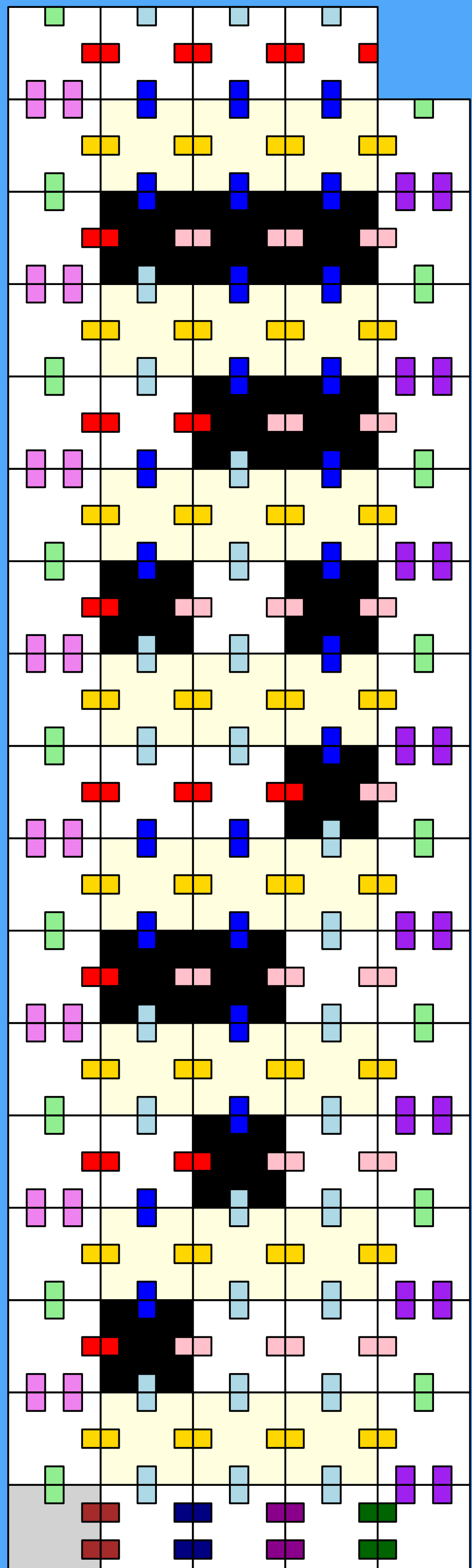

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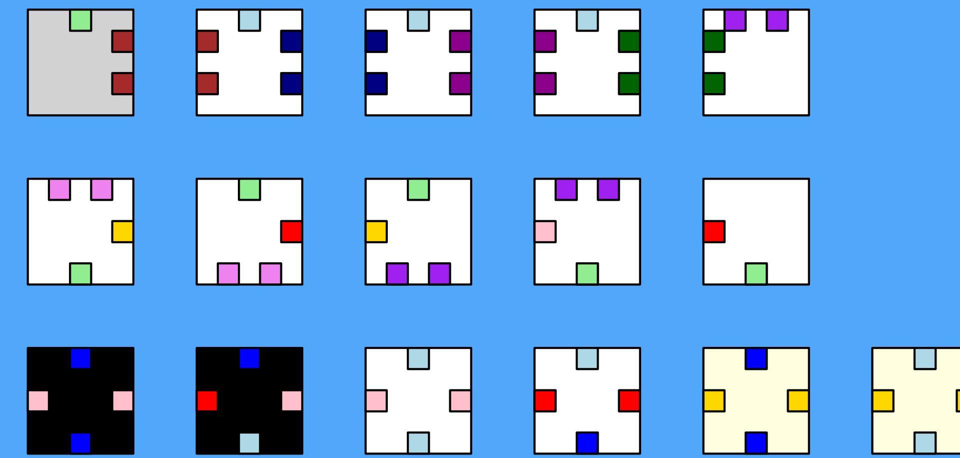

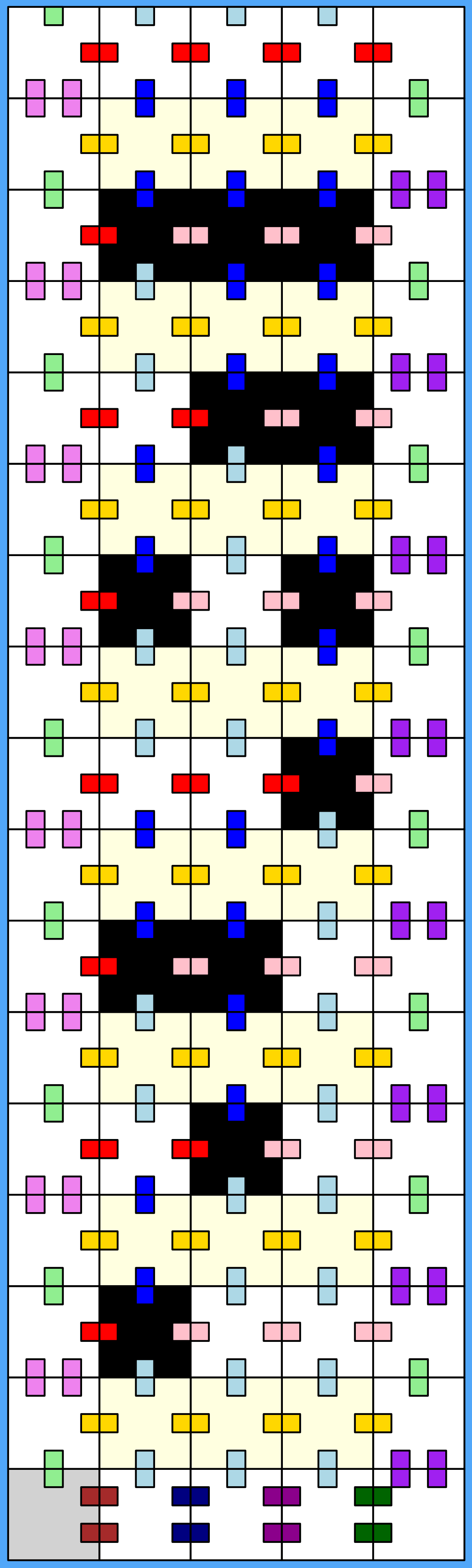

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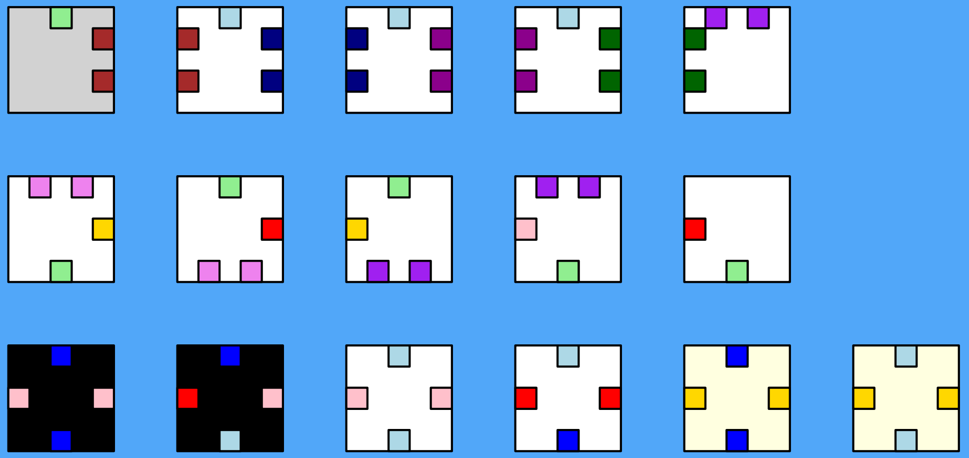

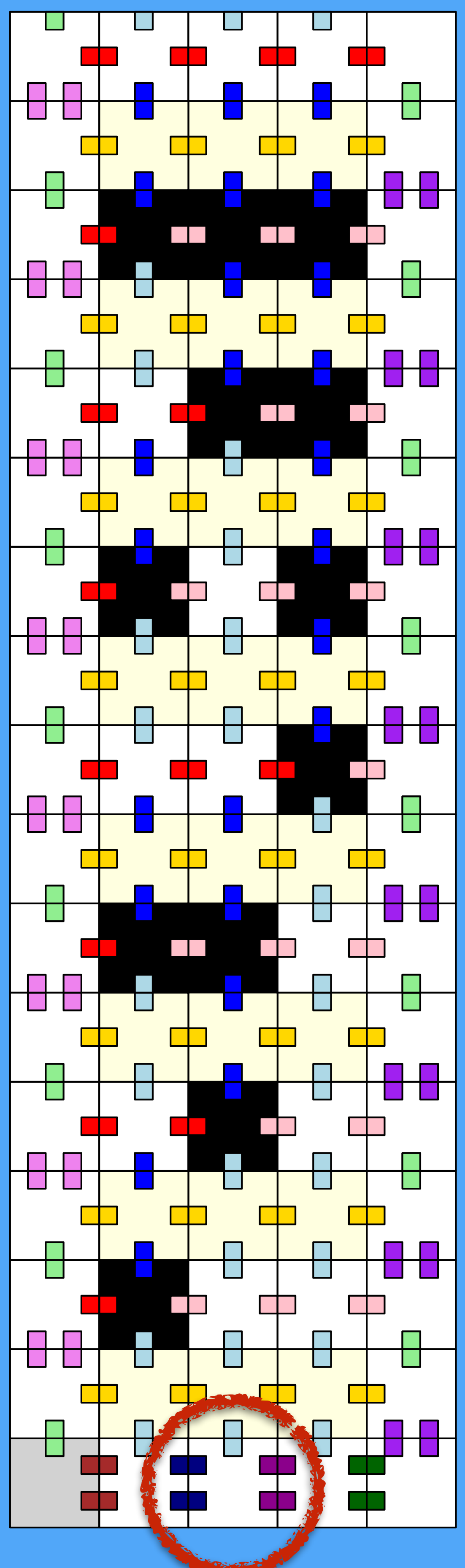

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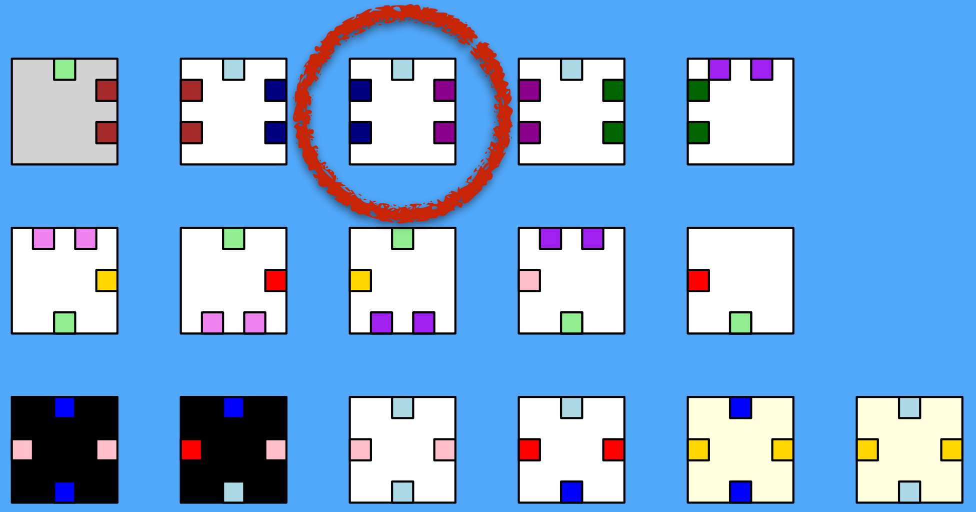

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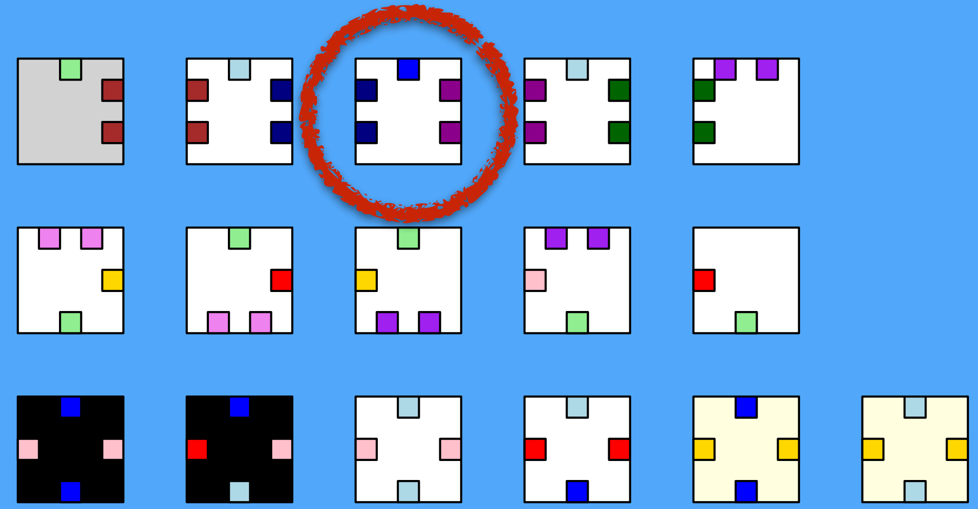
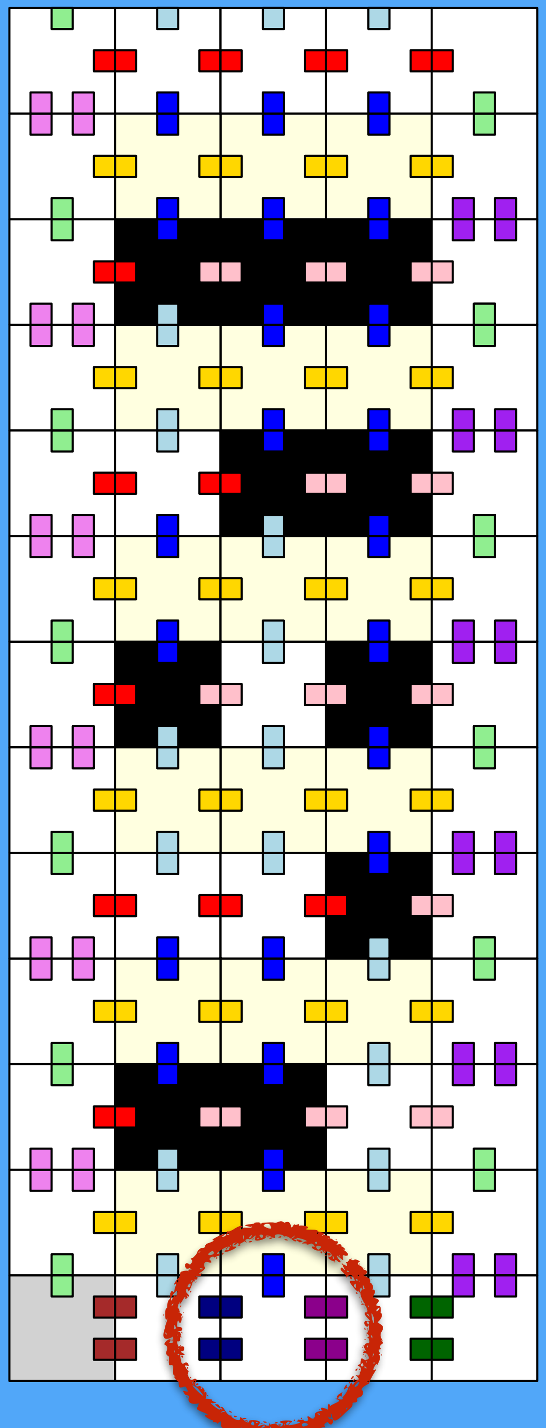
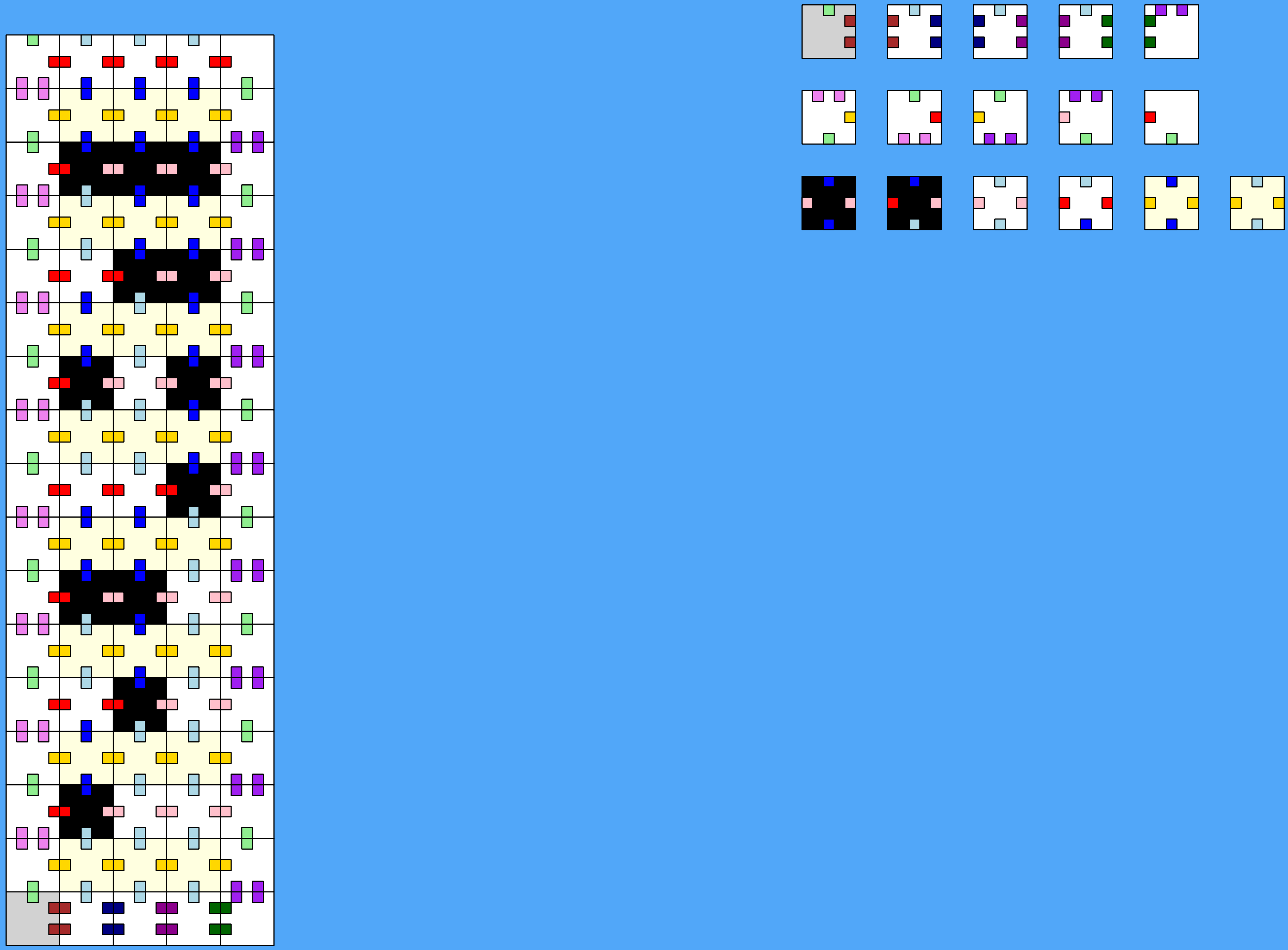
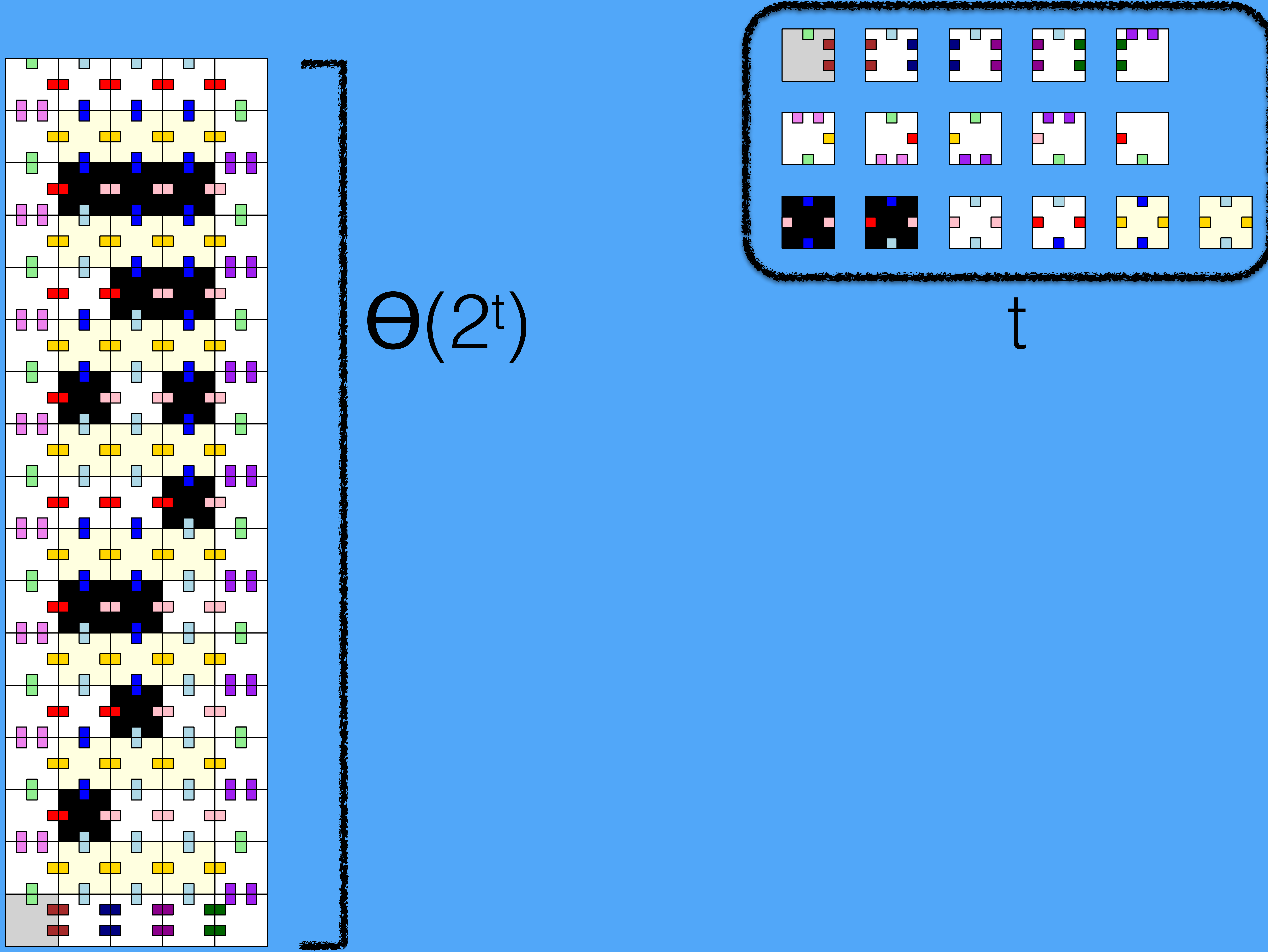
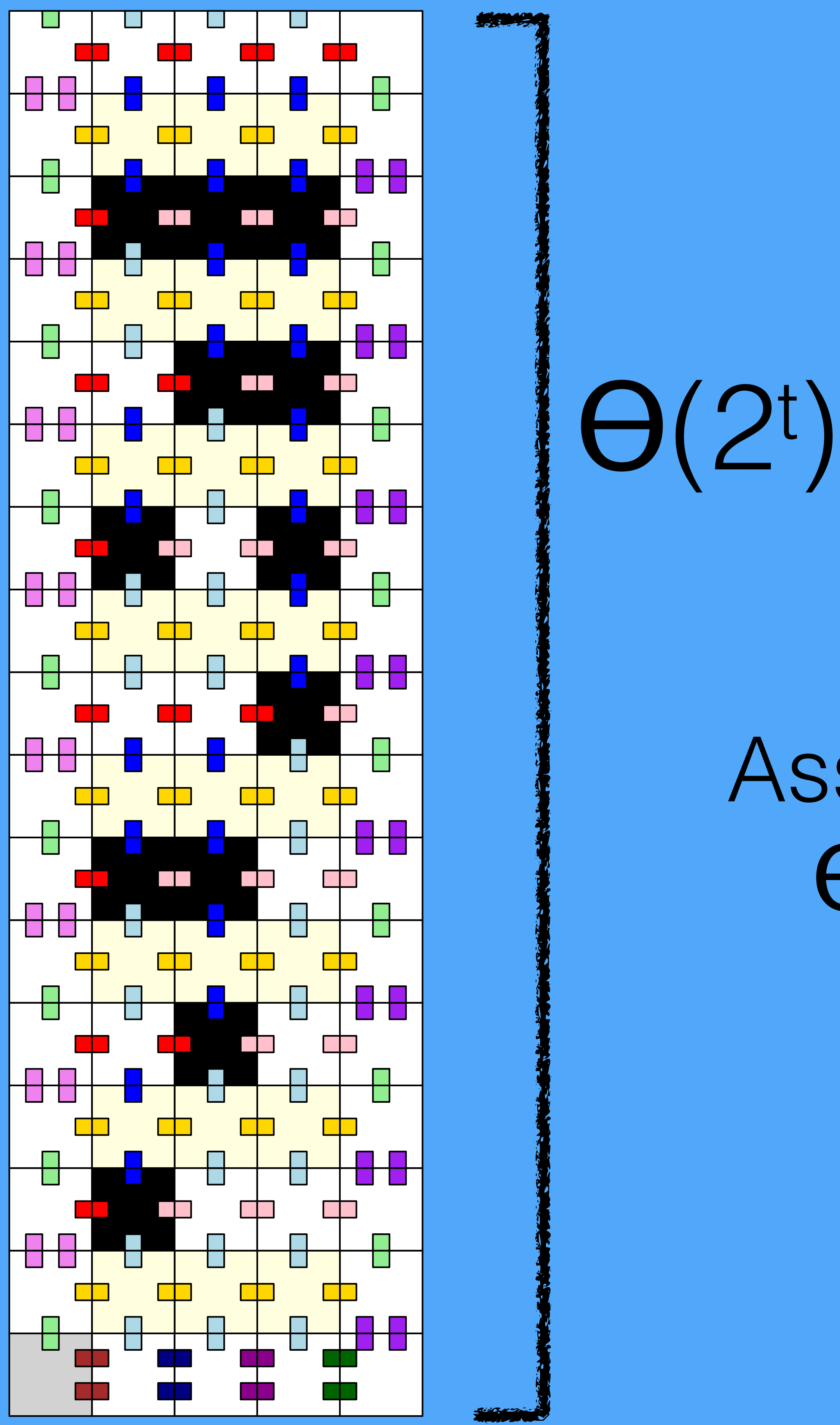
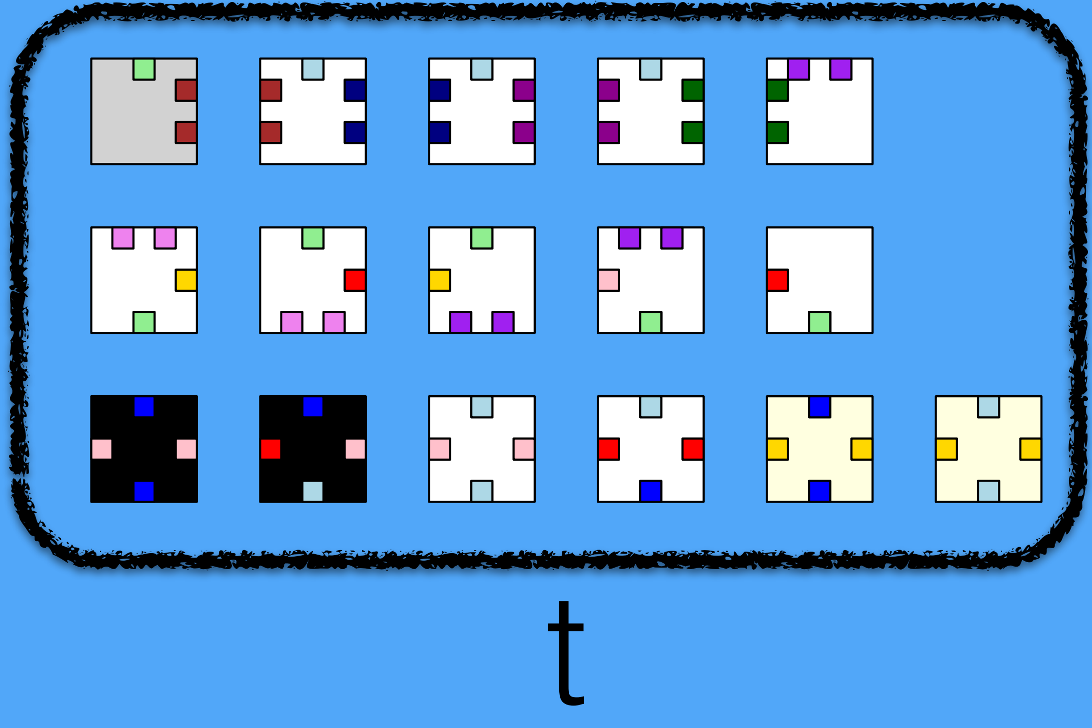







Θ(2^t)
t





Assembly of height n using $\Theta(\log(n))$-sized tile set.

Some Known Results

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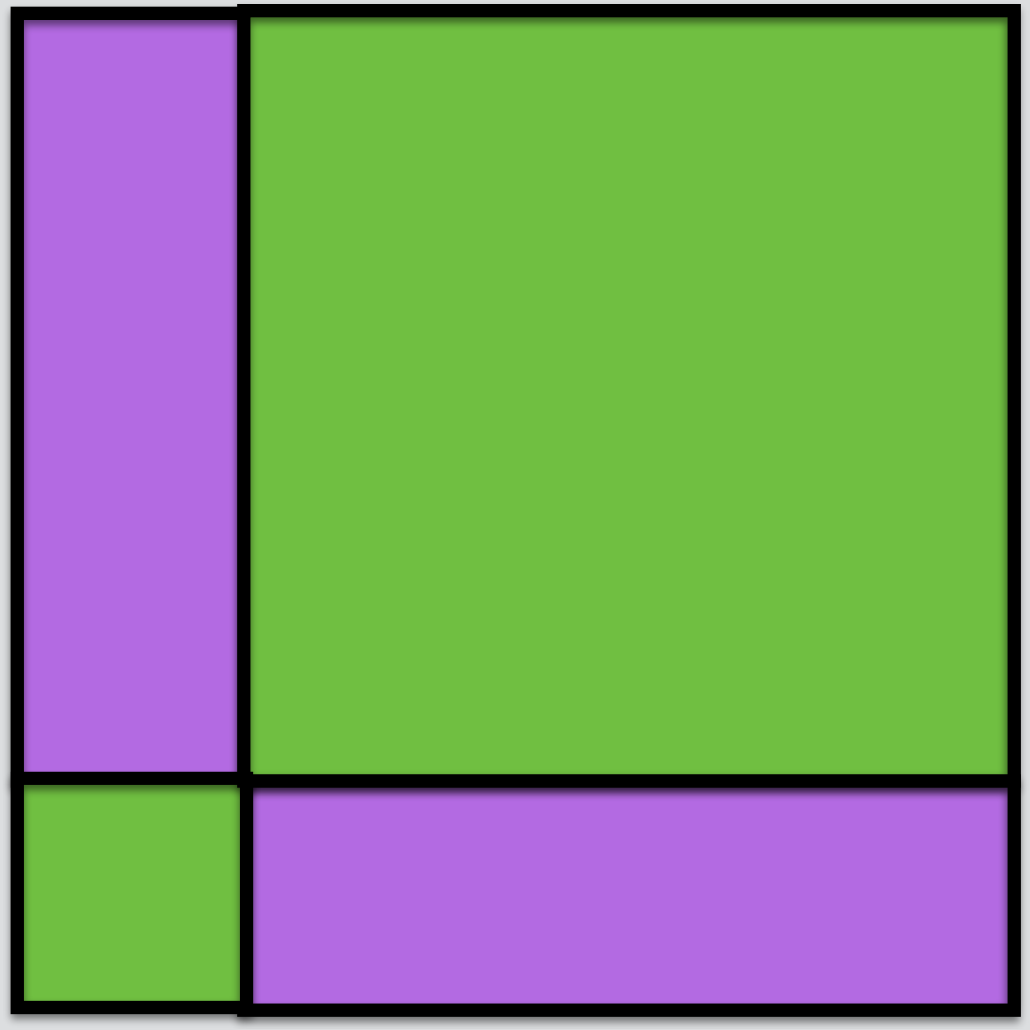

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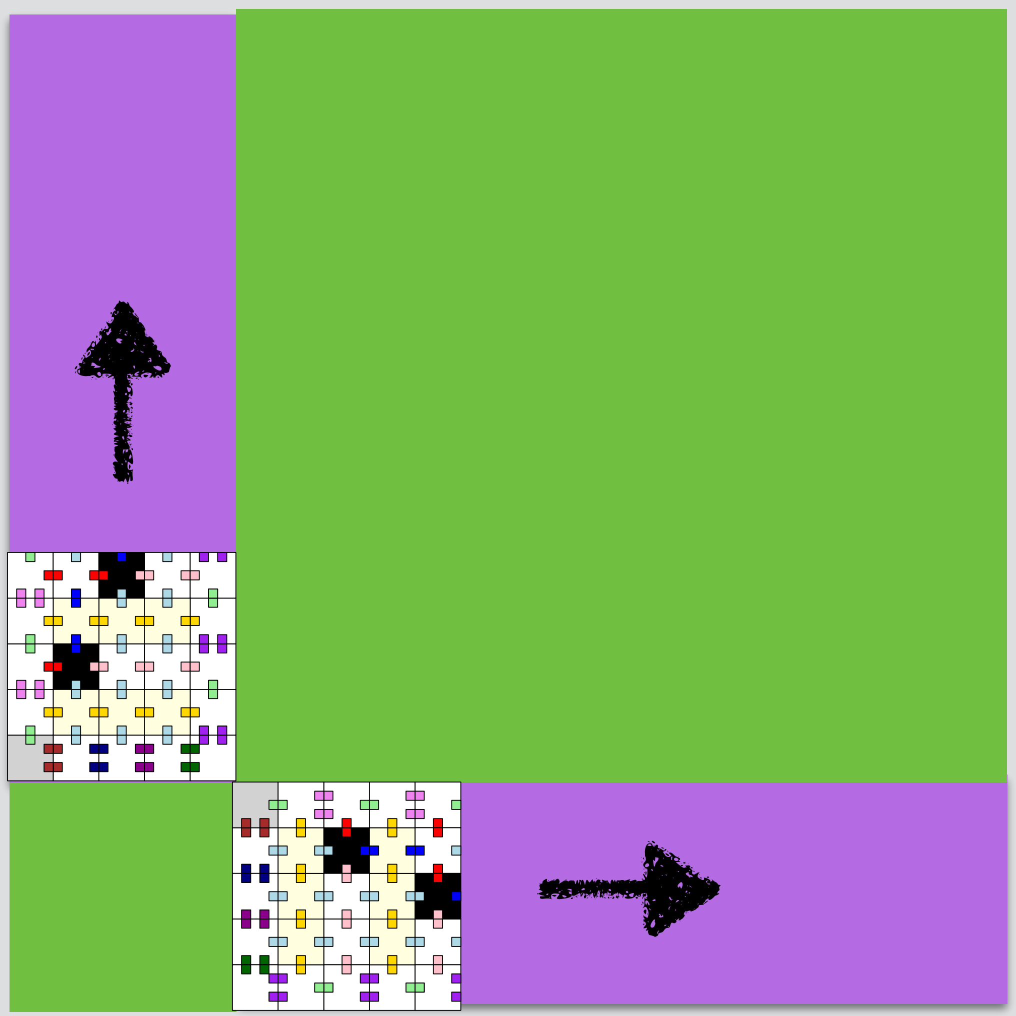

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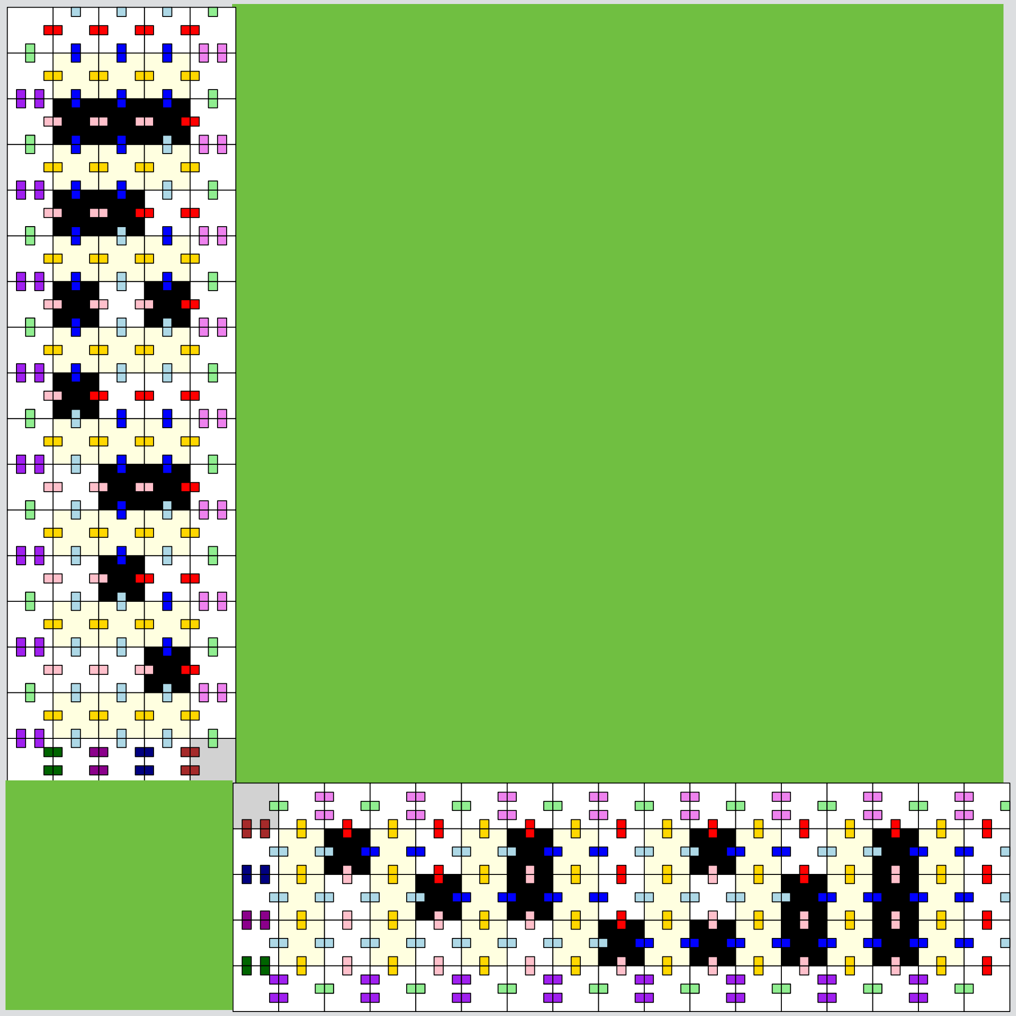

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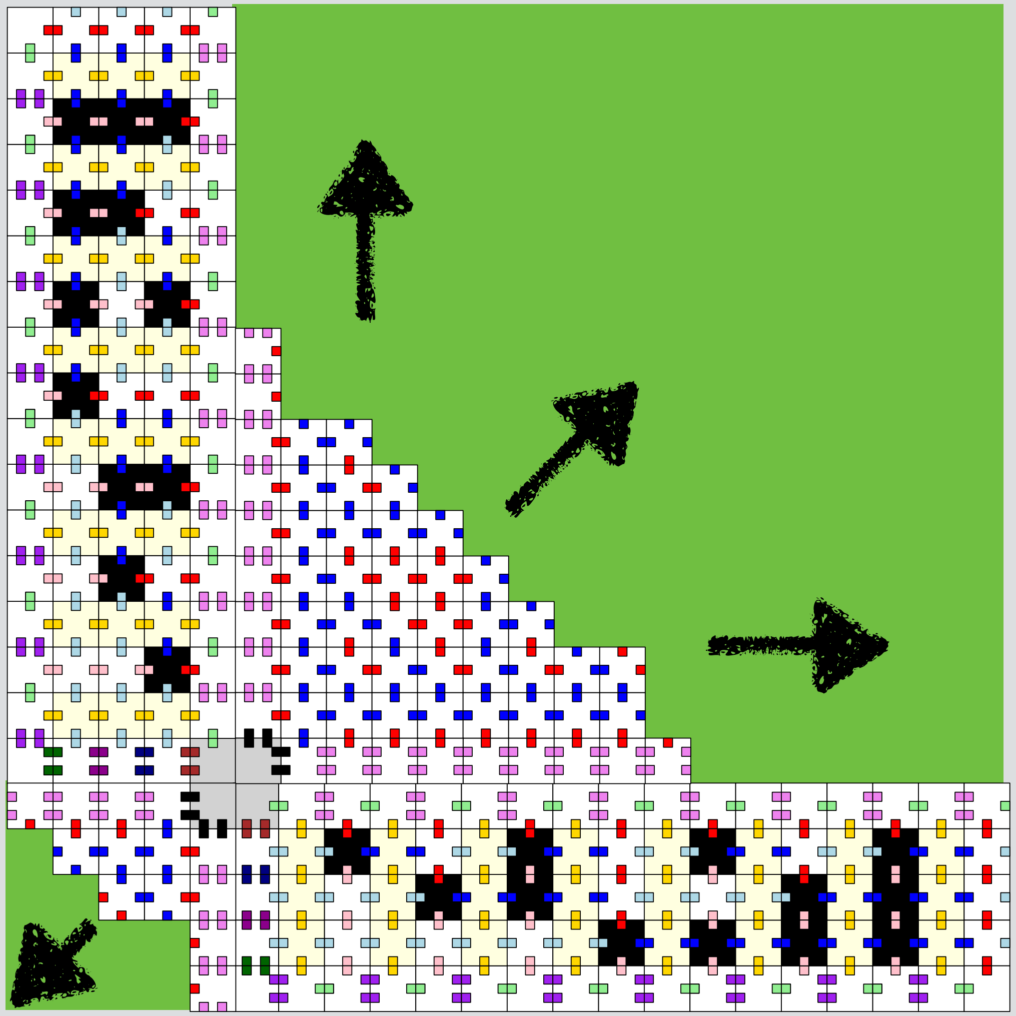

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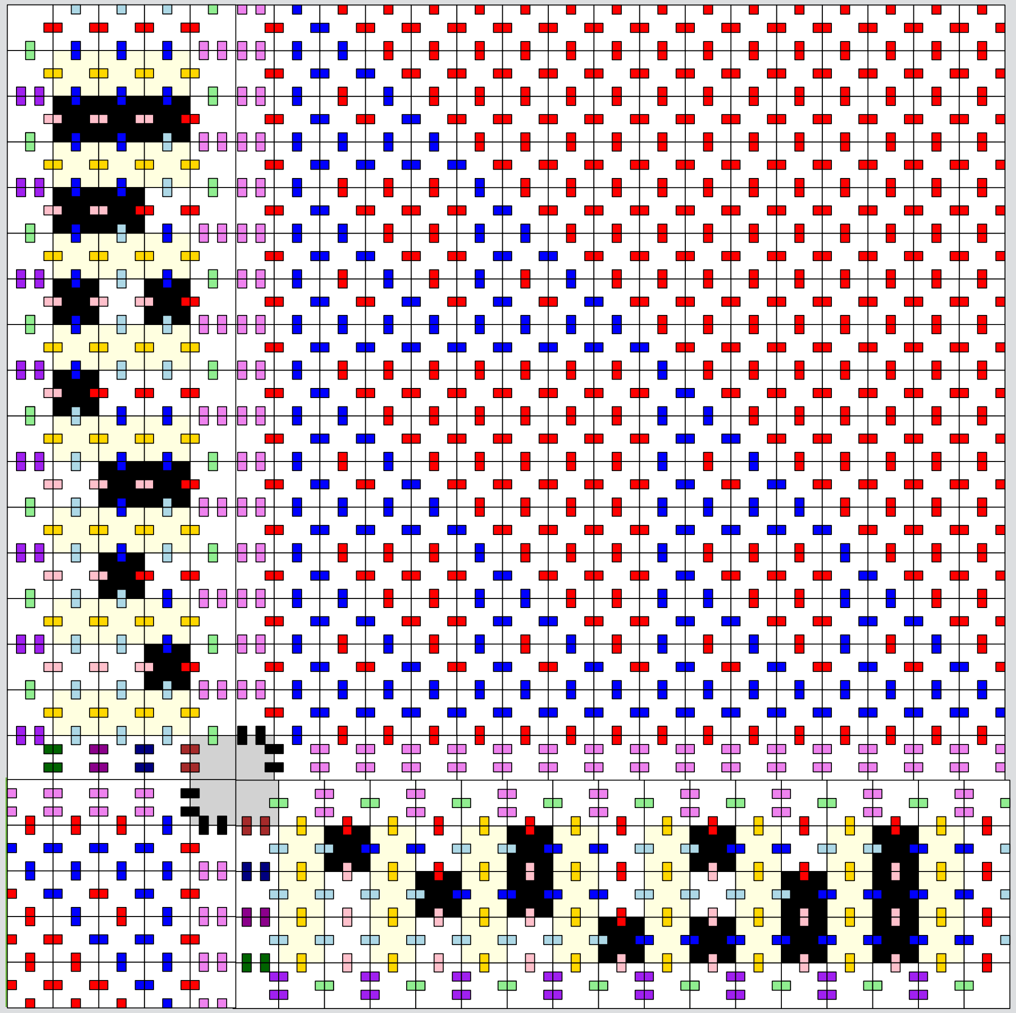

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- Assembling n x n square at any τ requires Ω(log(n)/loglog(n))-sized tile set w.h.p. [RW 2000]

    - Tile set of size t contains O(t*log(t)) bits of information, n contains log(n) bits w.h.p.

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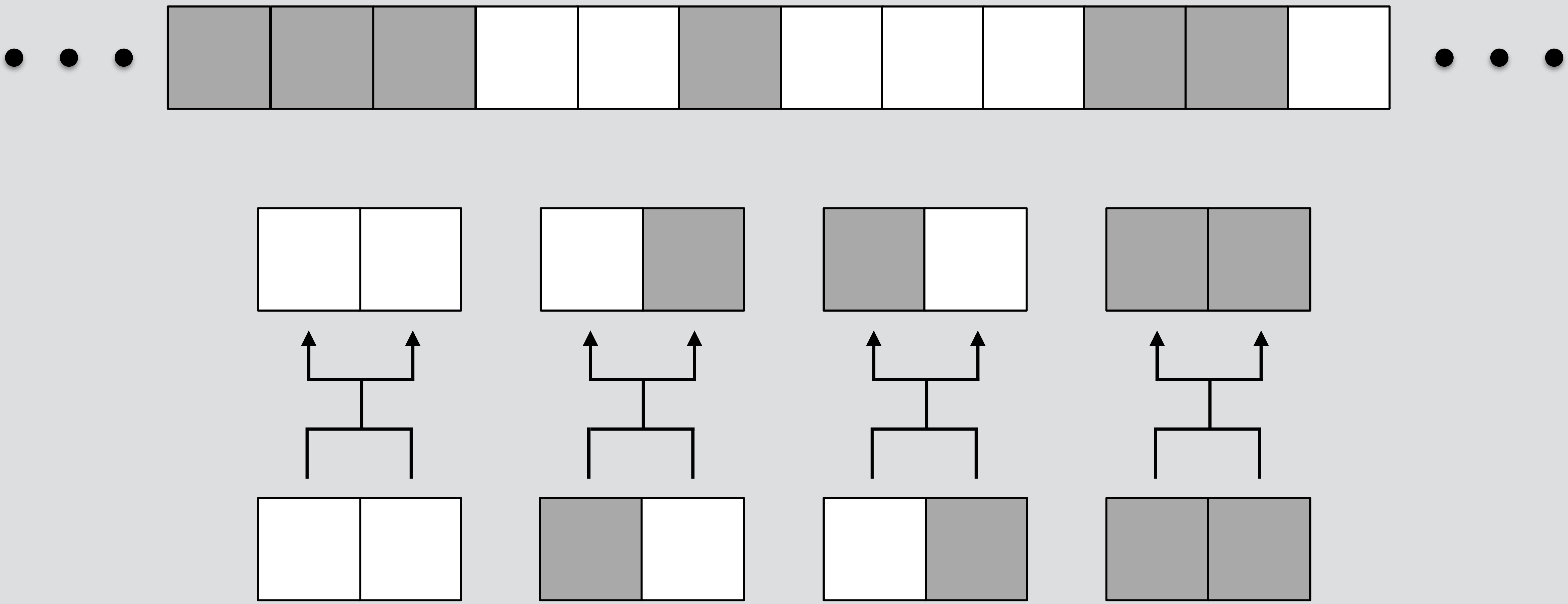

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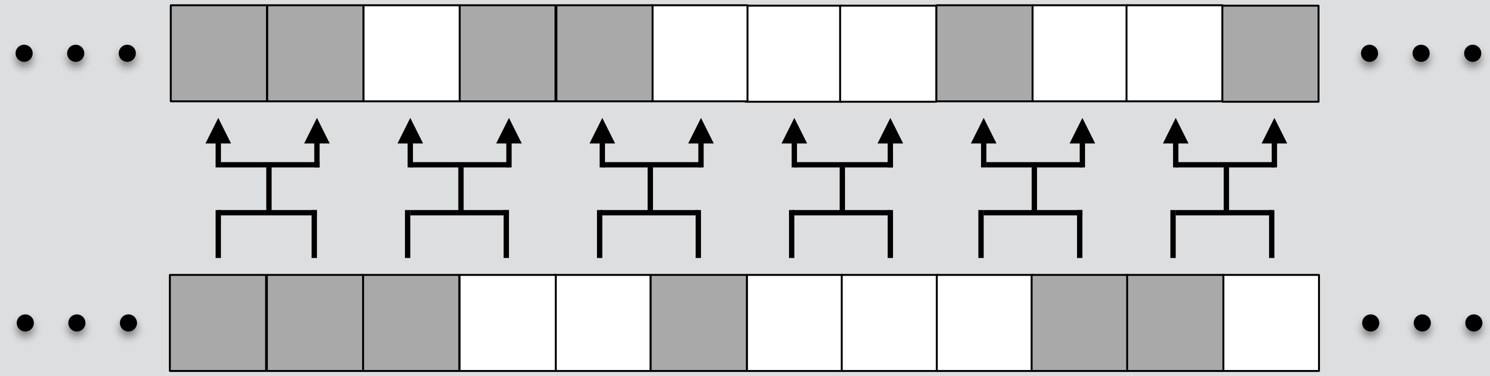

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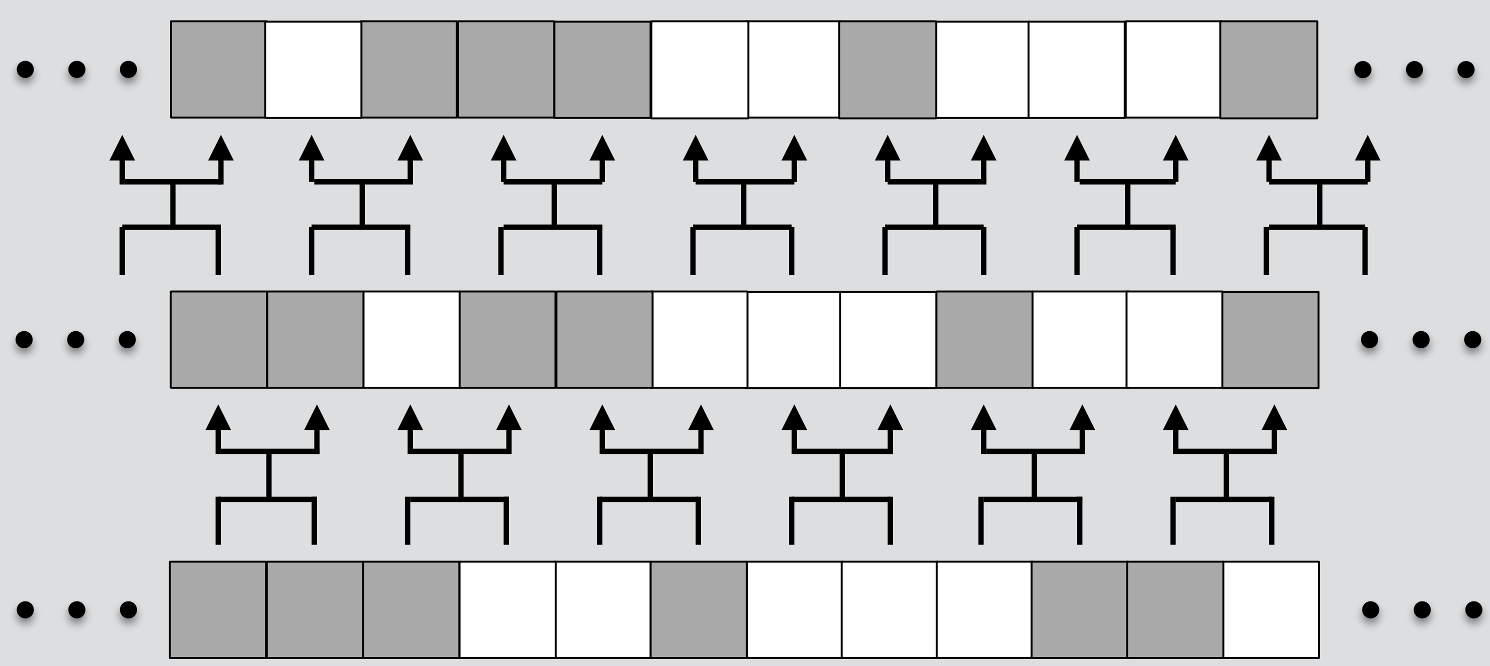

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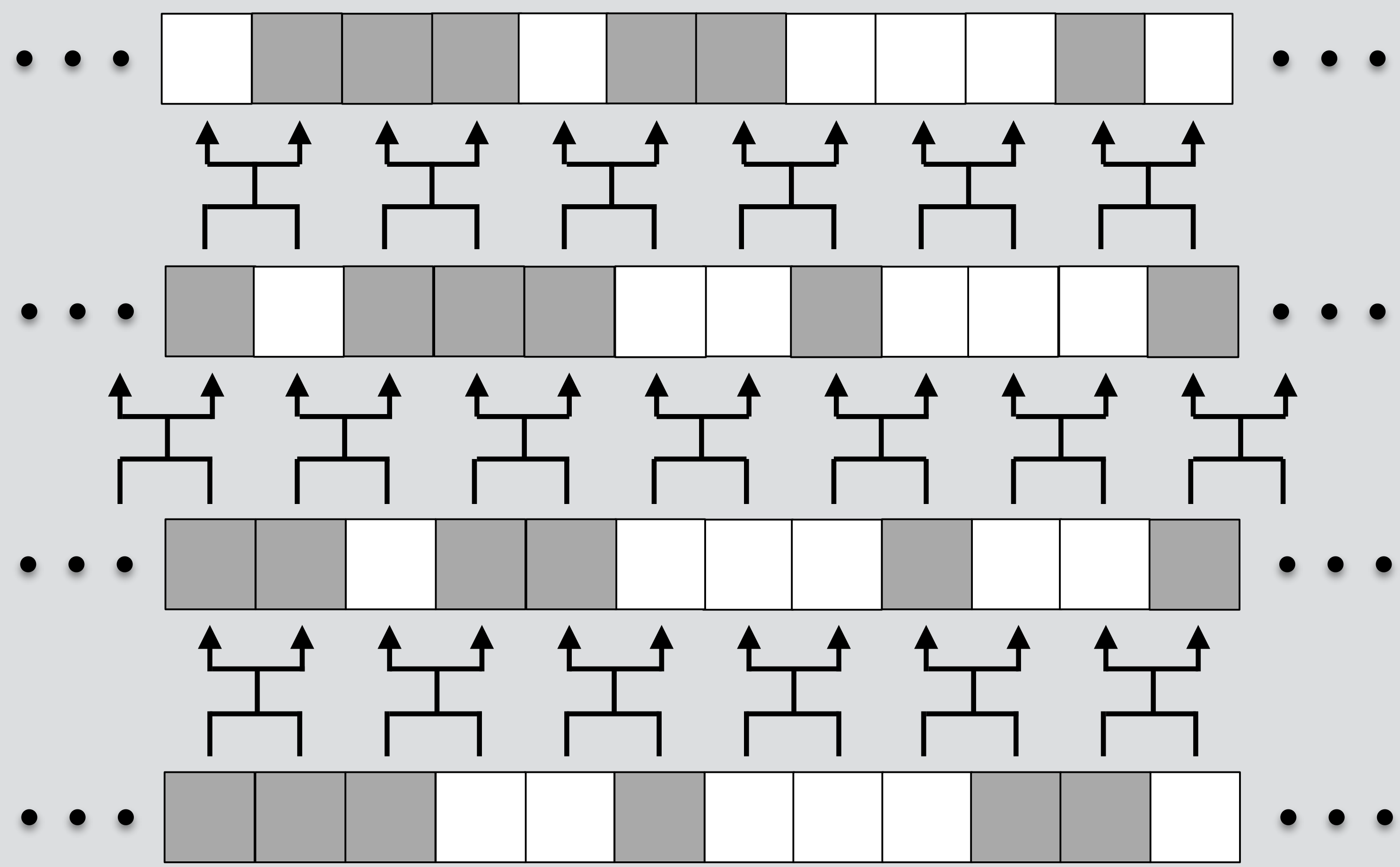

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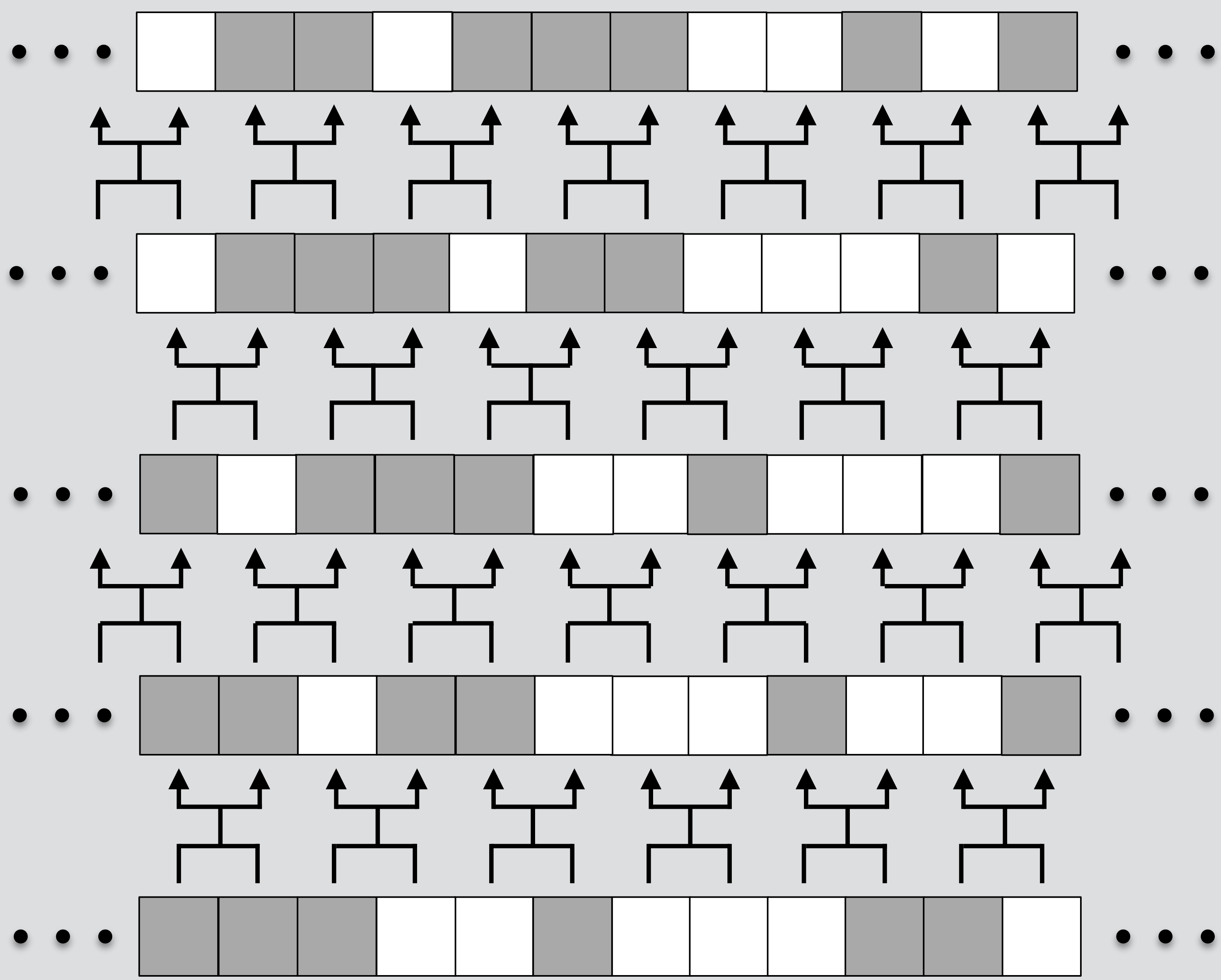

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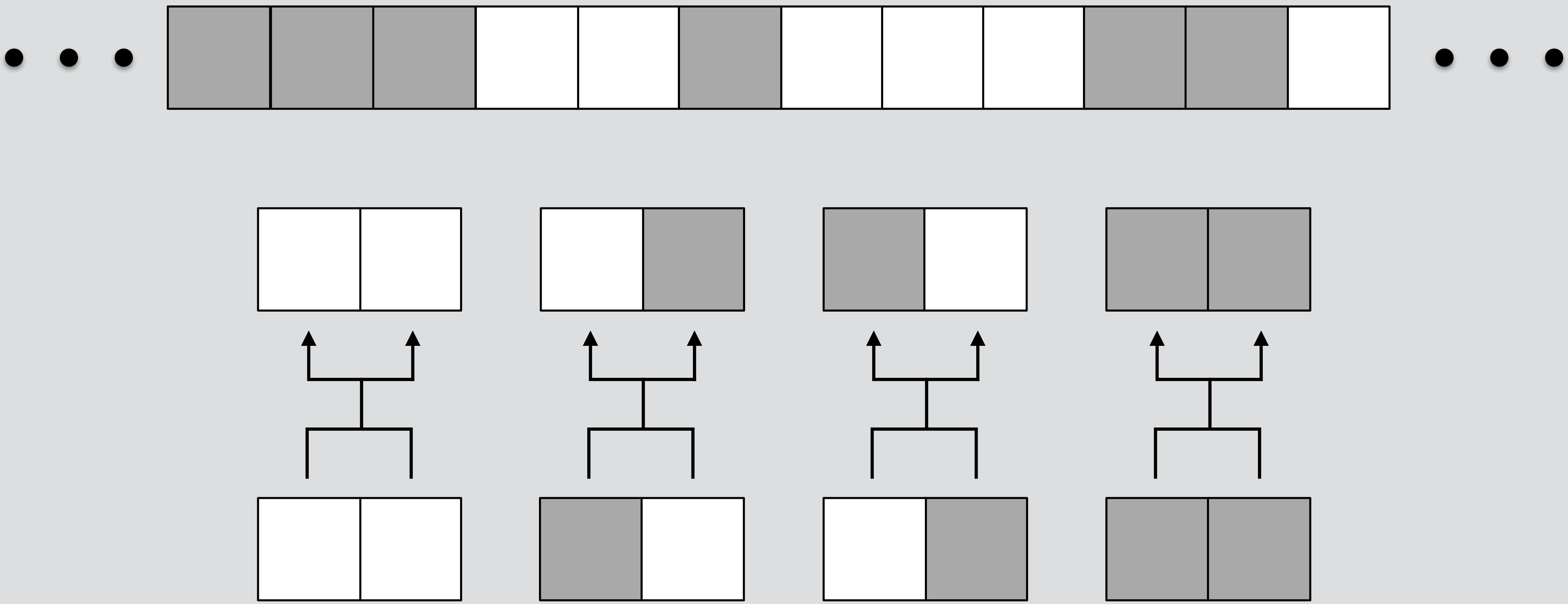

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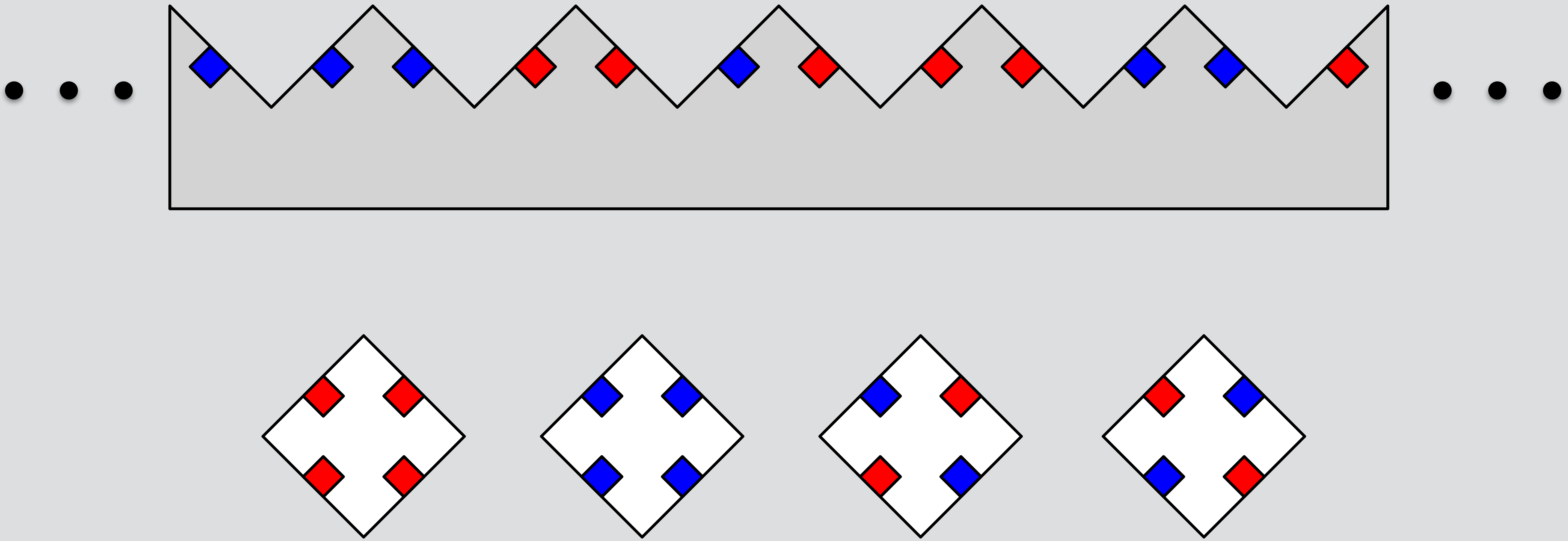

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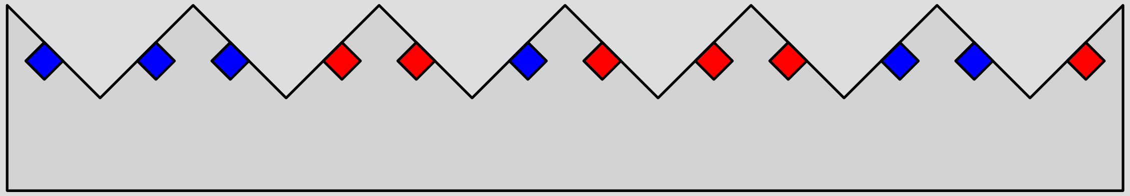

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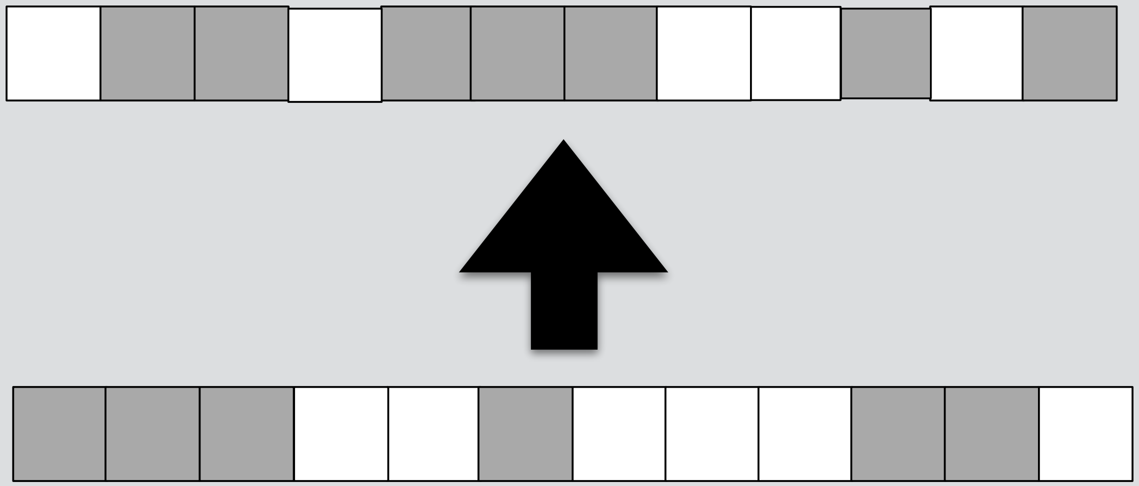

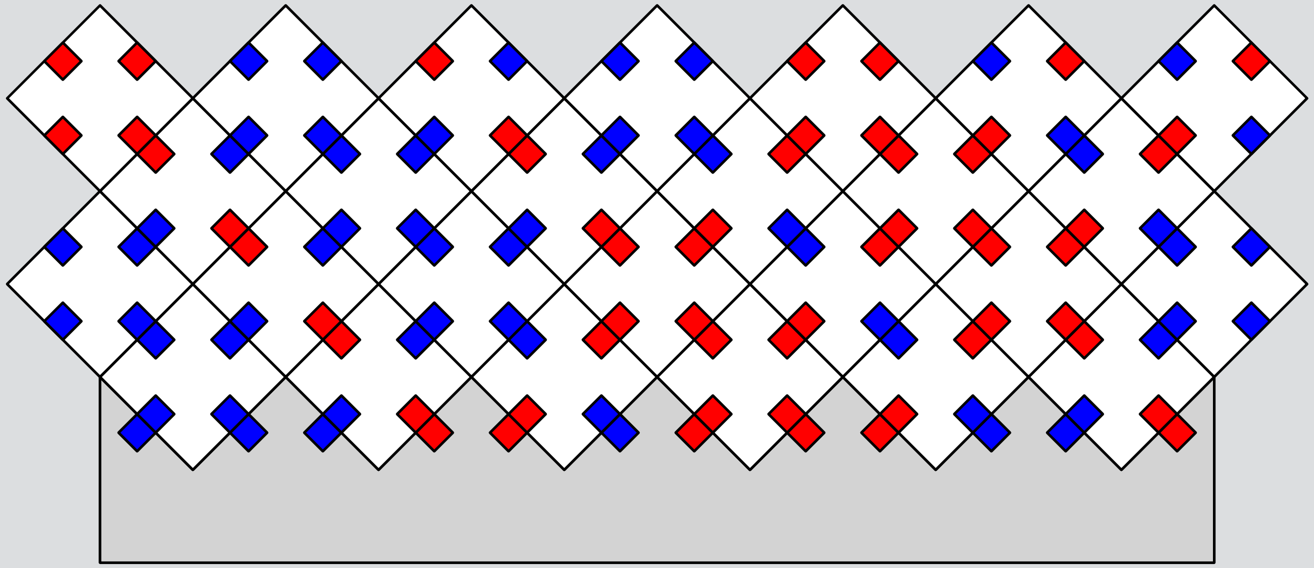

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    - Simulate *blocked cellular automata*.
    - Blocked cellular automata are Turing-universal.
        - Tweak [Lindgren, Nordahl 1990] TU proof.

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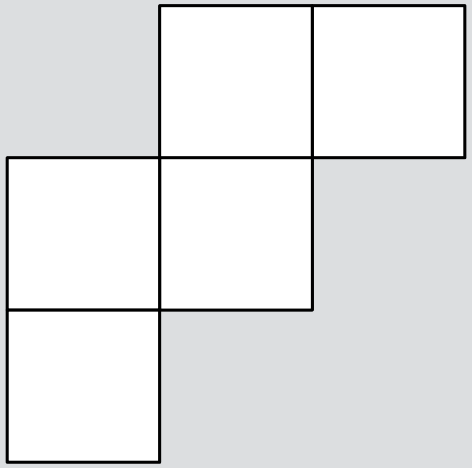
- Can construct a scaled version of any shape at $\tau = 2$ using O(K/log(K))-sized tile set. [Soloveichik, Winfree 2007]
    - K = Kolmogorov complexity of polyomino.



Polyomino

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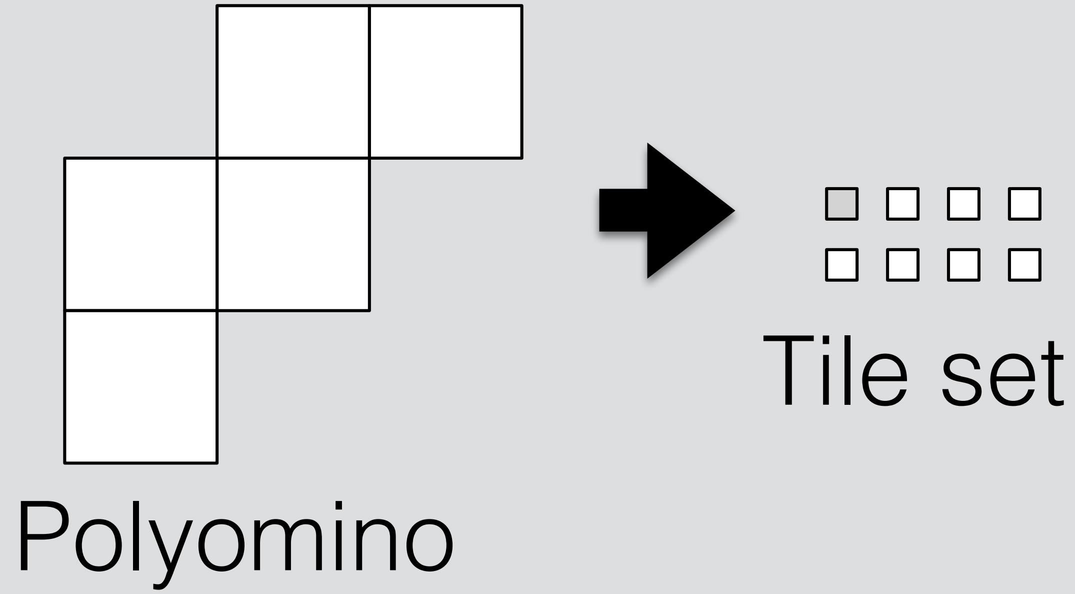

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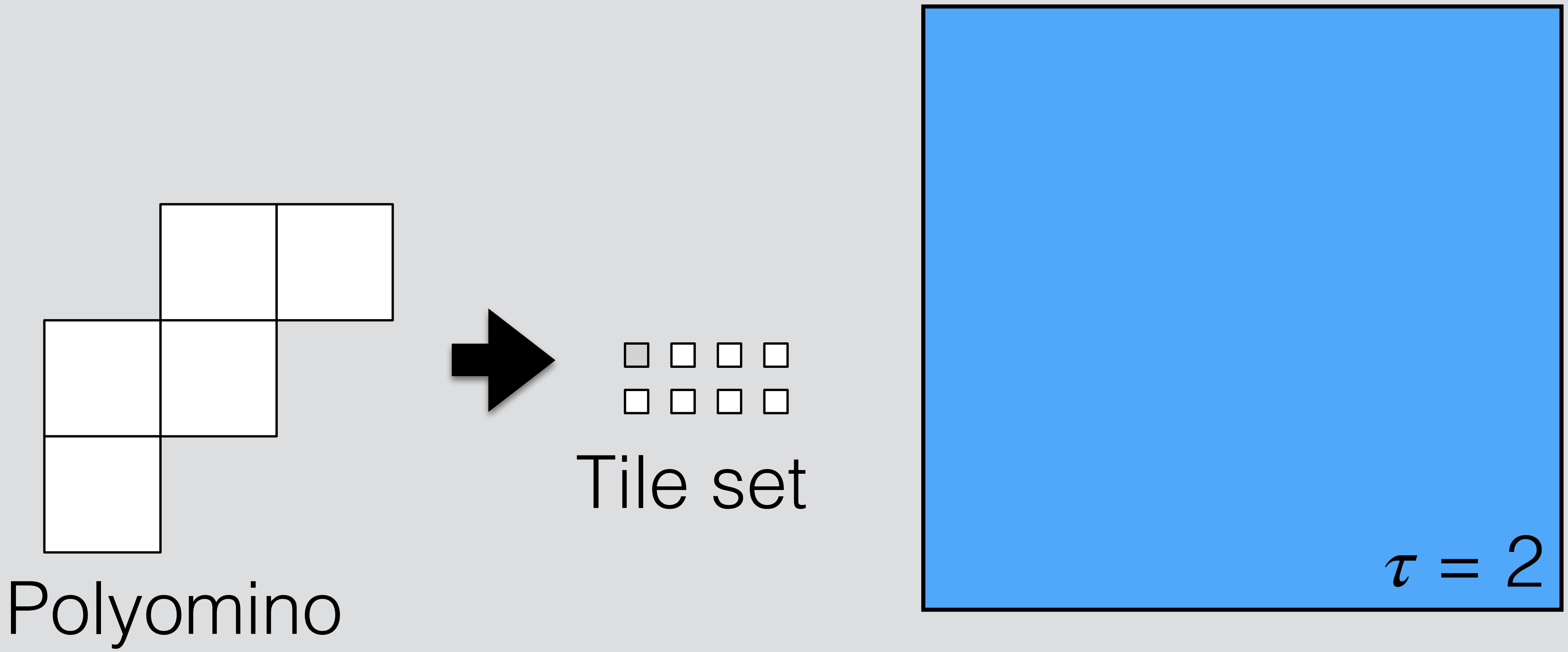

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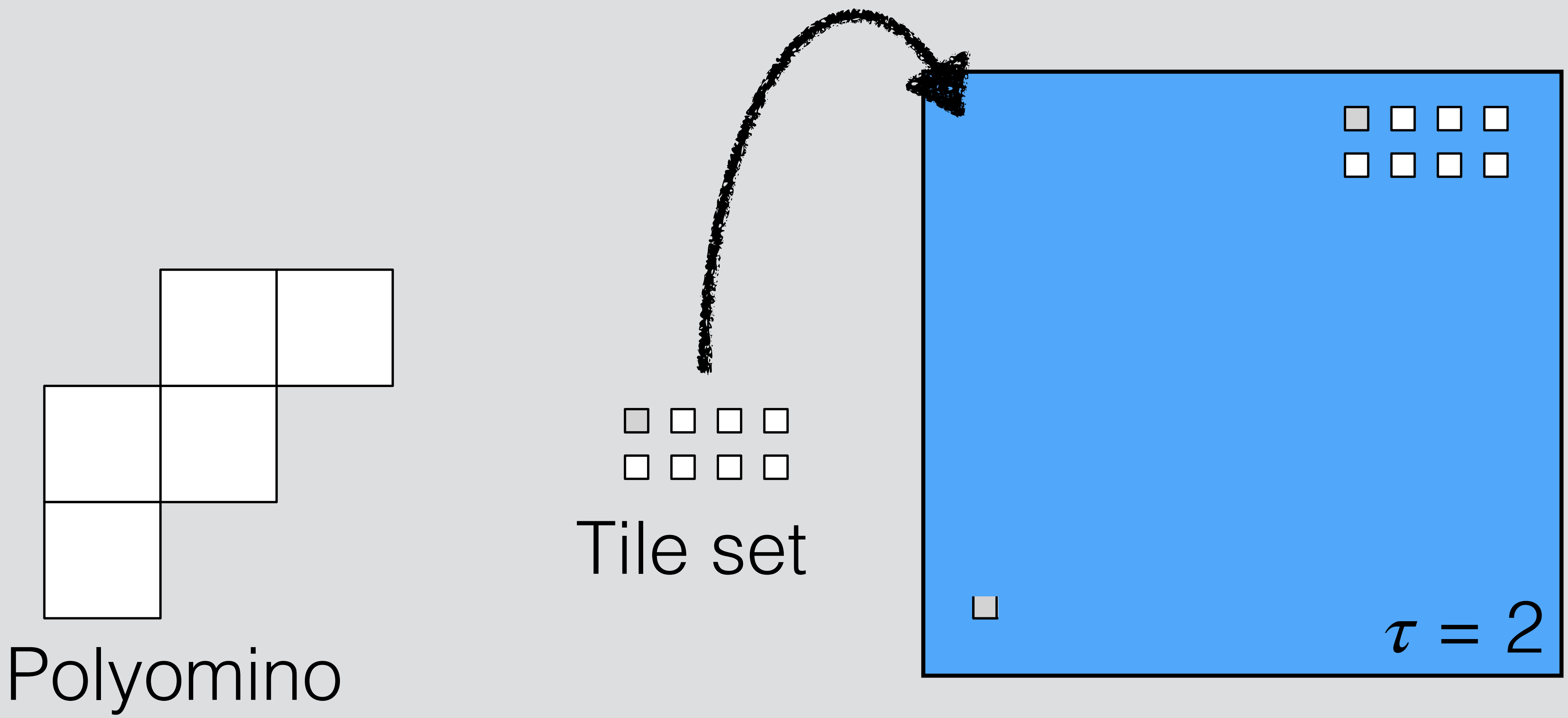

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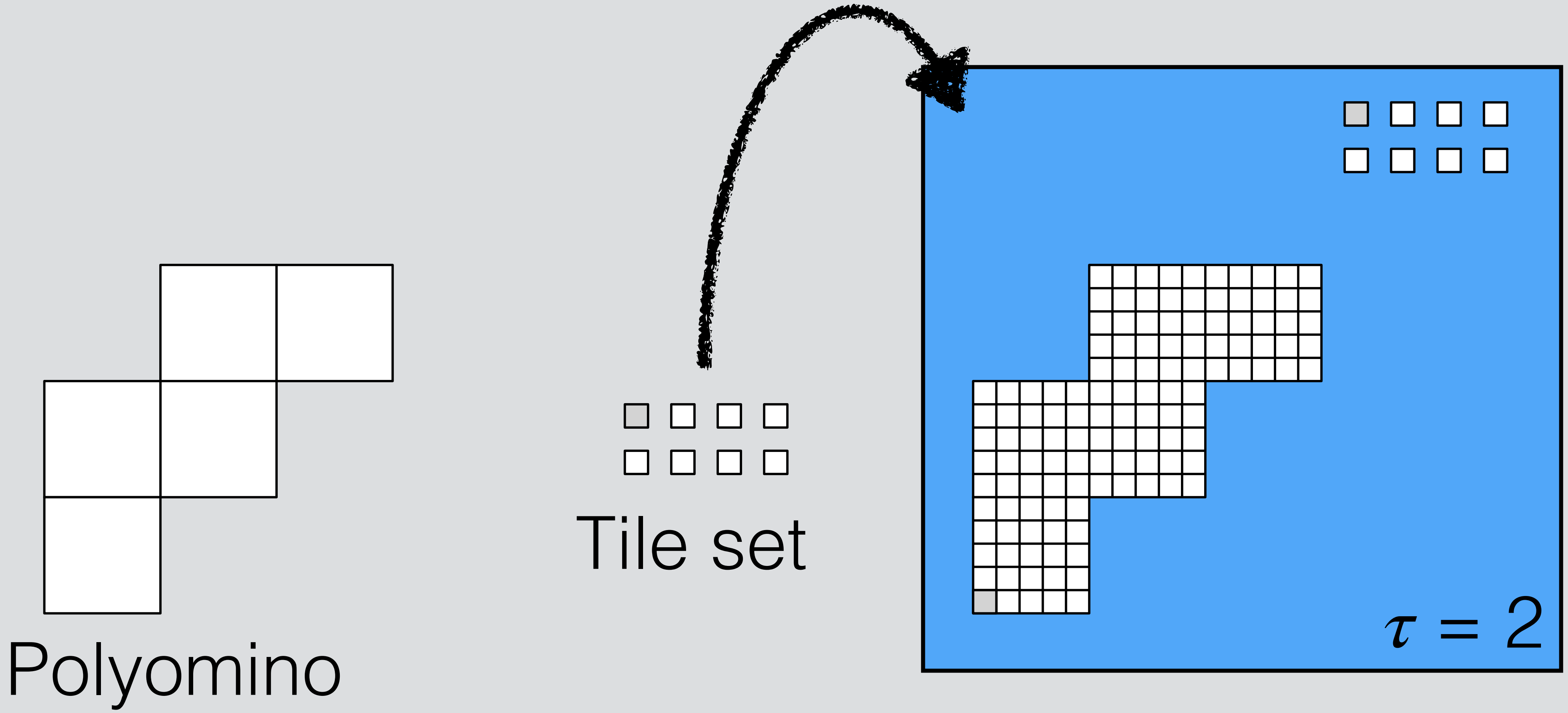

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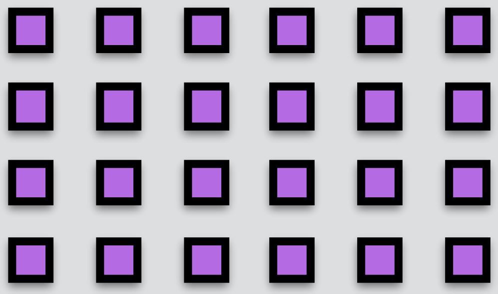

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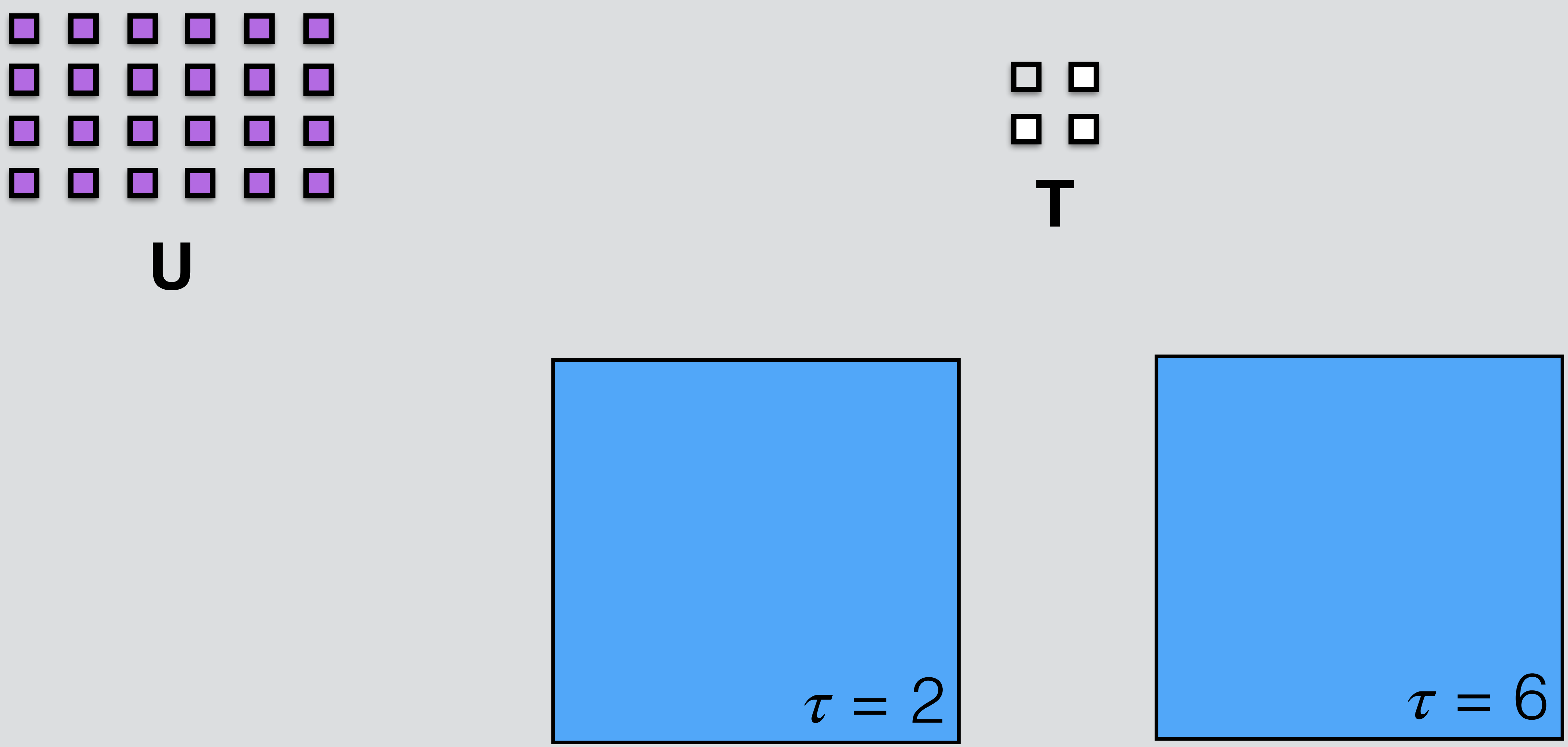

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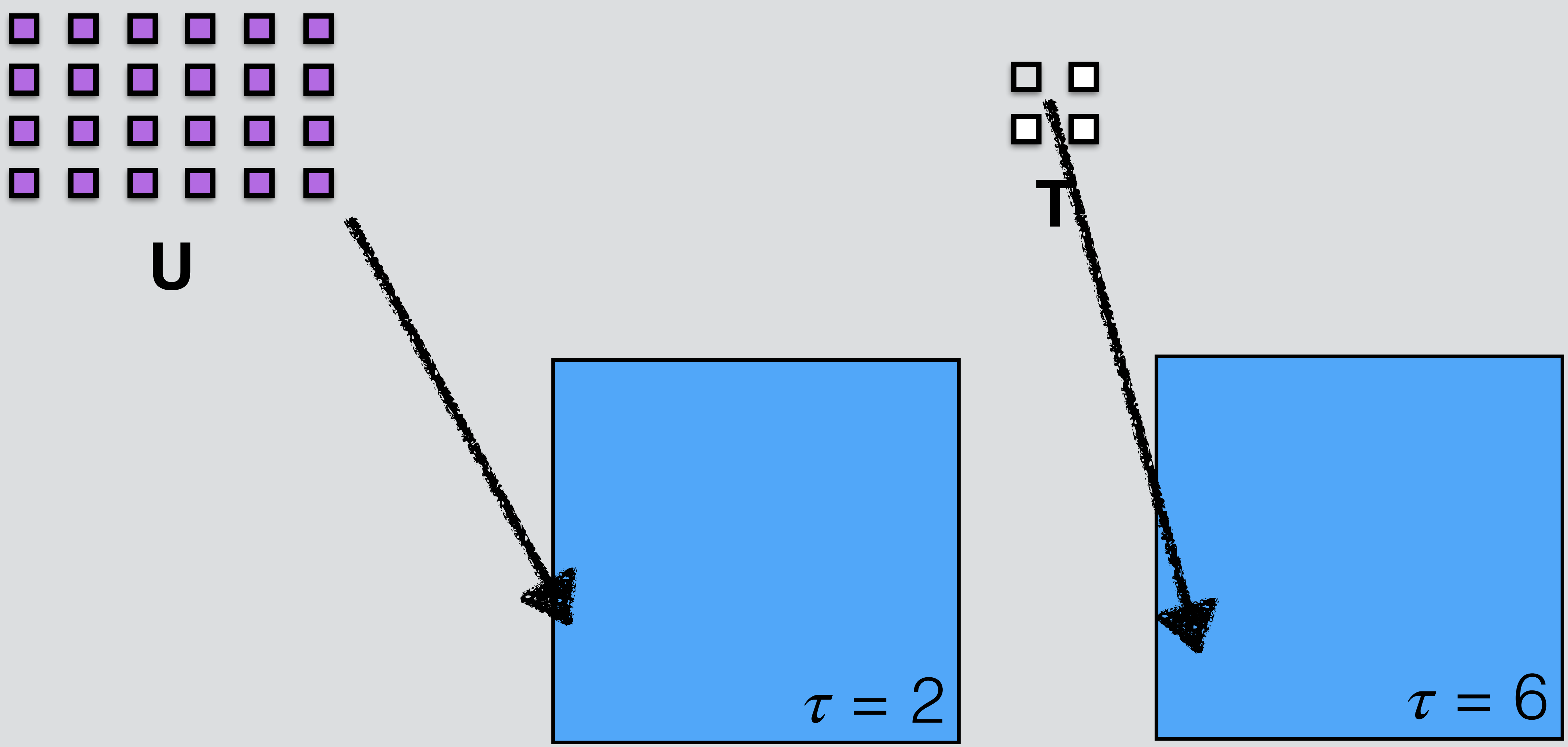

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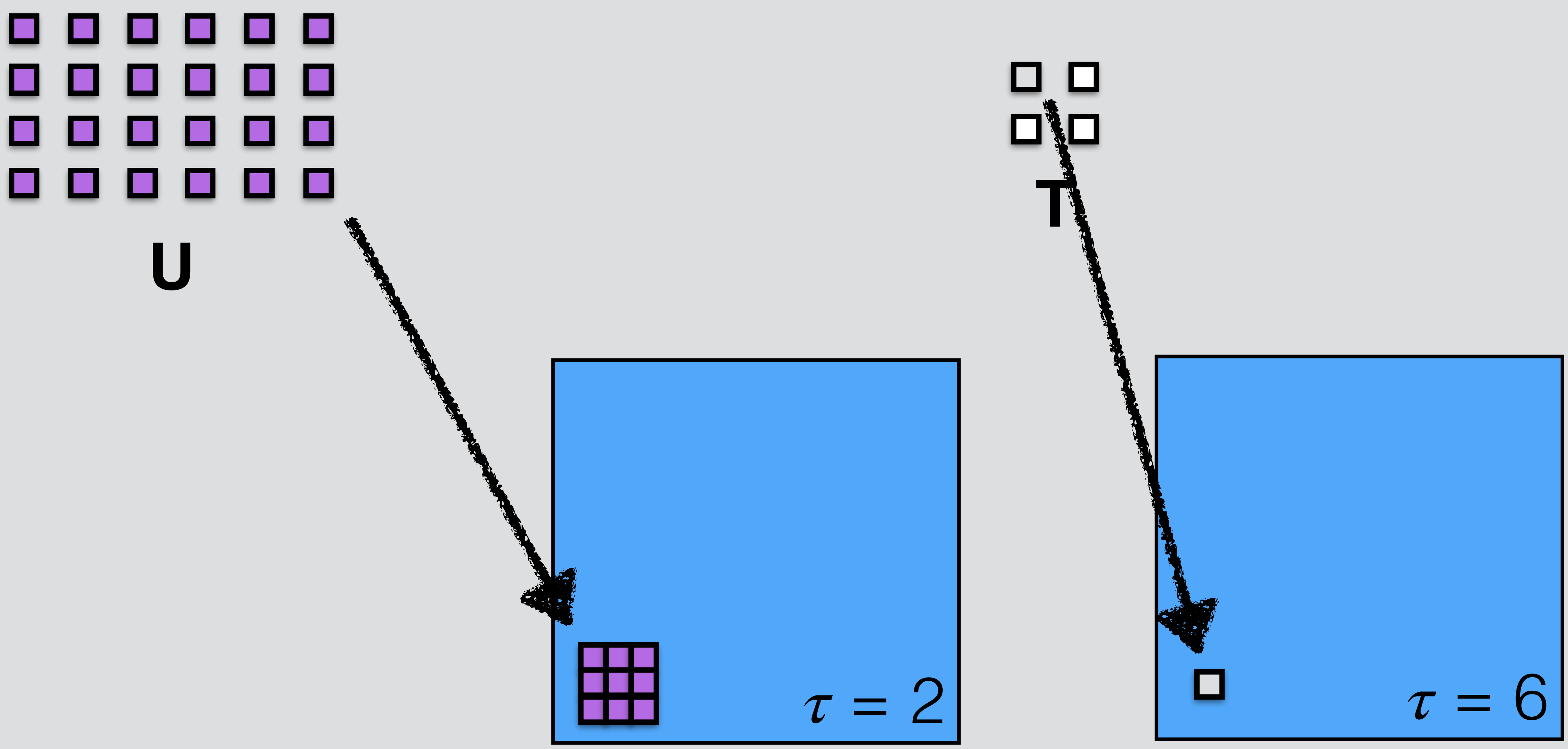

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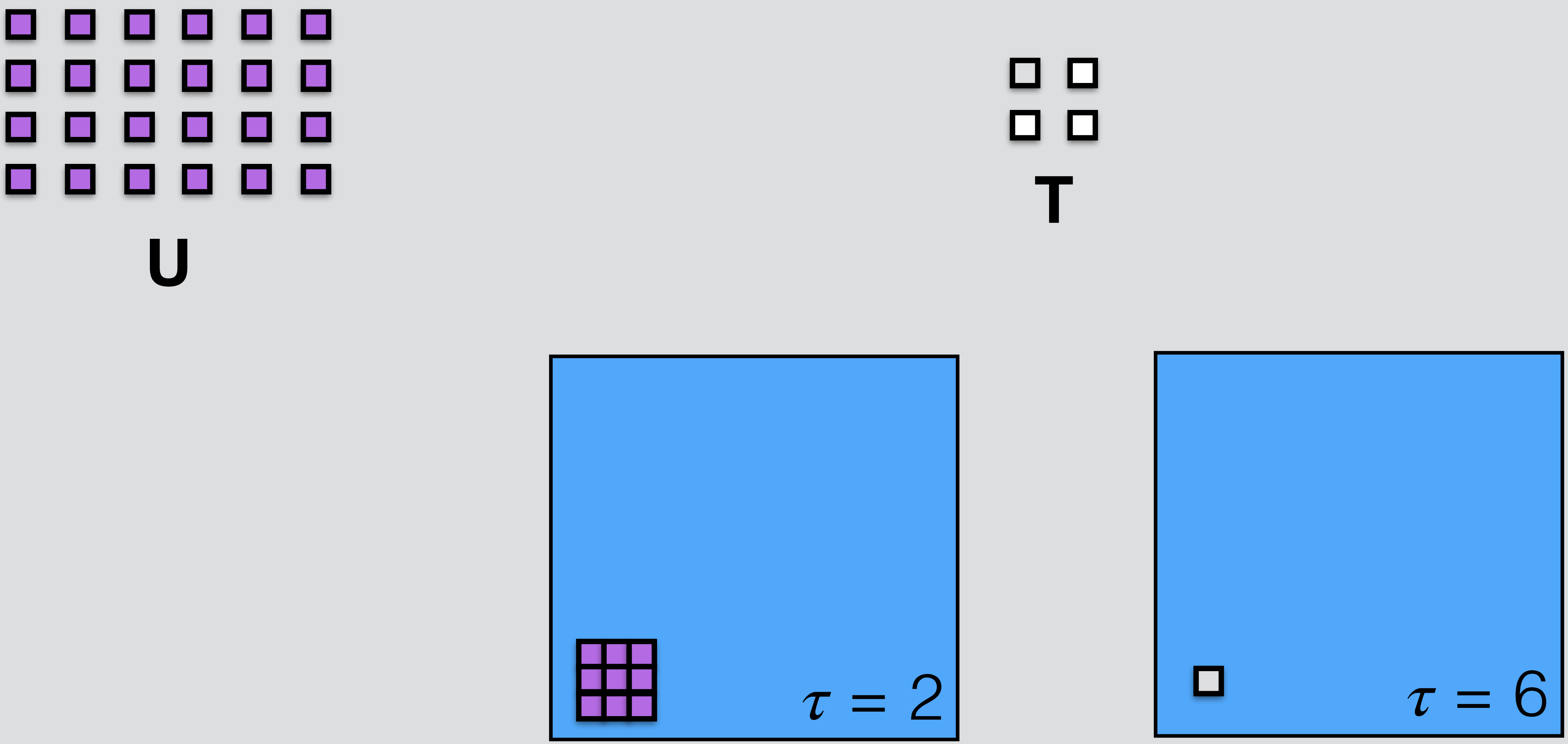

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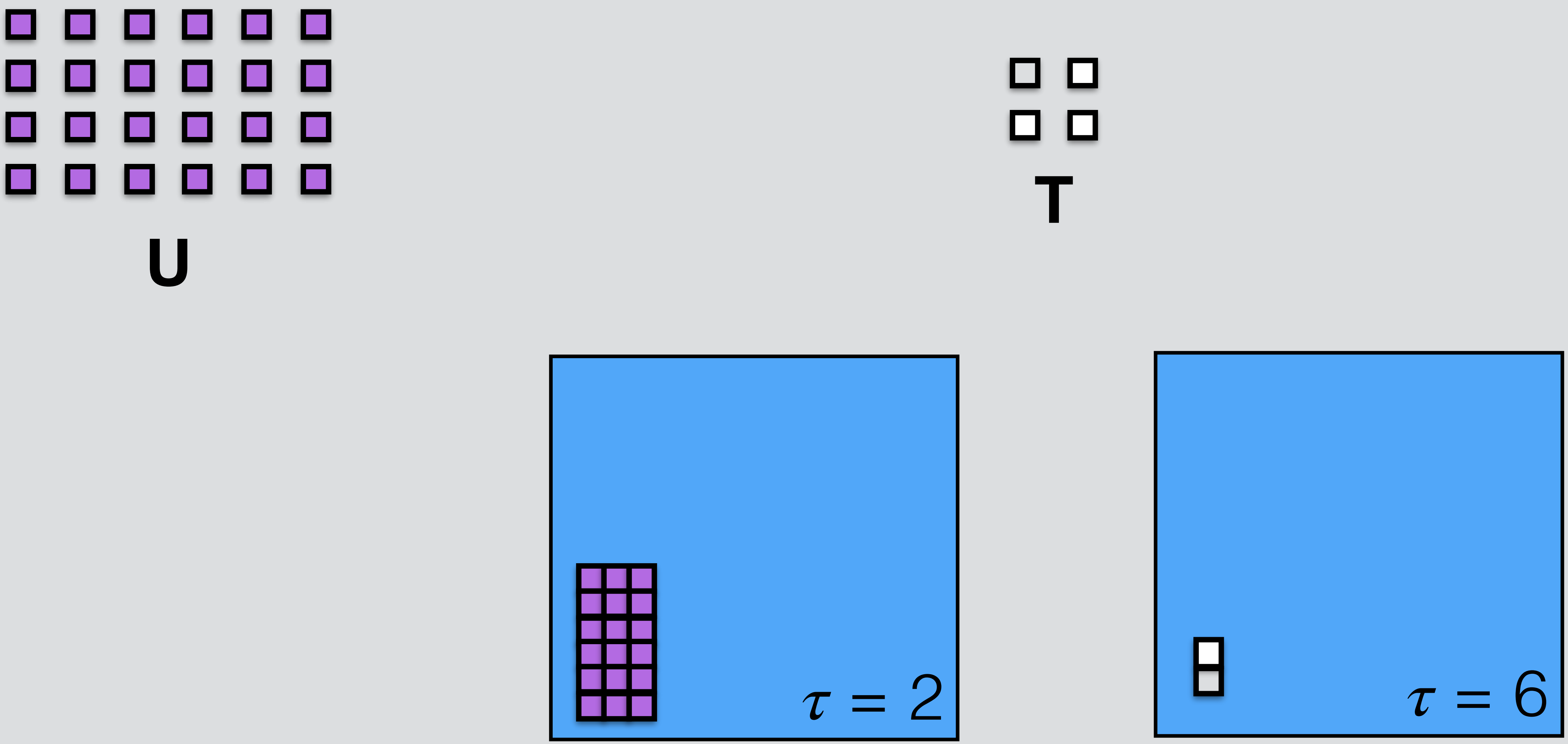

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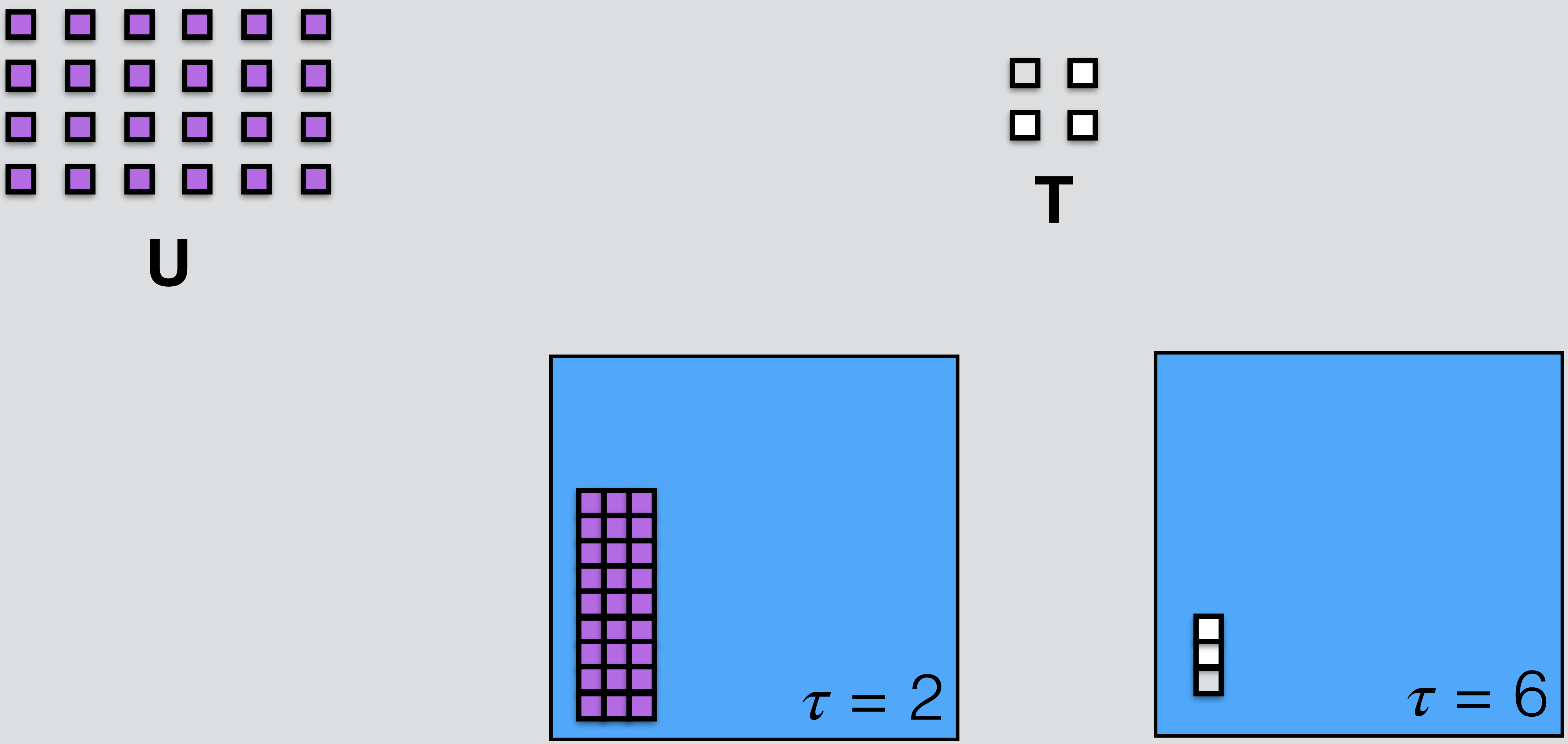

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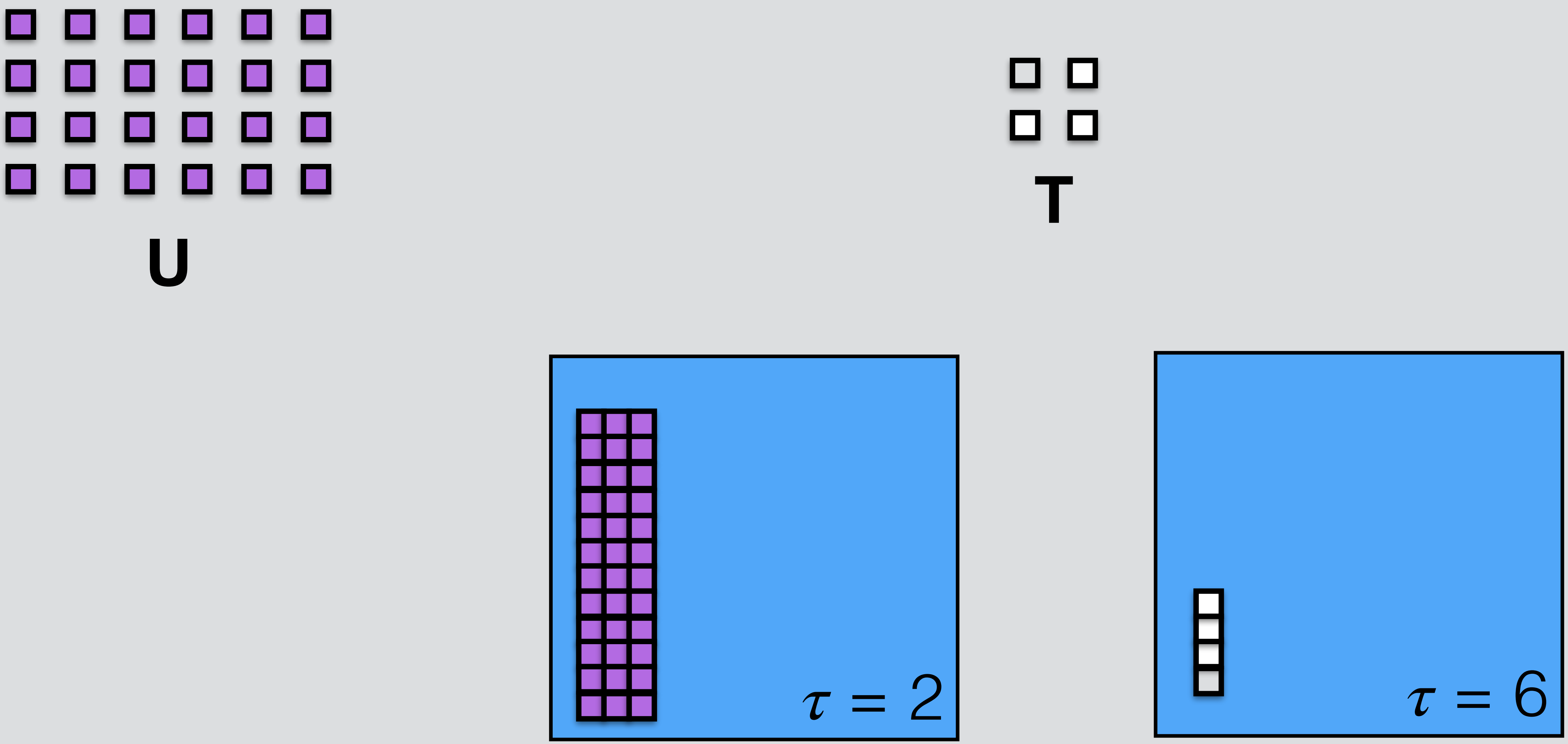

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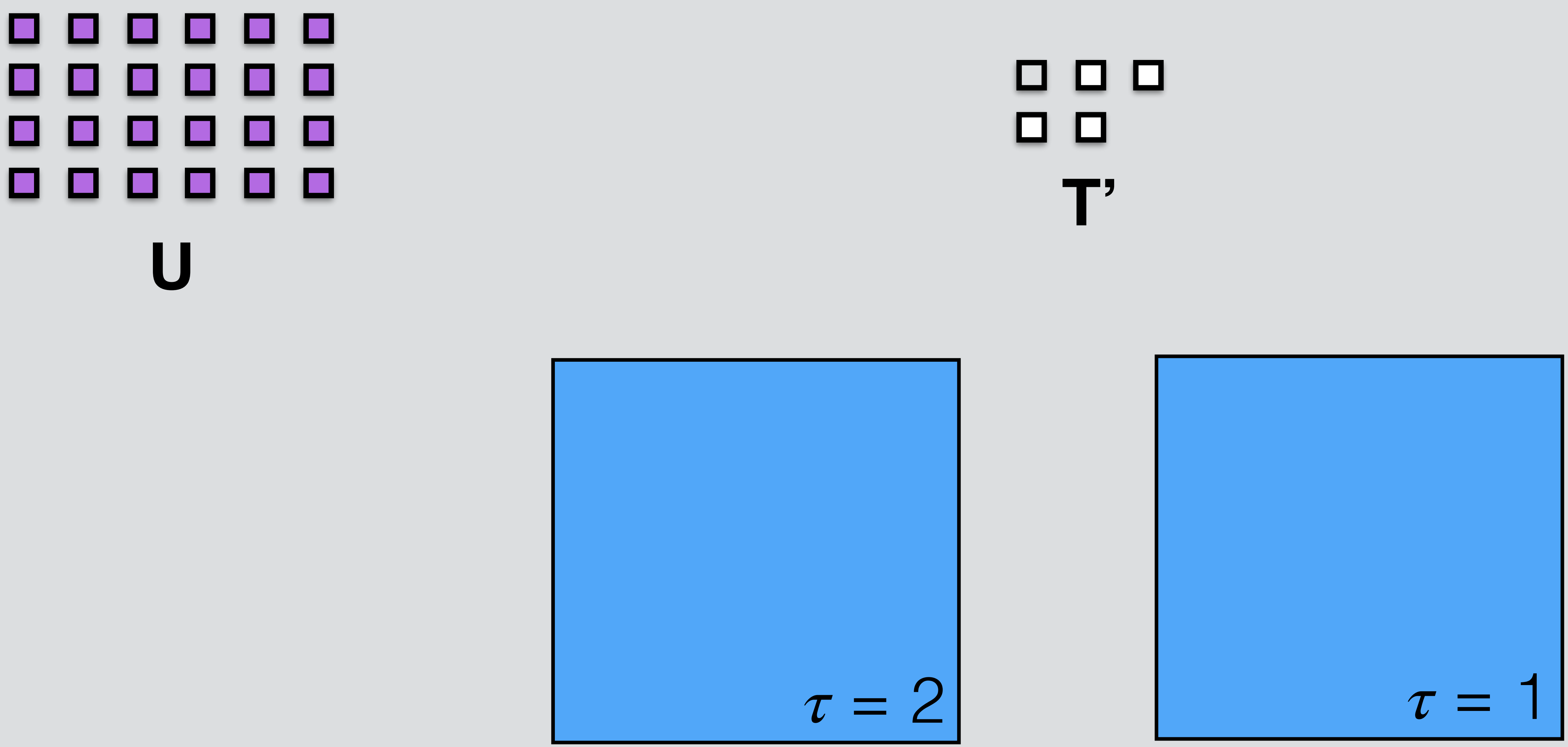

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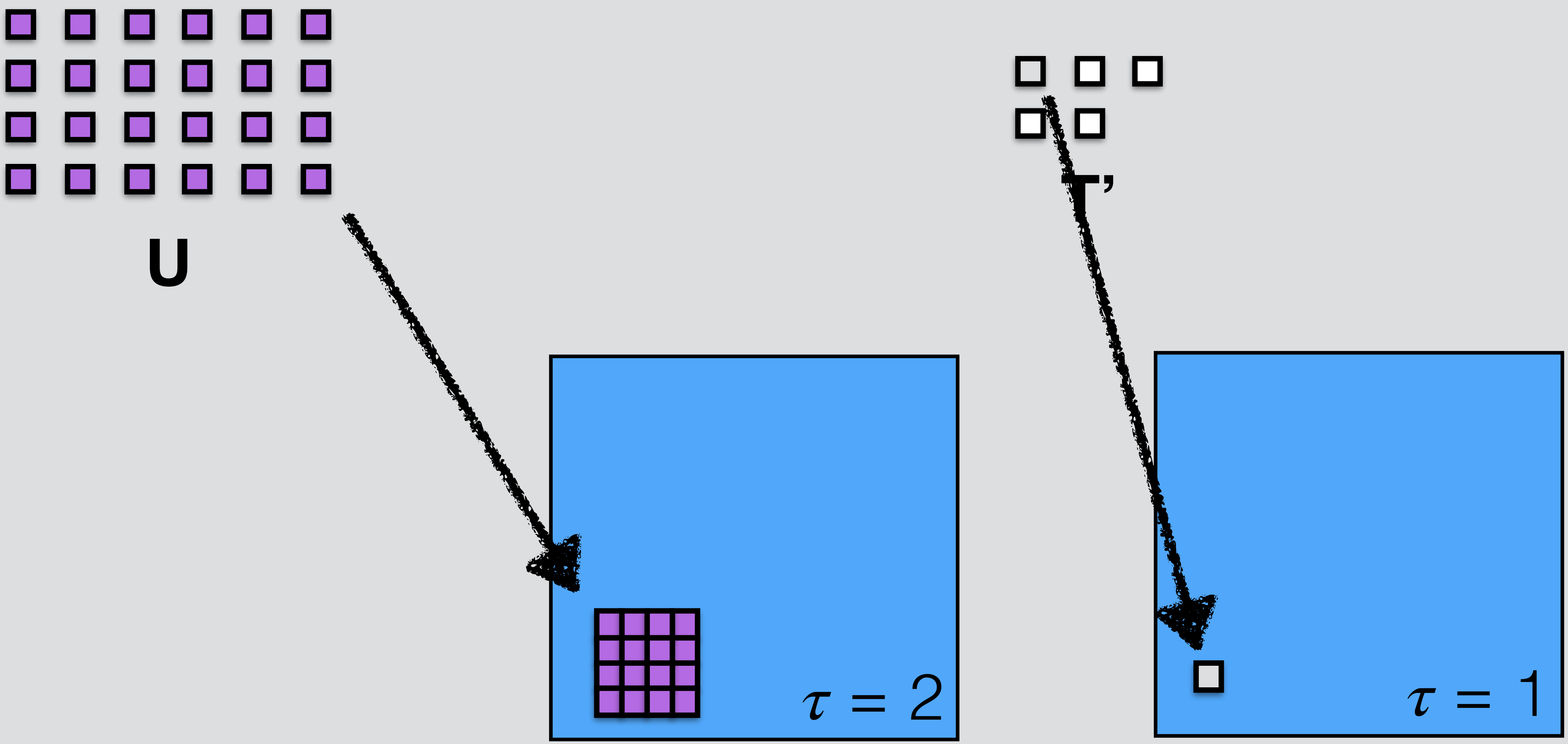

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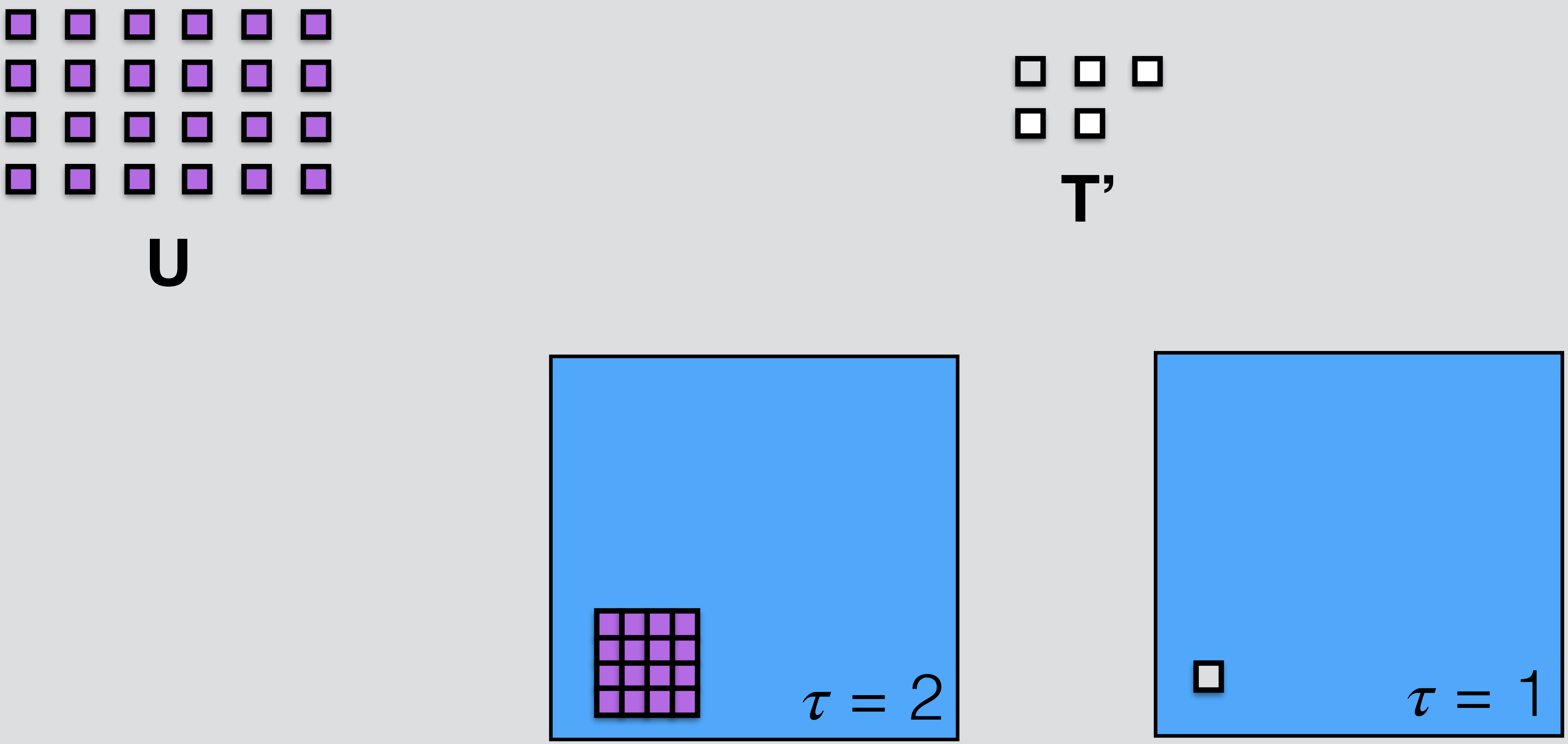

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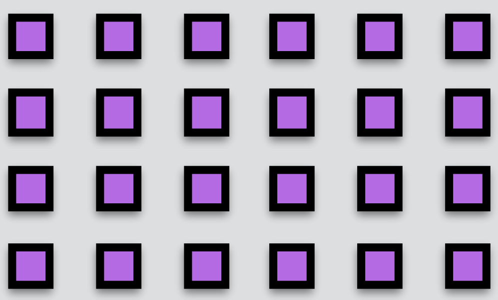

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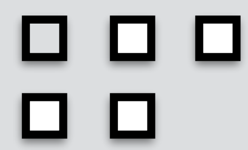
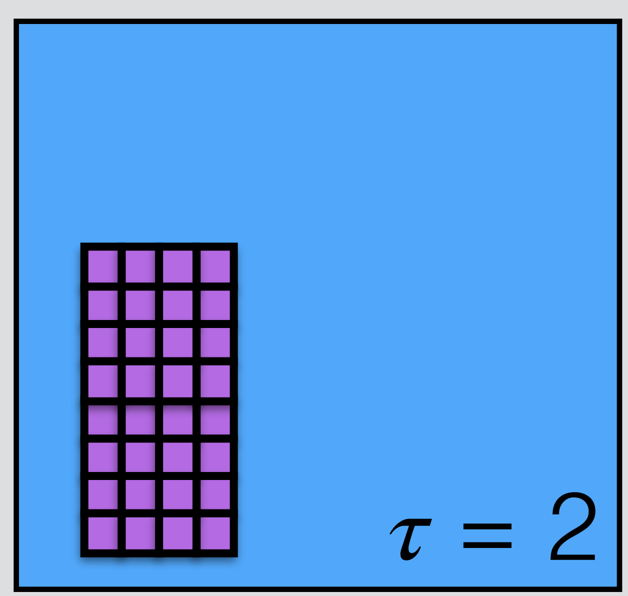
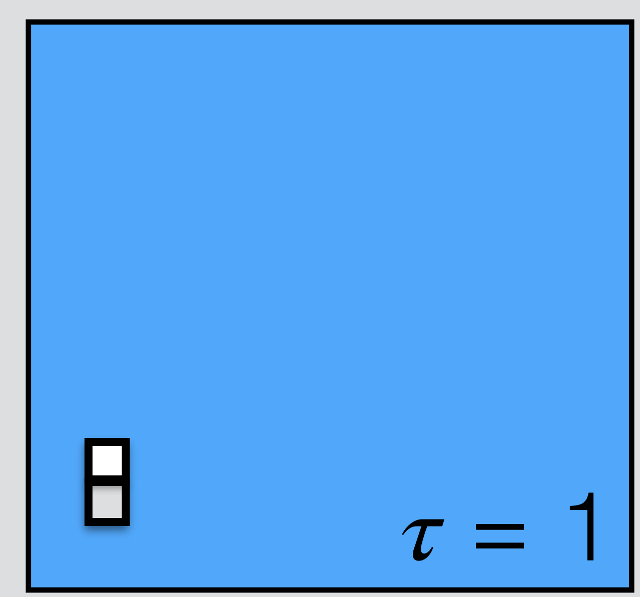


T'





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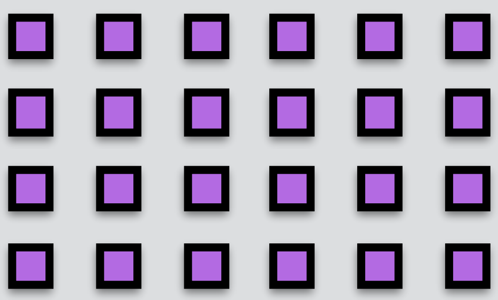

U

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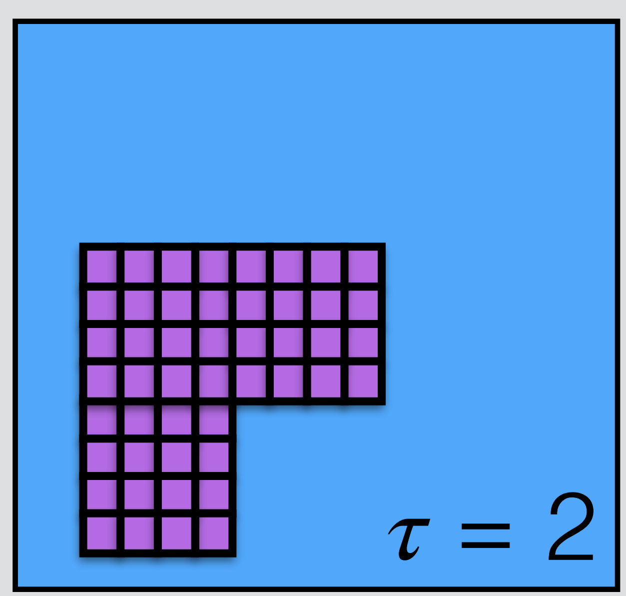

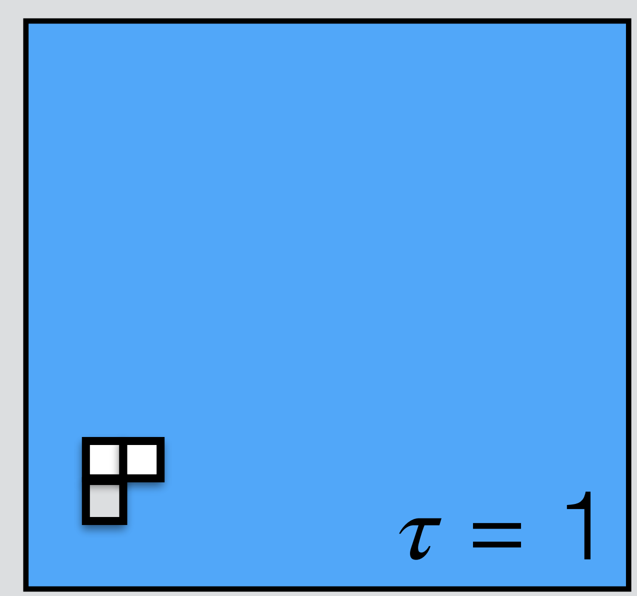

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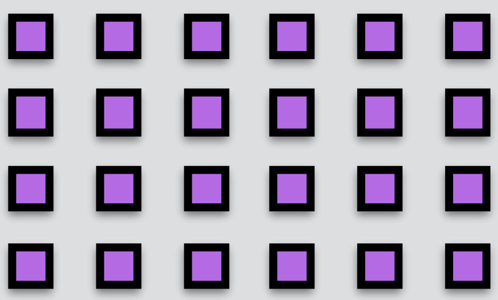

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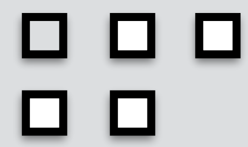
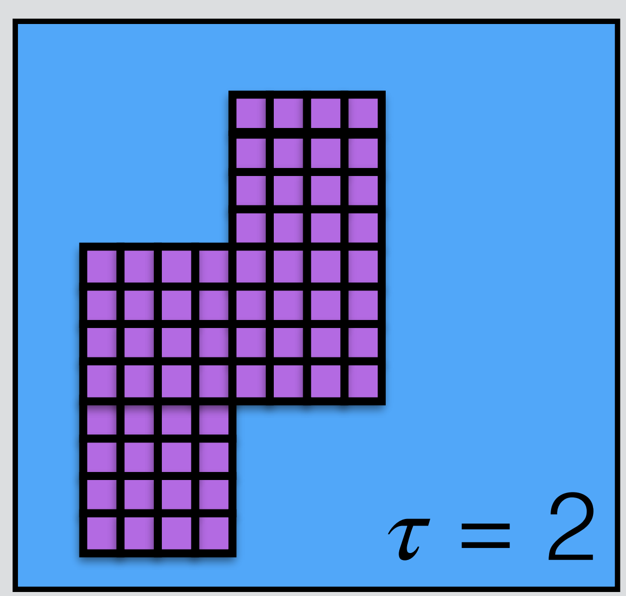
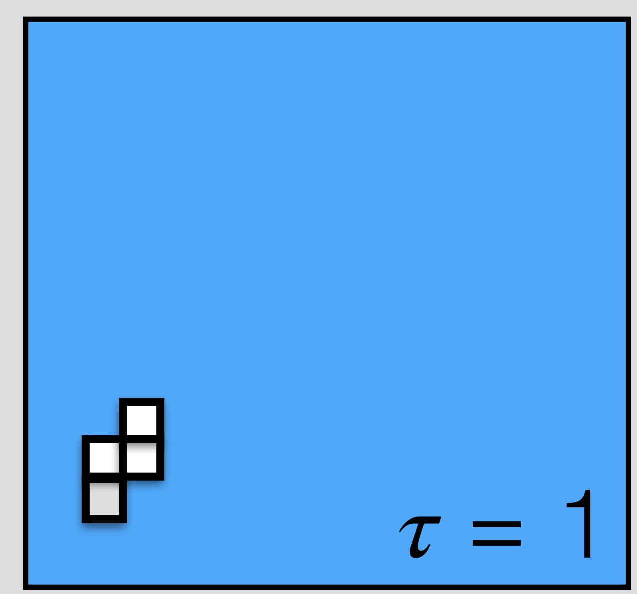


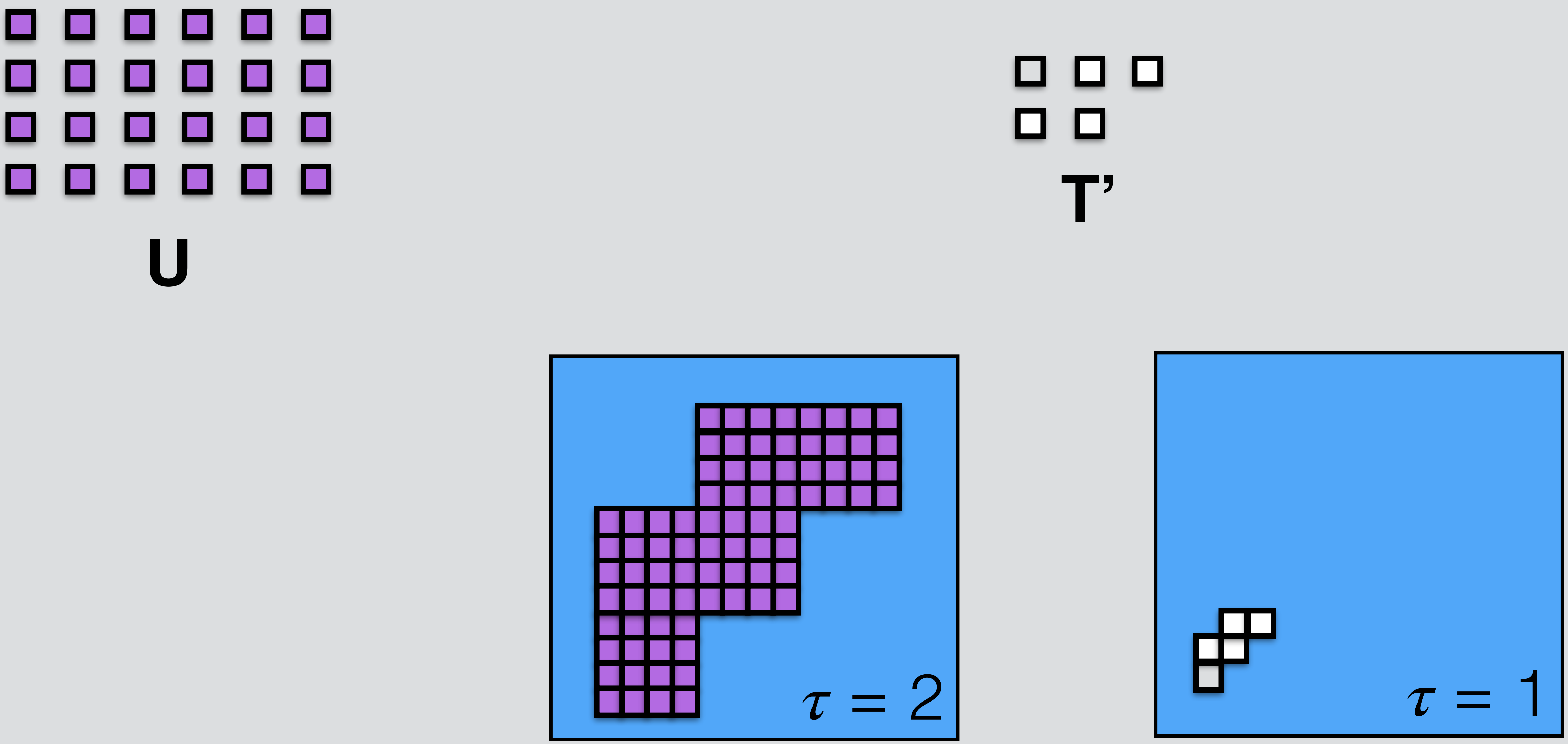
T'





Some Known Results

- *Intrinsic Universality:* There exists a single tile set **U** at τ = 2 that simulates any tile assembly system, given an appropriate seed assembly. [Doty et al. 2012]



Role of Tile Shape

- Assembly is mostly combinatorial (glue-based).
- t tiles contain O(t*log(t)) bits of information.
- Assembling shapes requires arbitrarily large tile sets.

Role of Tile Shape

- Assembling shapes requires arbitrarily large tile sets.
- This is bad in practice:
    - Relative concentrations are low (slow assembly).
    - Number of glues is high (hard to engineer).
- This is theoretically unsatisfying: geometry of self-assembly is trivialized/ignored.

Our Work:
One-Tile Systems

Our Work

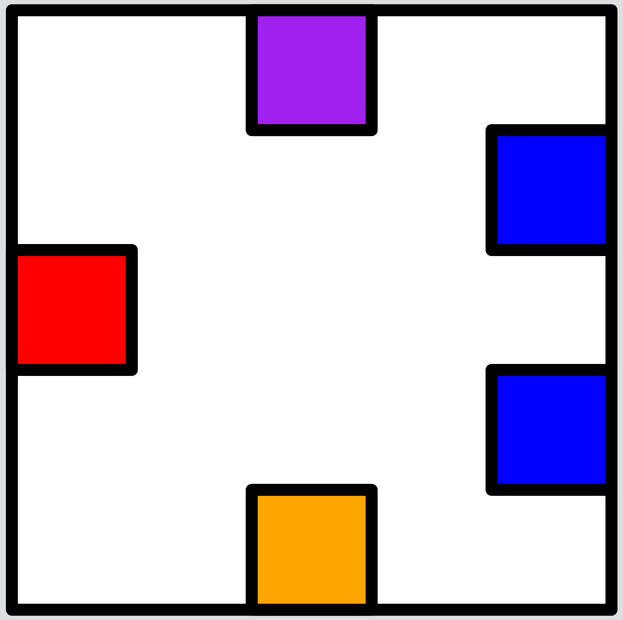
- Generalize square tiles to polygonal tiles.
    - Removes $\Omega(\log(n)/\log\log(n))$ lower bound.
- Prove results on a single polygonal tile can do:
    - With rotation.
    - Without rotation.

Our Work

- Generalize square tiles to polygonal tiles.
    - Removes $\Omega(\log(n)/\log\log(n))$ lower bound.
- Prove results on a single polygonal tile can do:
    - With rotation. **Everything!**
    - Without rotation. **A few things.**

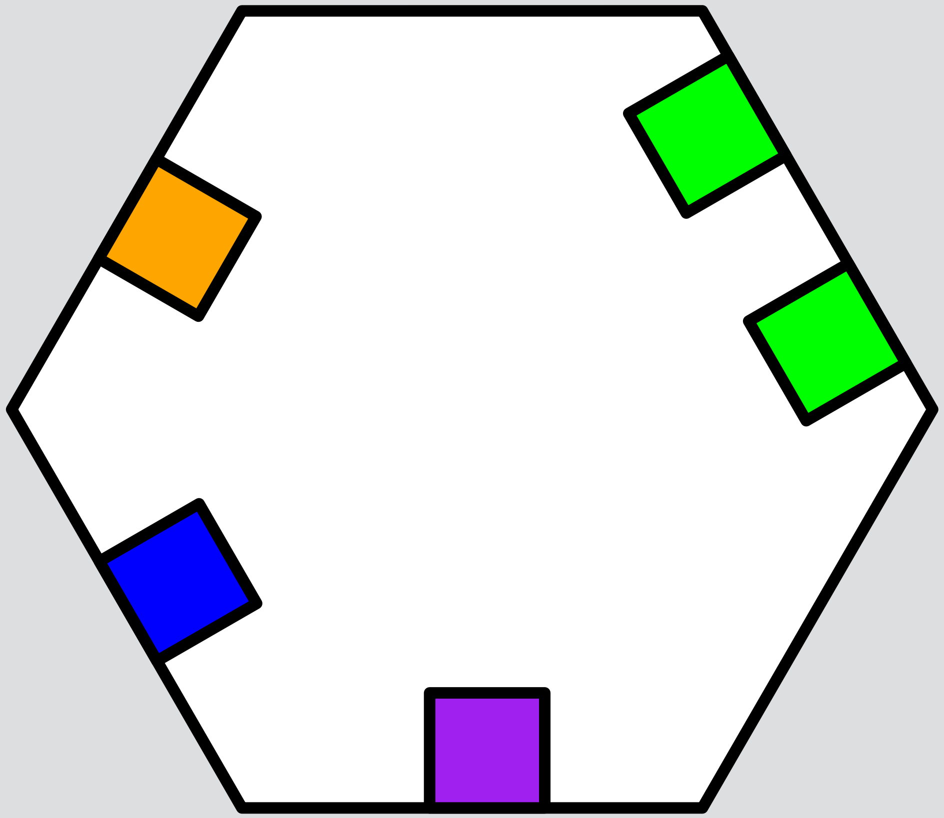
Square Tiles

- Glues on each side.
- Glues have color, strength.
- Tiles bond edgewise.

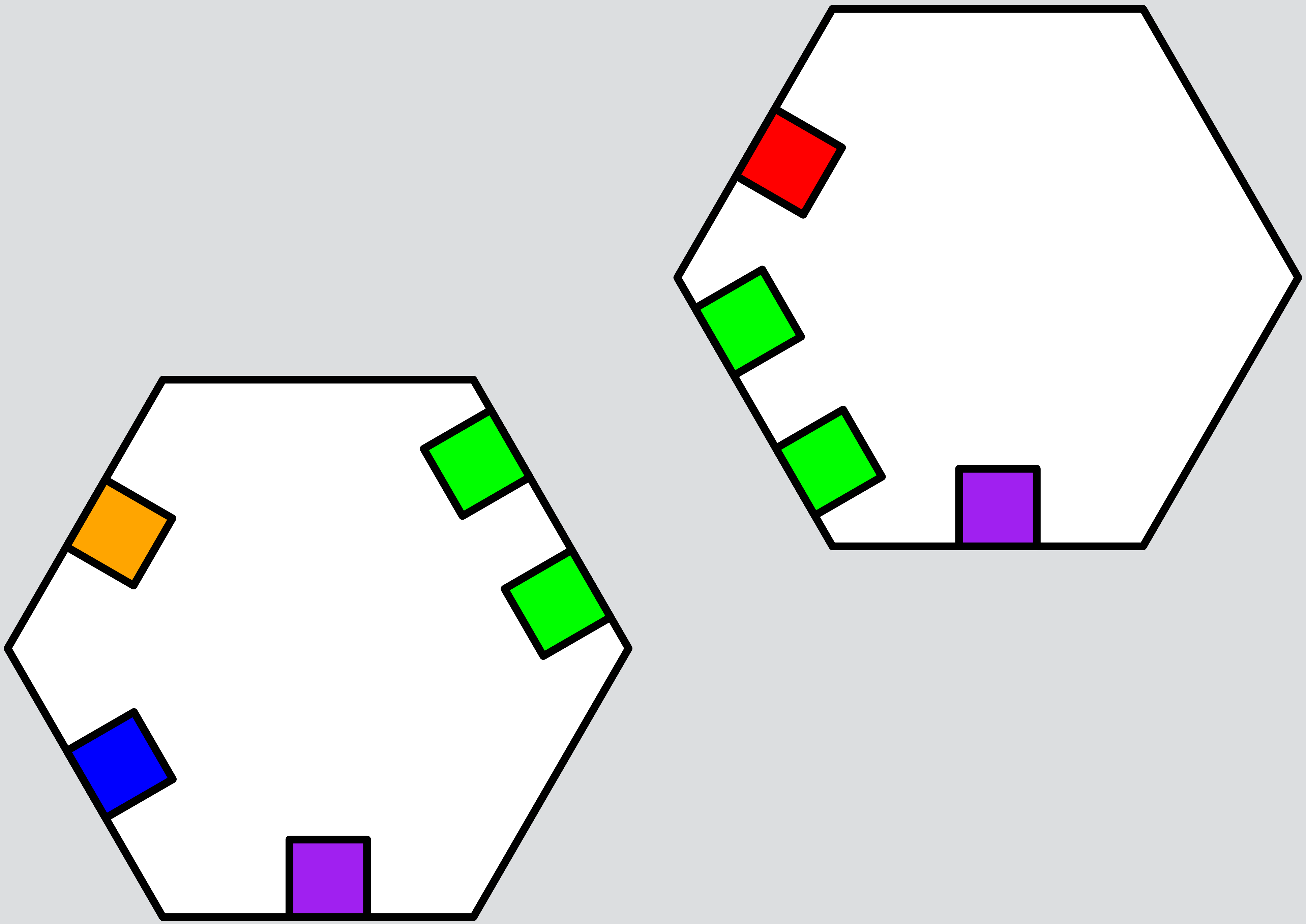


Hexagonal Tiles

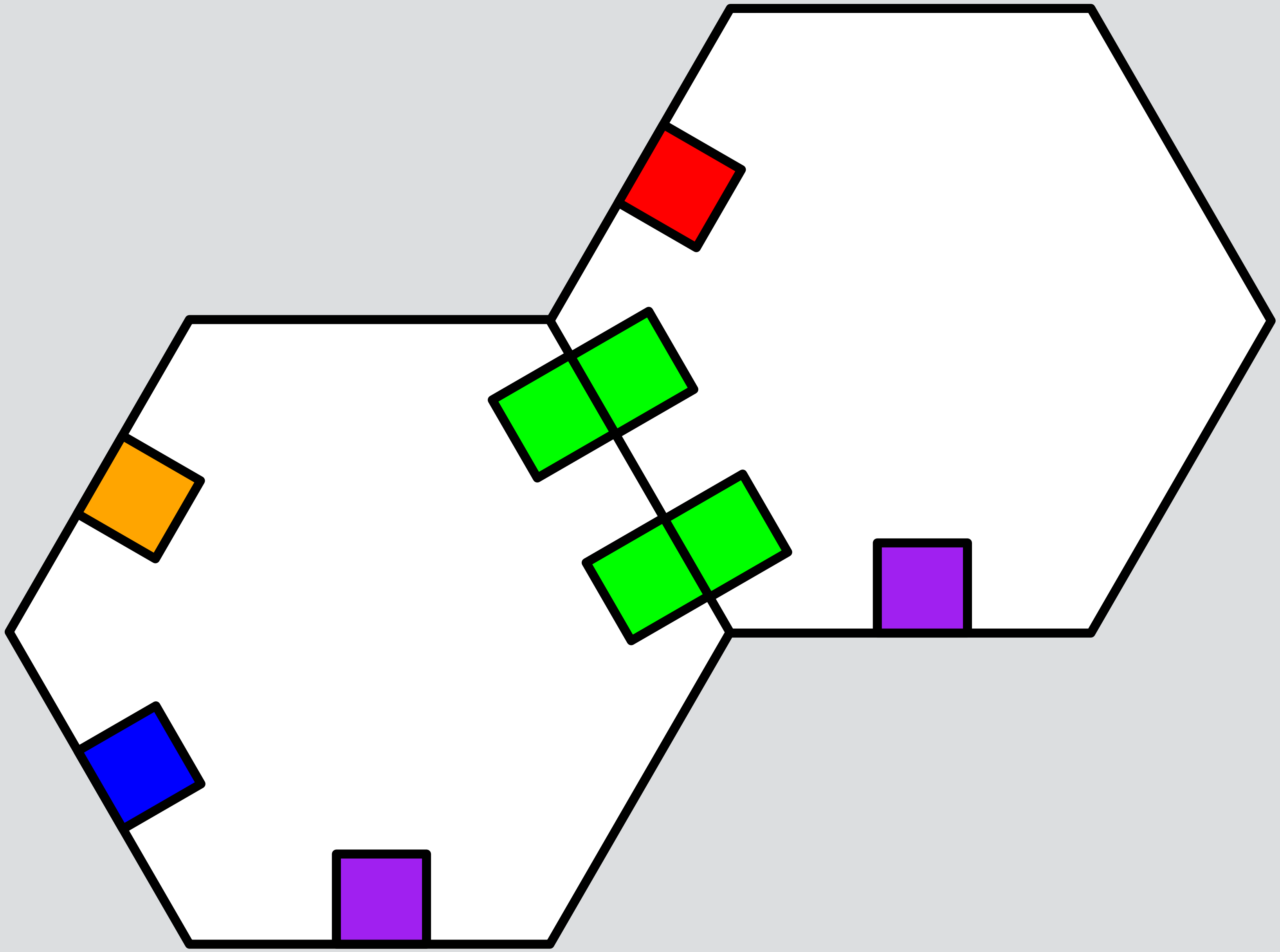
- Glues on each side.
- Glues have color, strength.
- Tiles bond edgewise.



Hexagonal Tiles



Hexagonal Tiles



Polygonal Tiles

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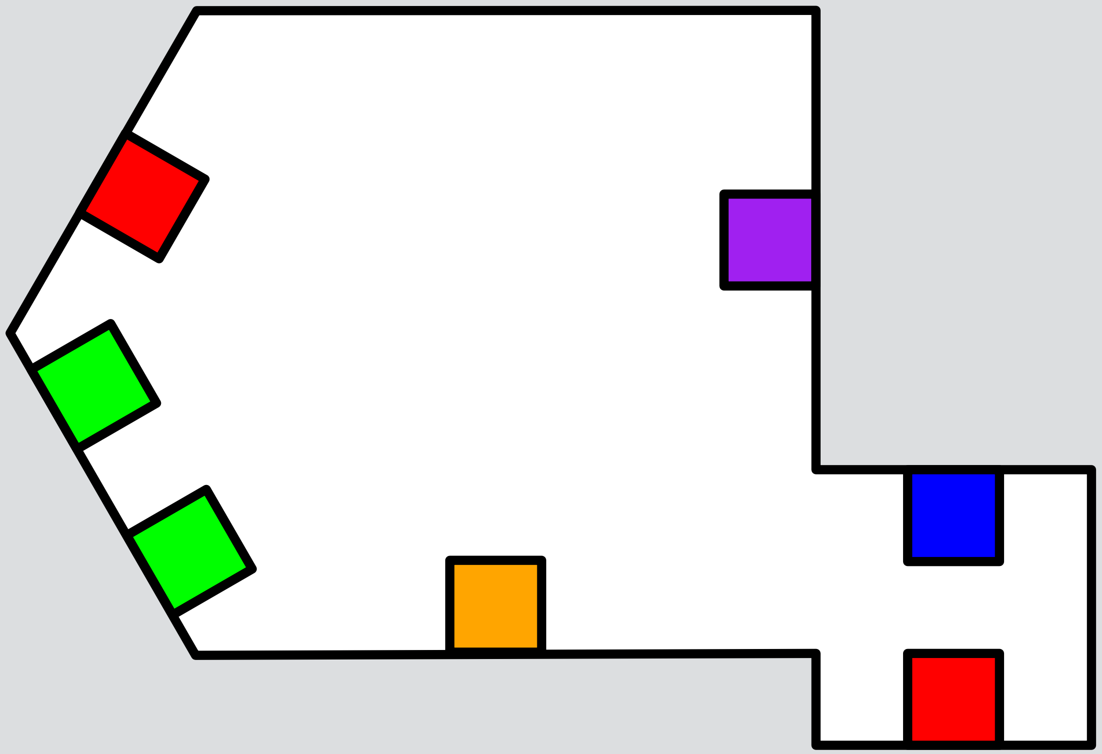
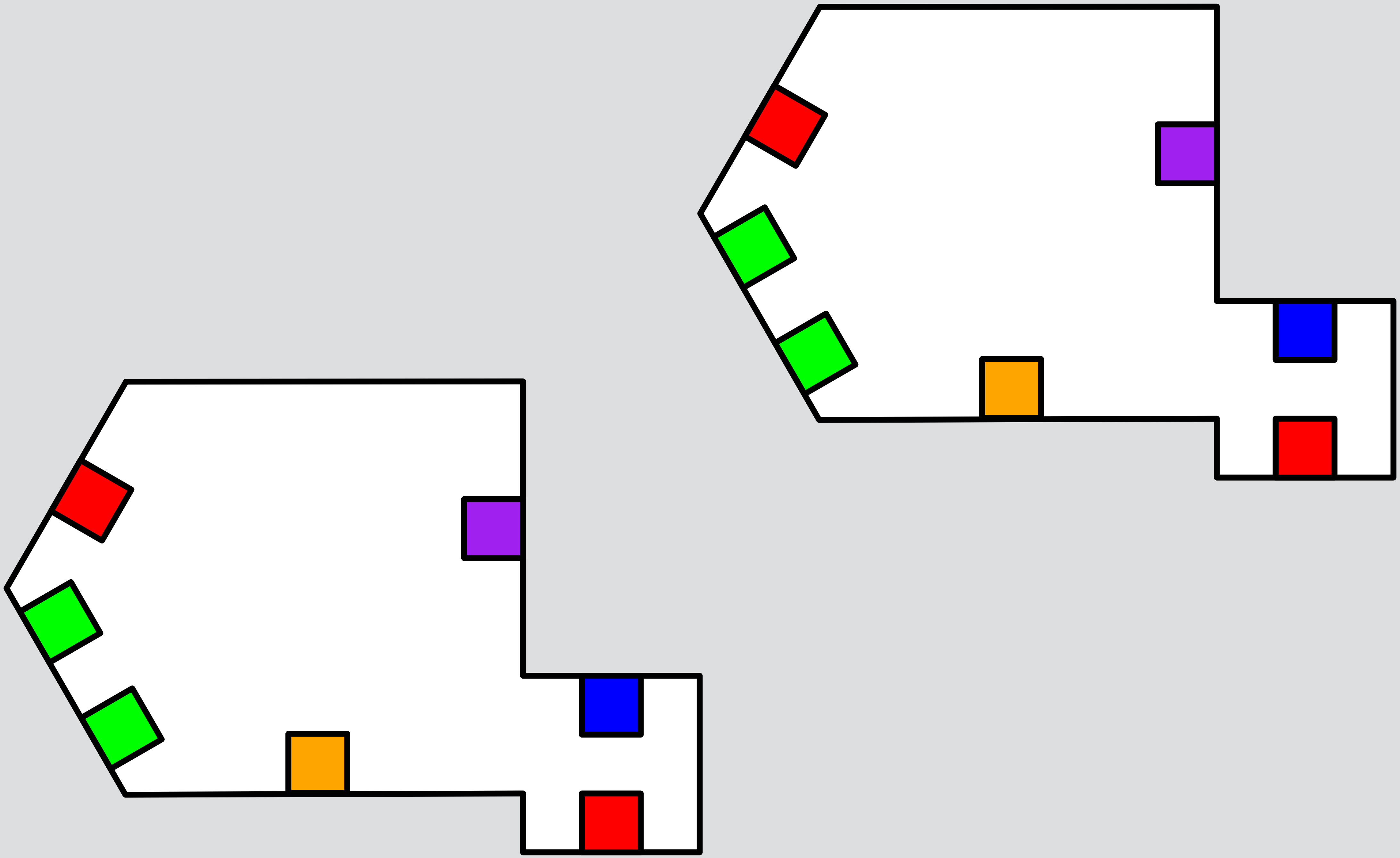


Polygonal Tiles



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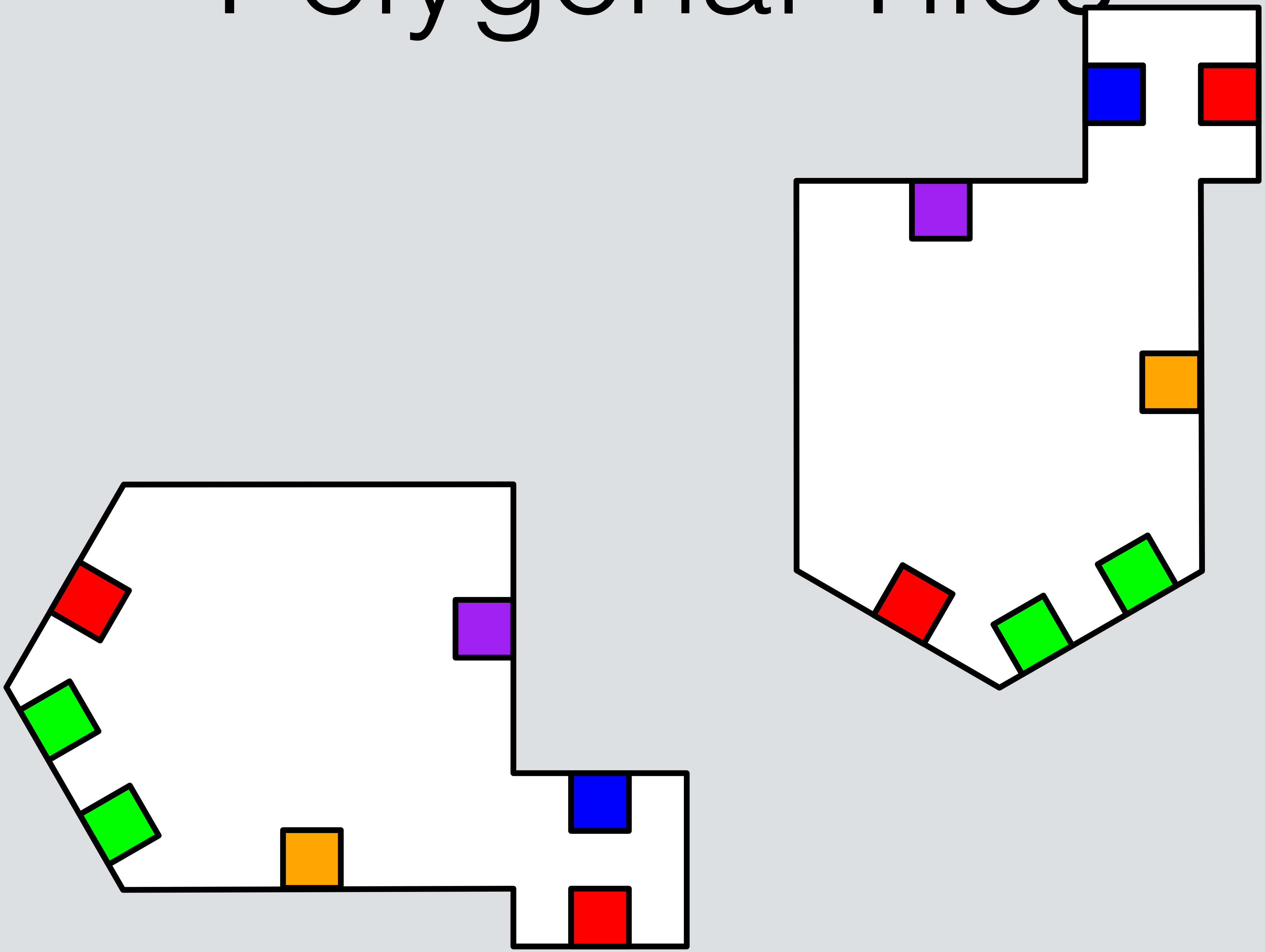

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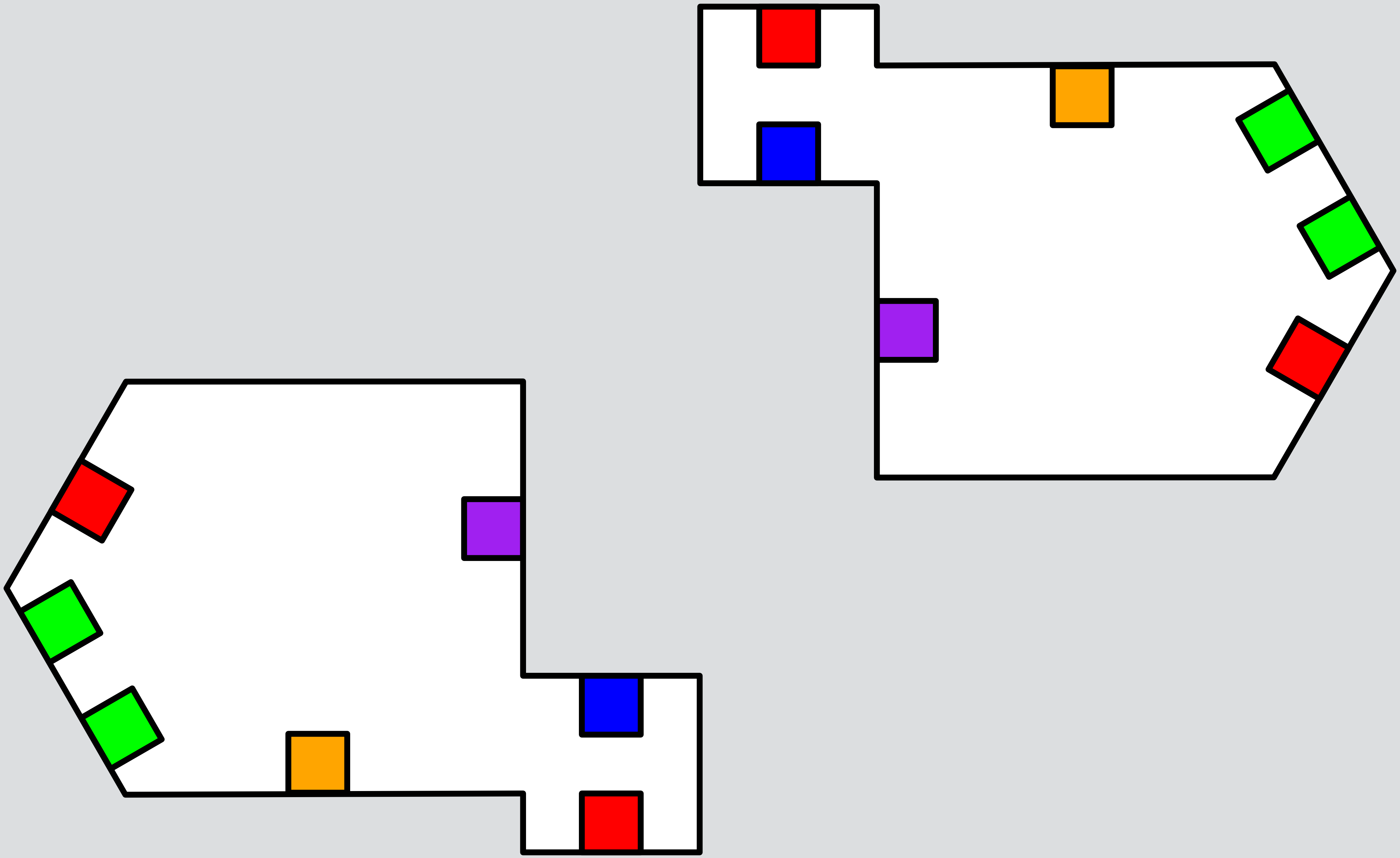
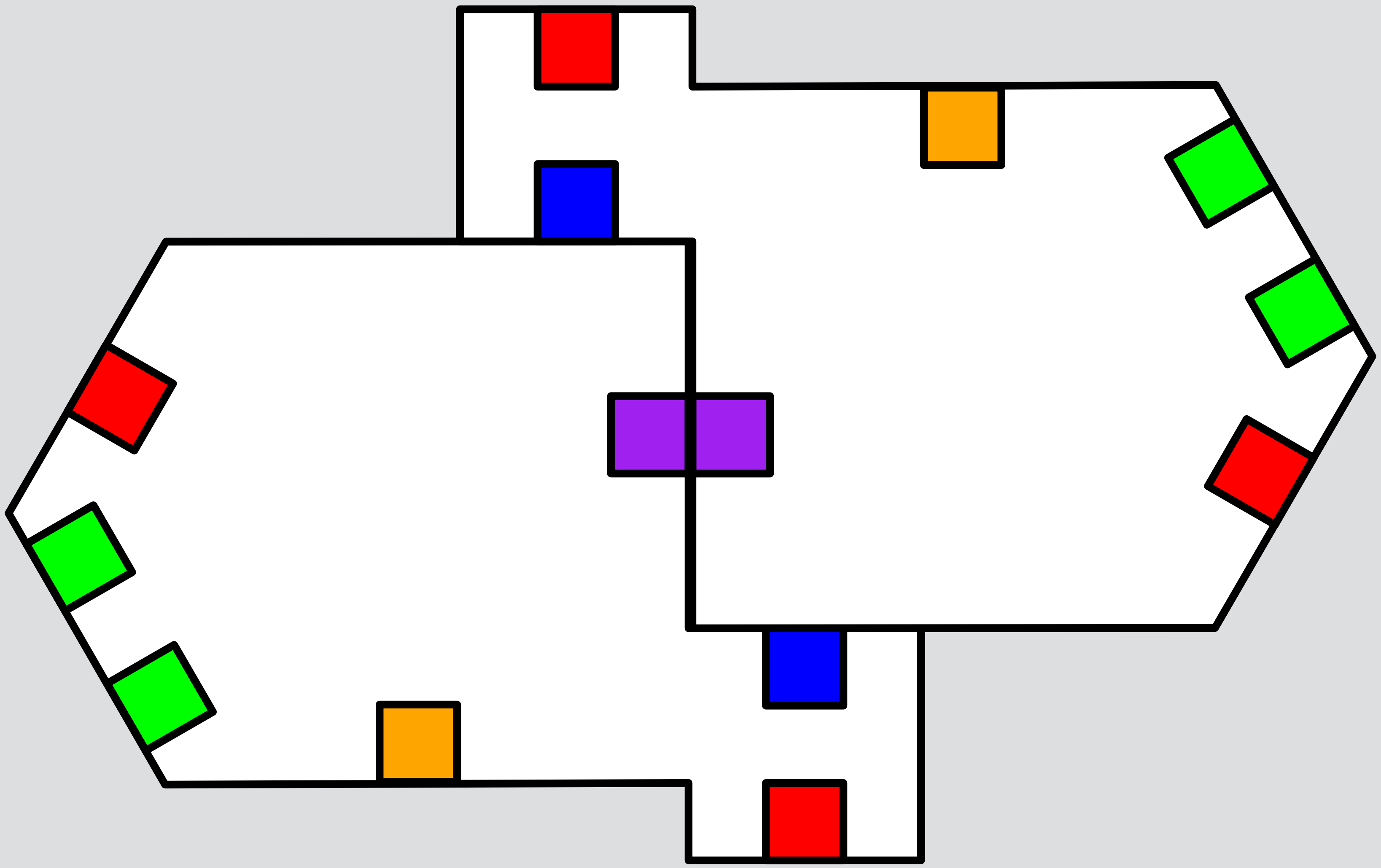


Polygonal Tiles



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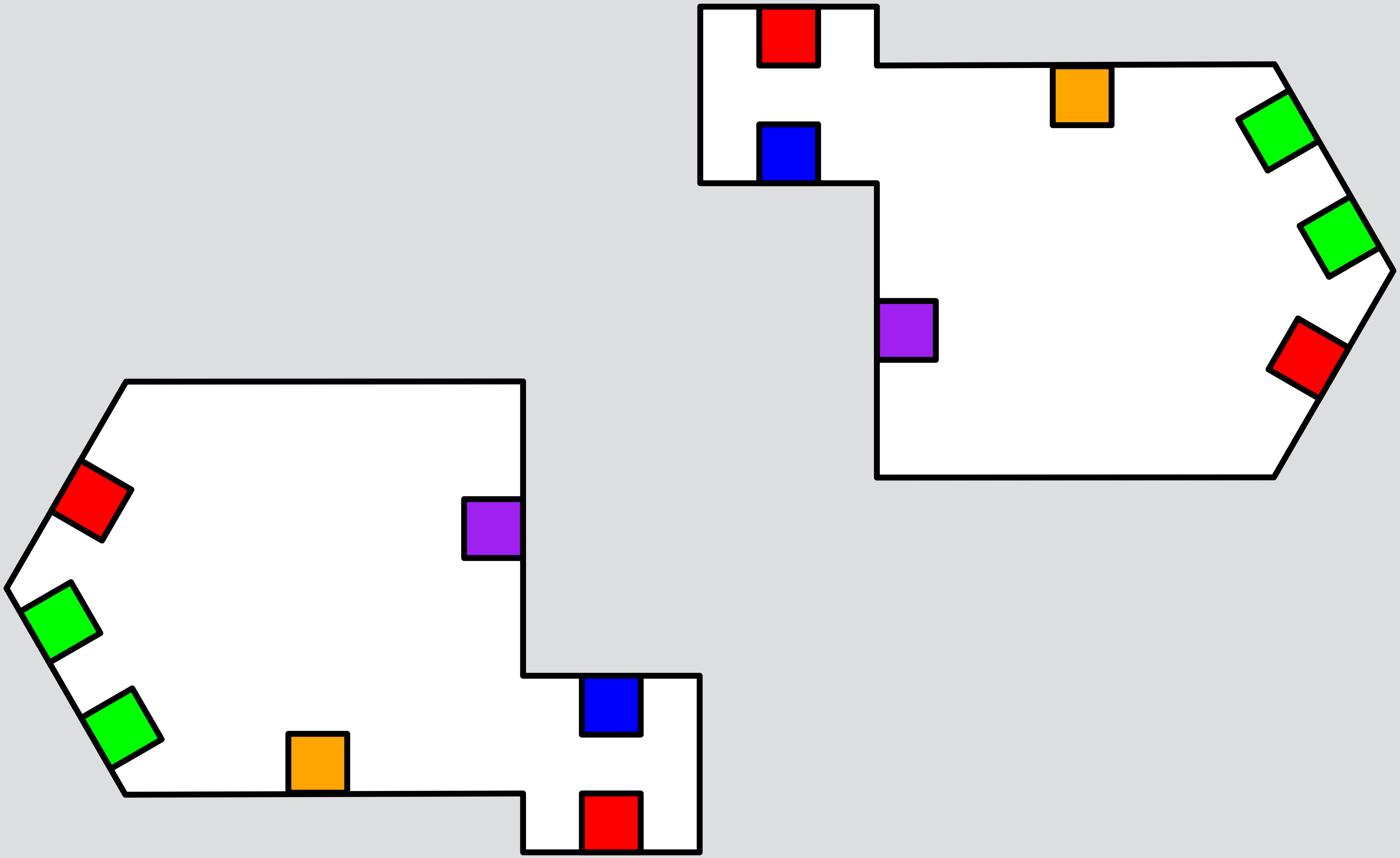

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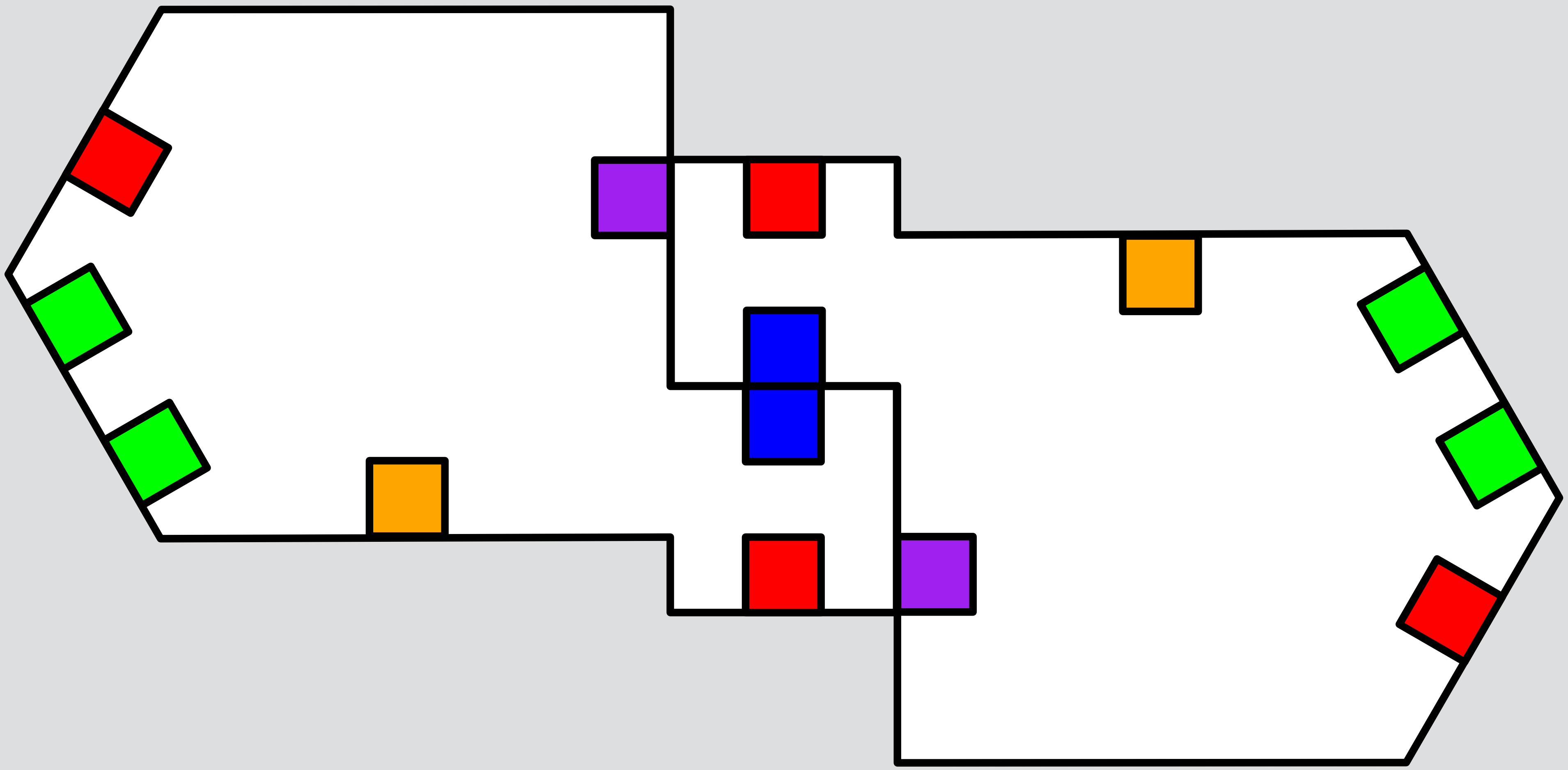

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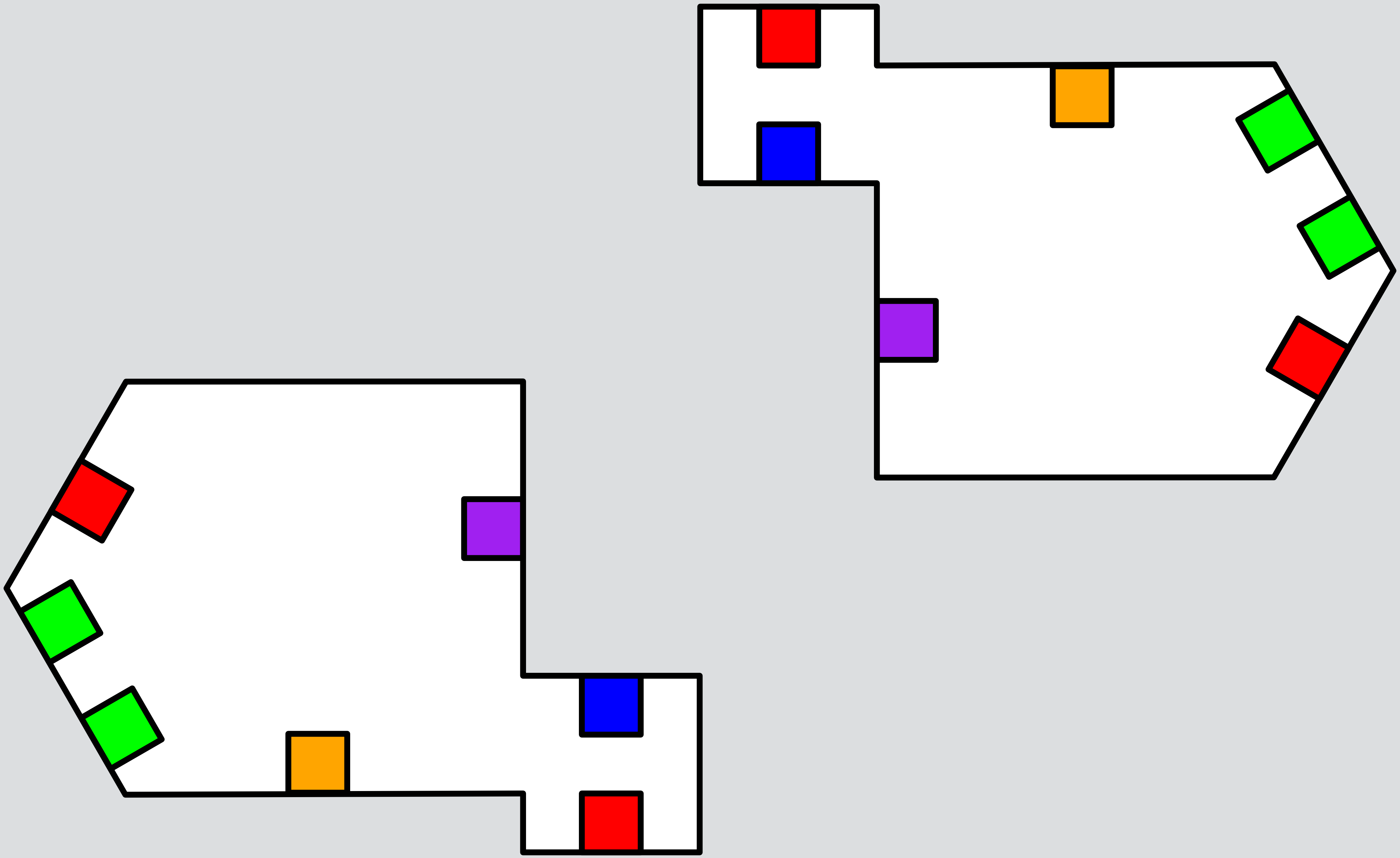

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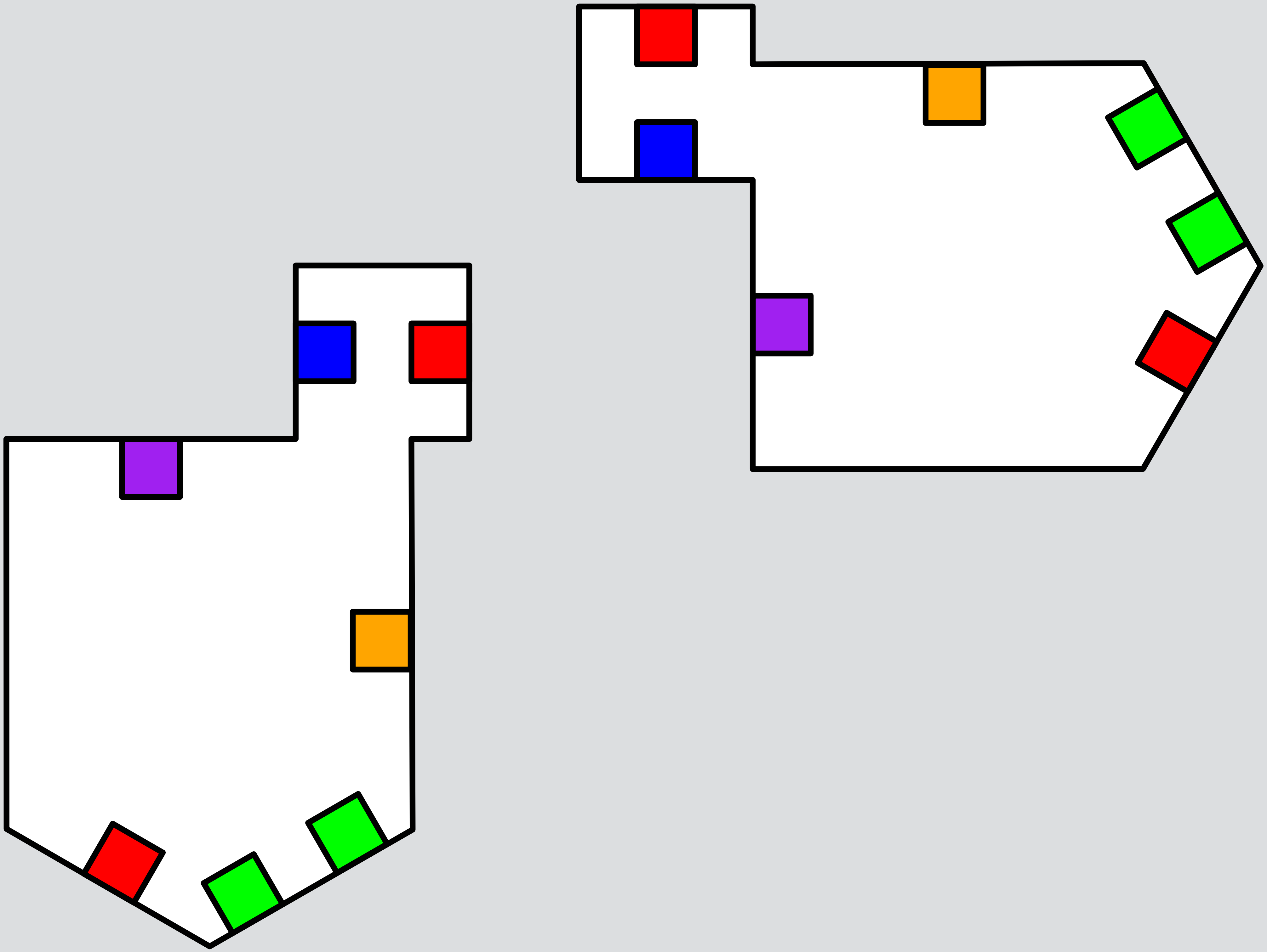
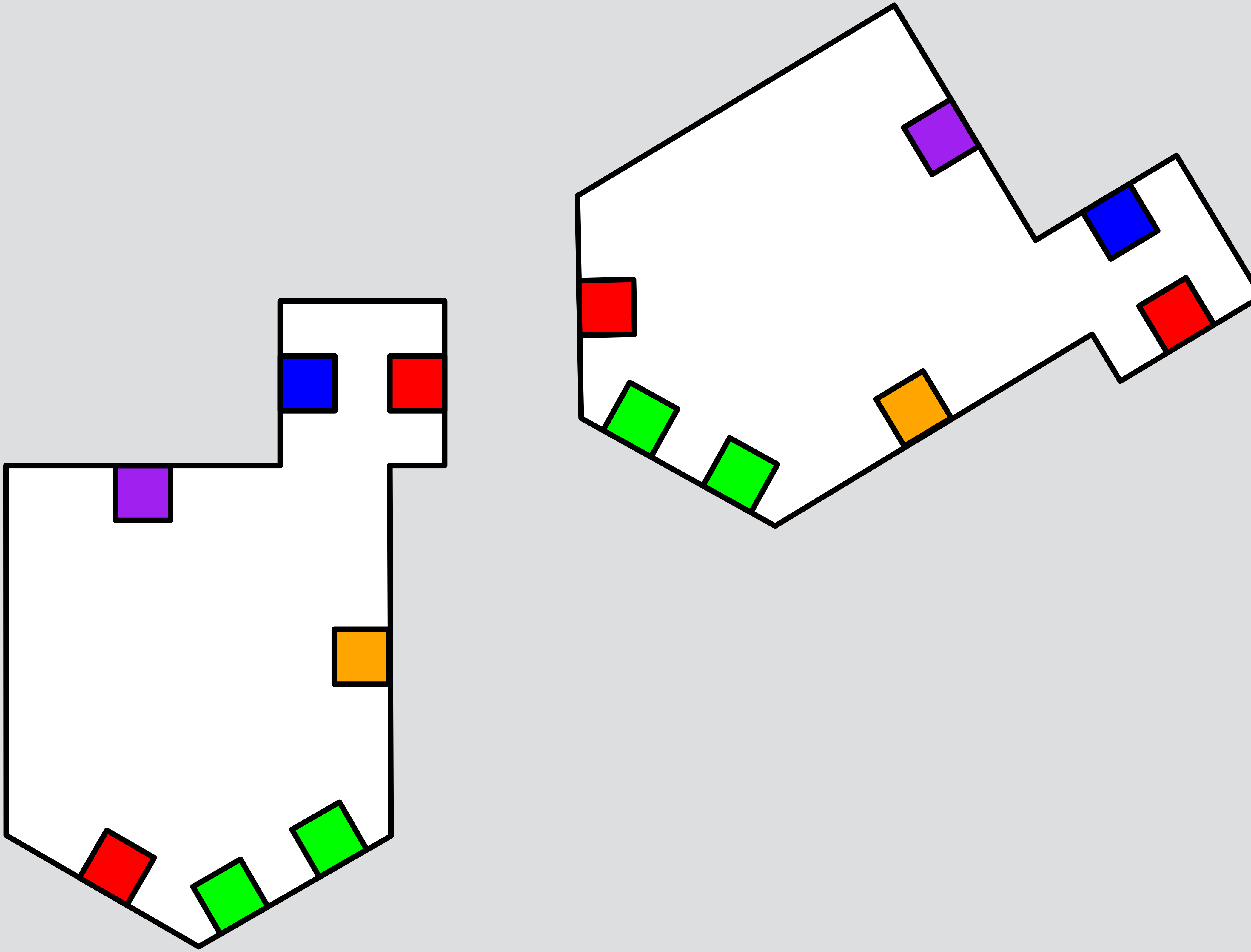
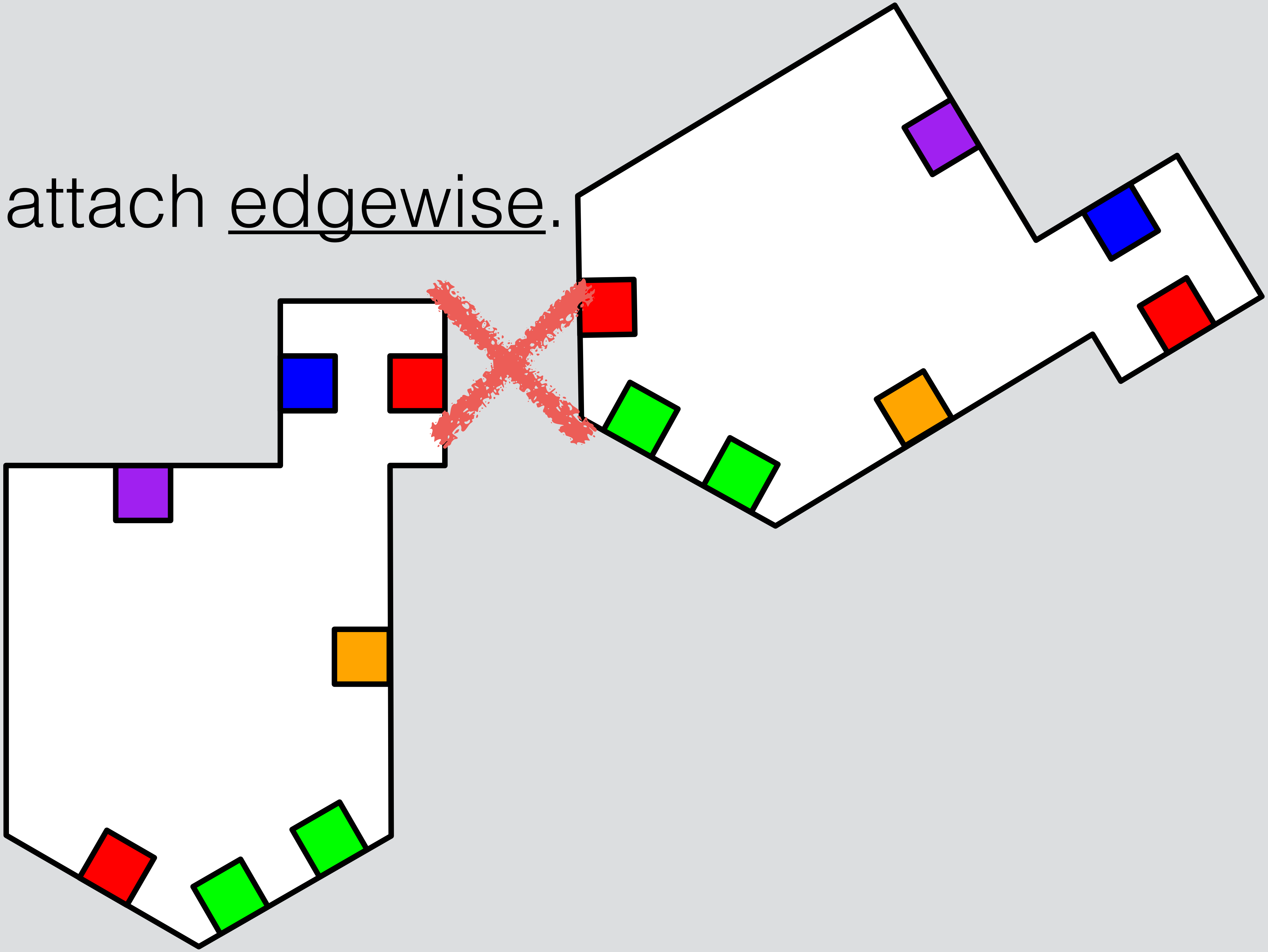


Polygonal Tiles



Polygonal Tiles

Cannot attach <u>edgewise</u>.

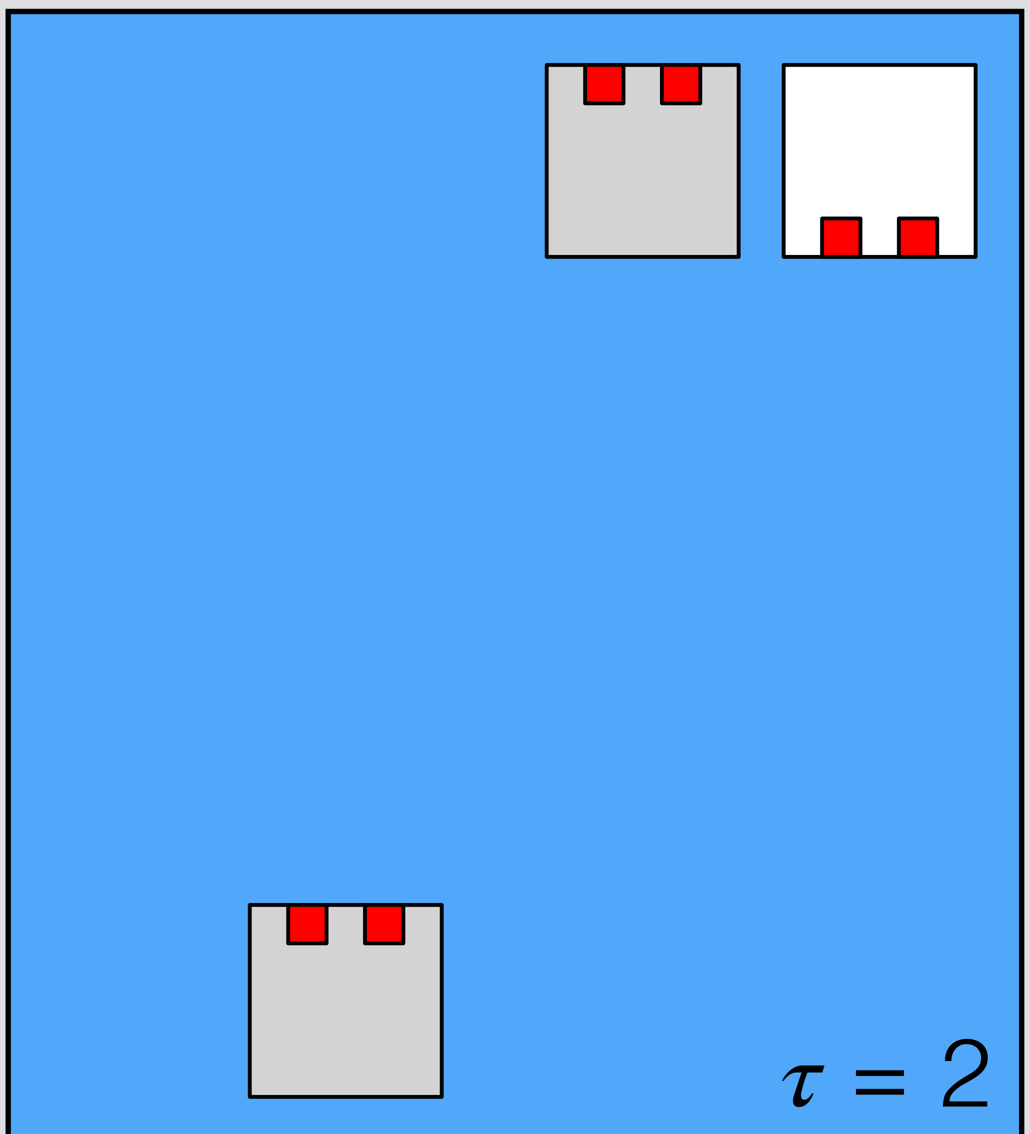


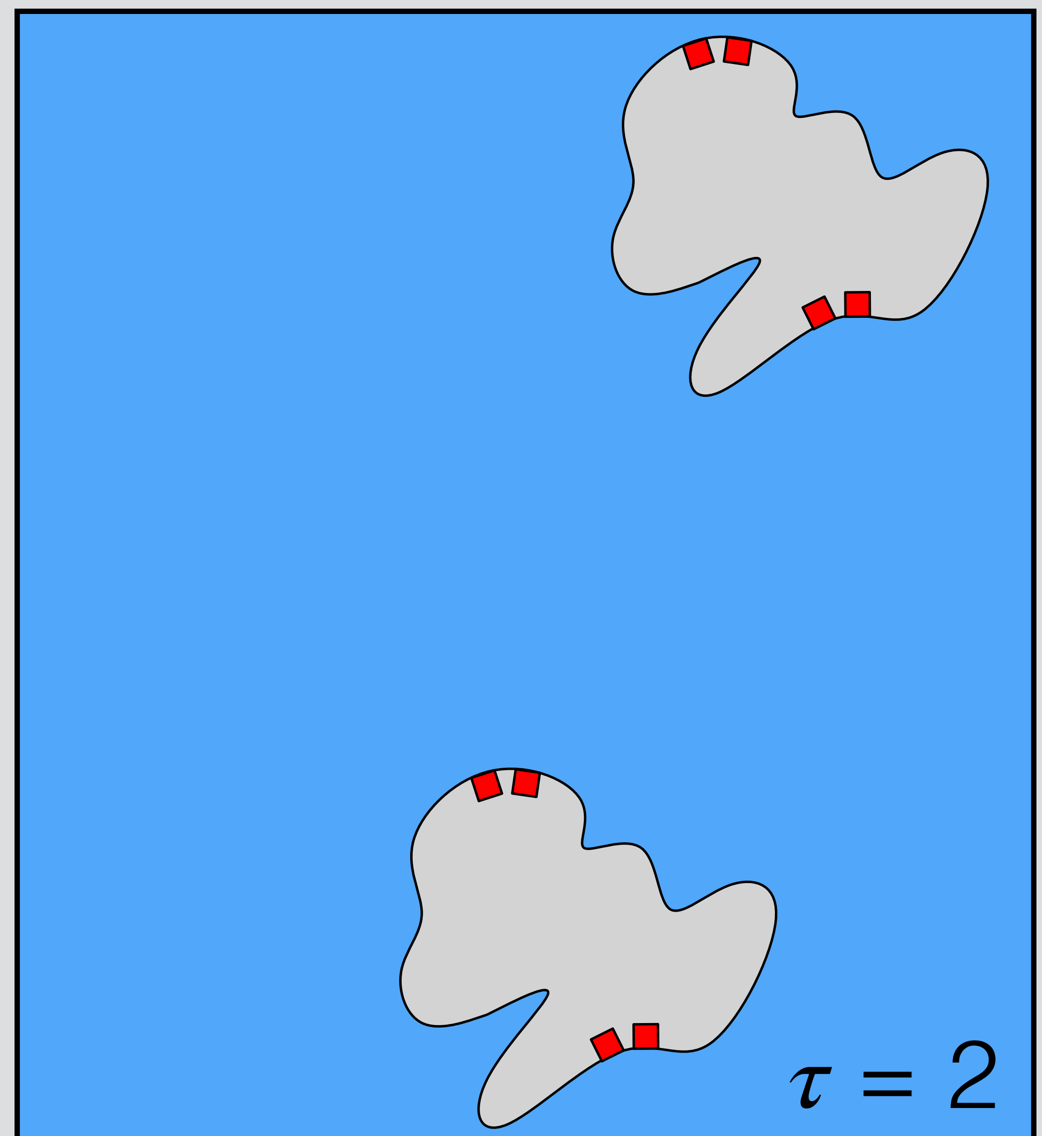
Main Result: Universality

- aTAM: square tiles that <u>cannot</u> rotate.
- pfbTAM: polygonal tiles that <u>can</u> rotate.
- Theorem: any aTAM tile set **T** at τ, there is a single-tile pfbTAM tile set at τ simulating **T**.

Proof of Universality

Challenge #1: strength-τ glues.





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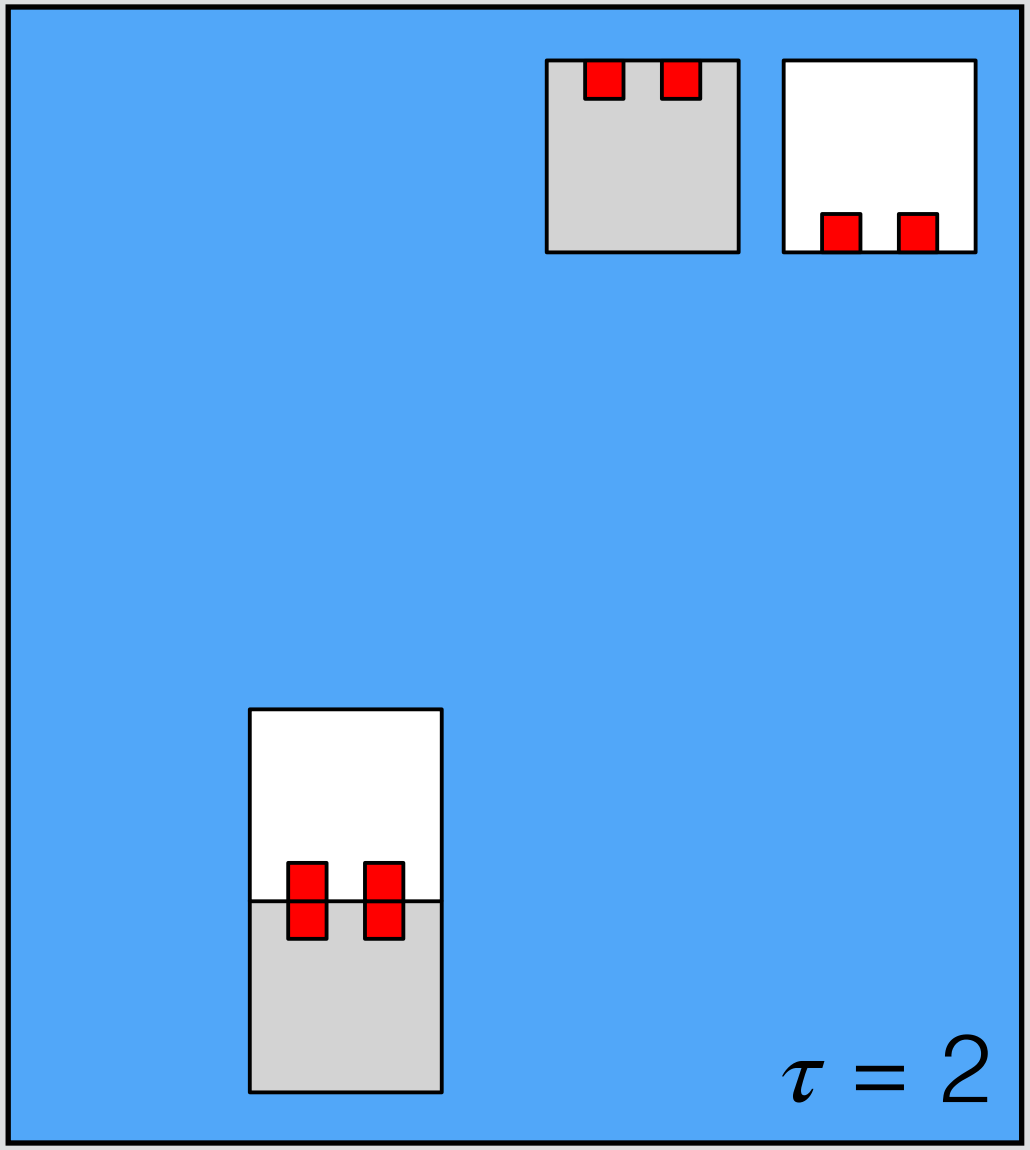

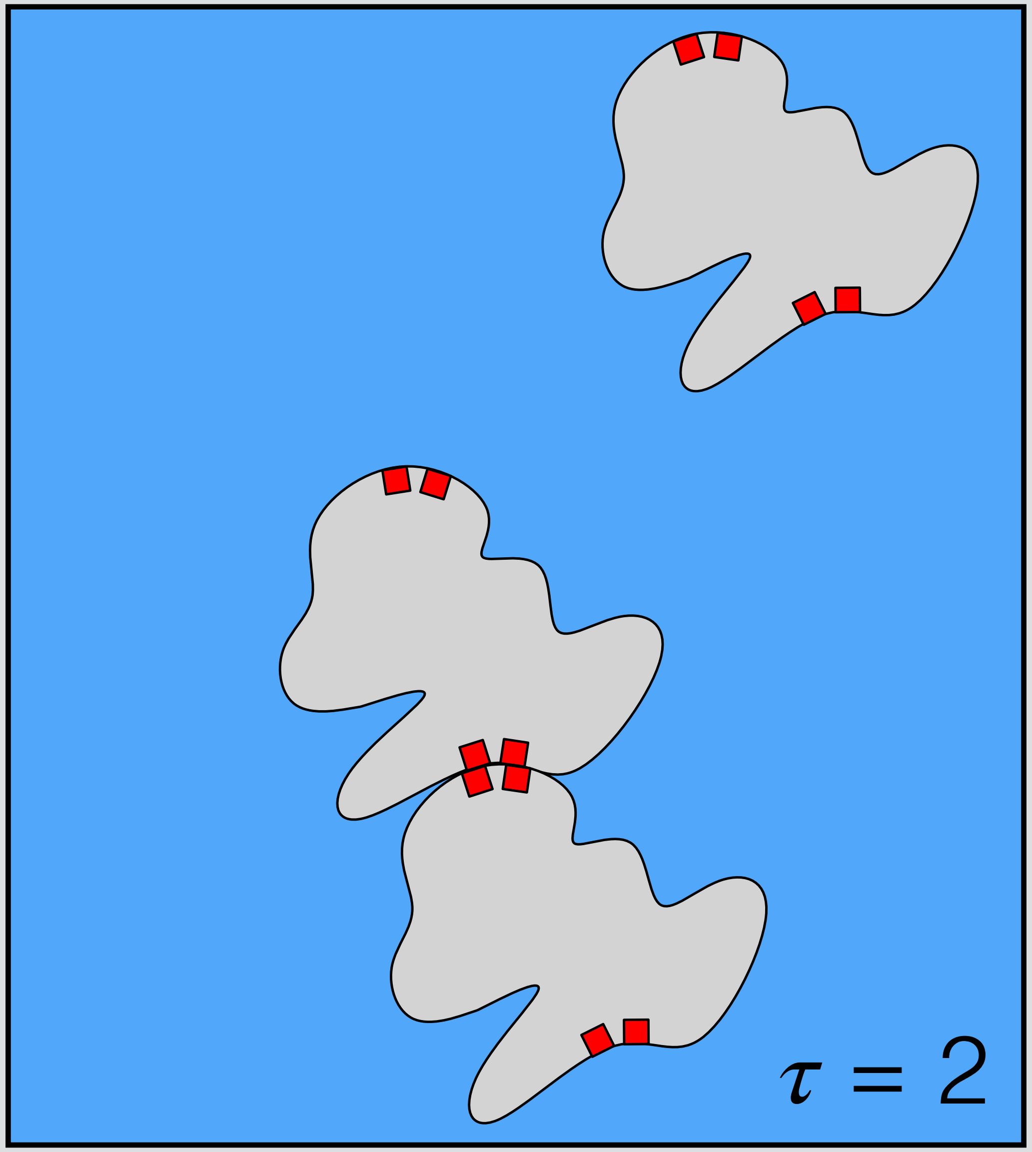

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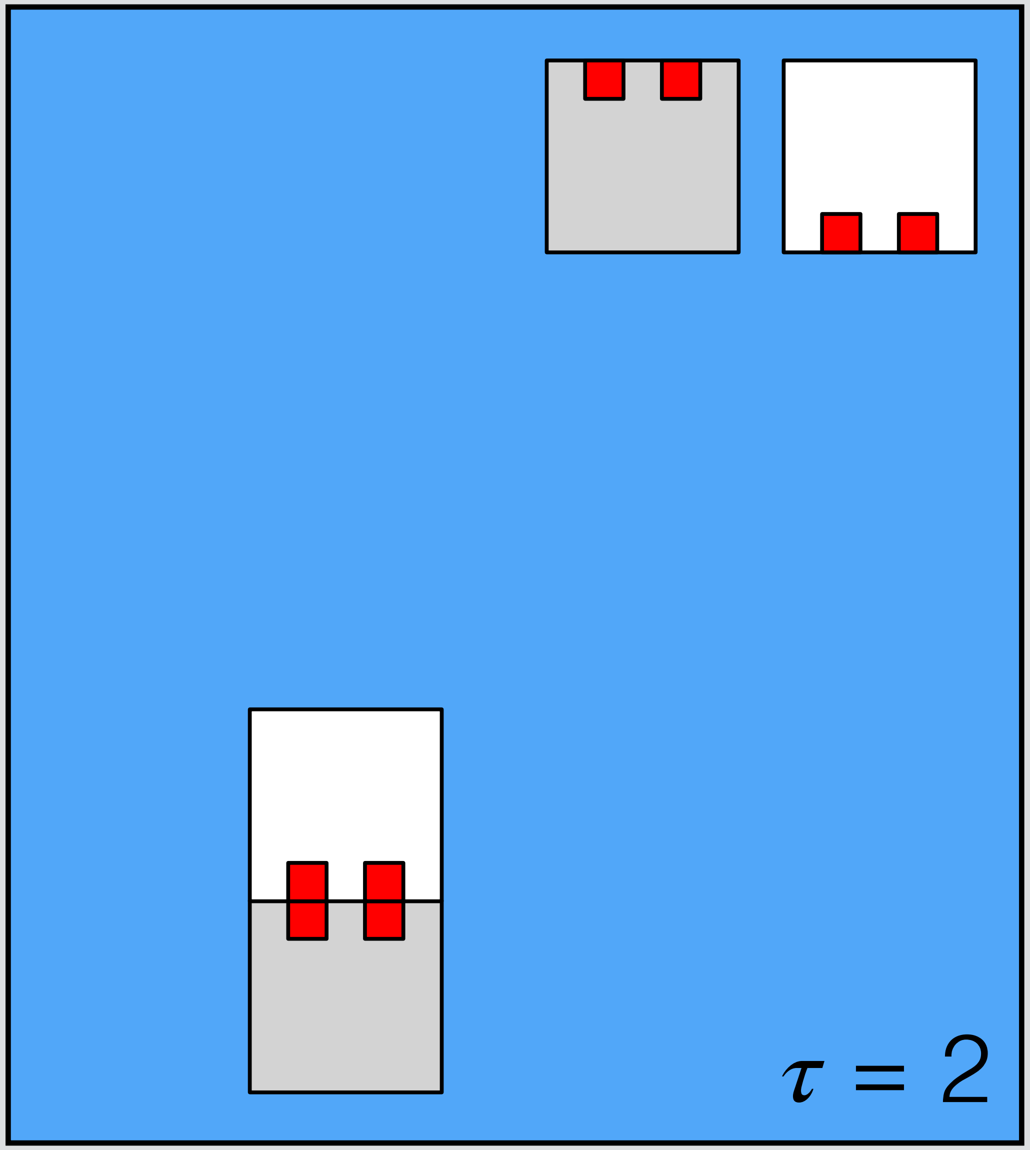

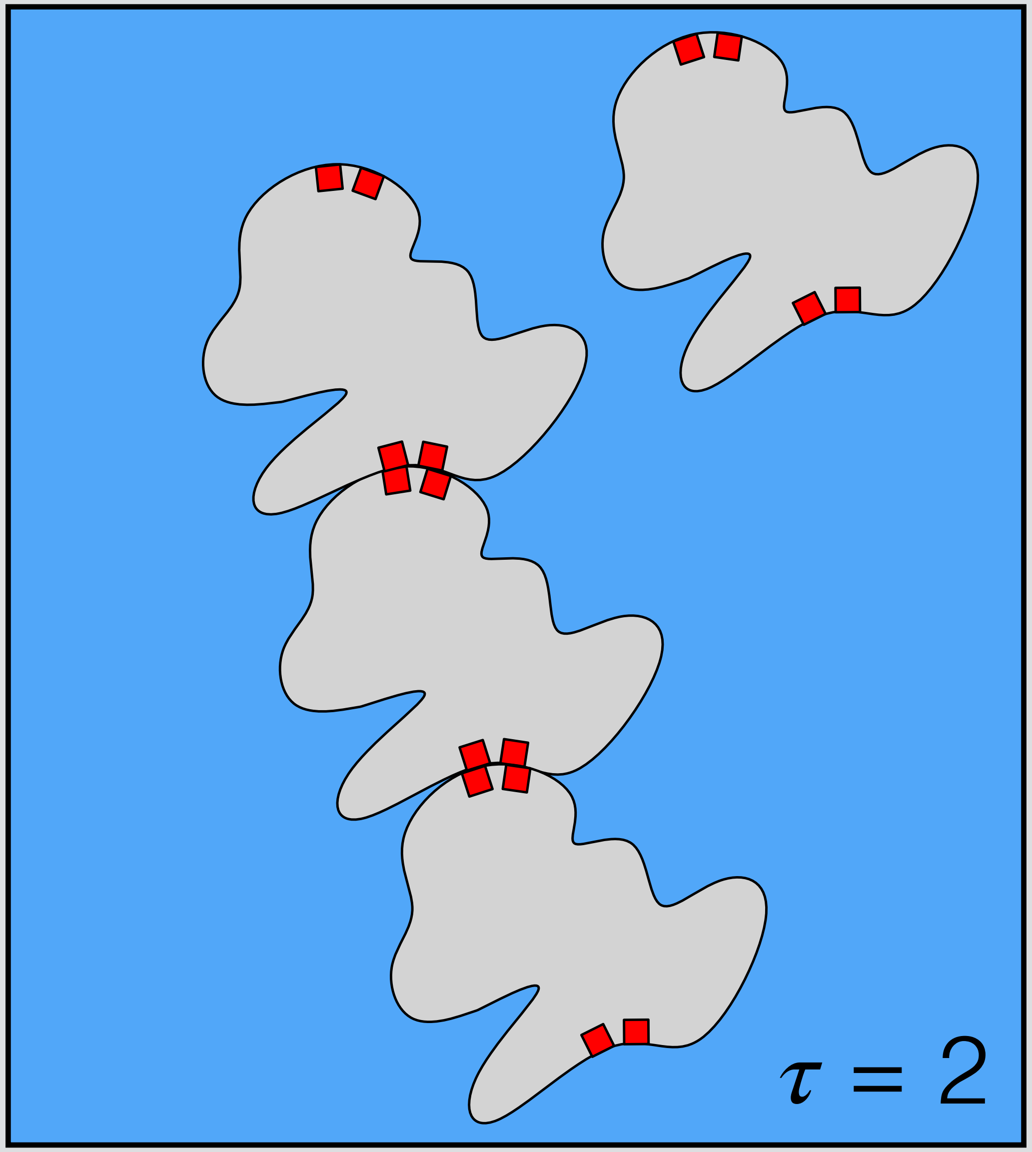

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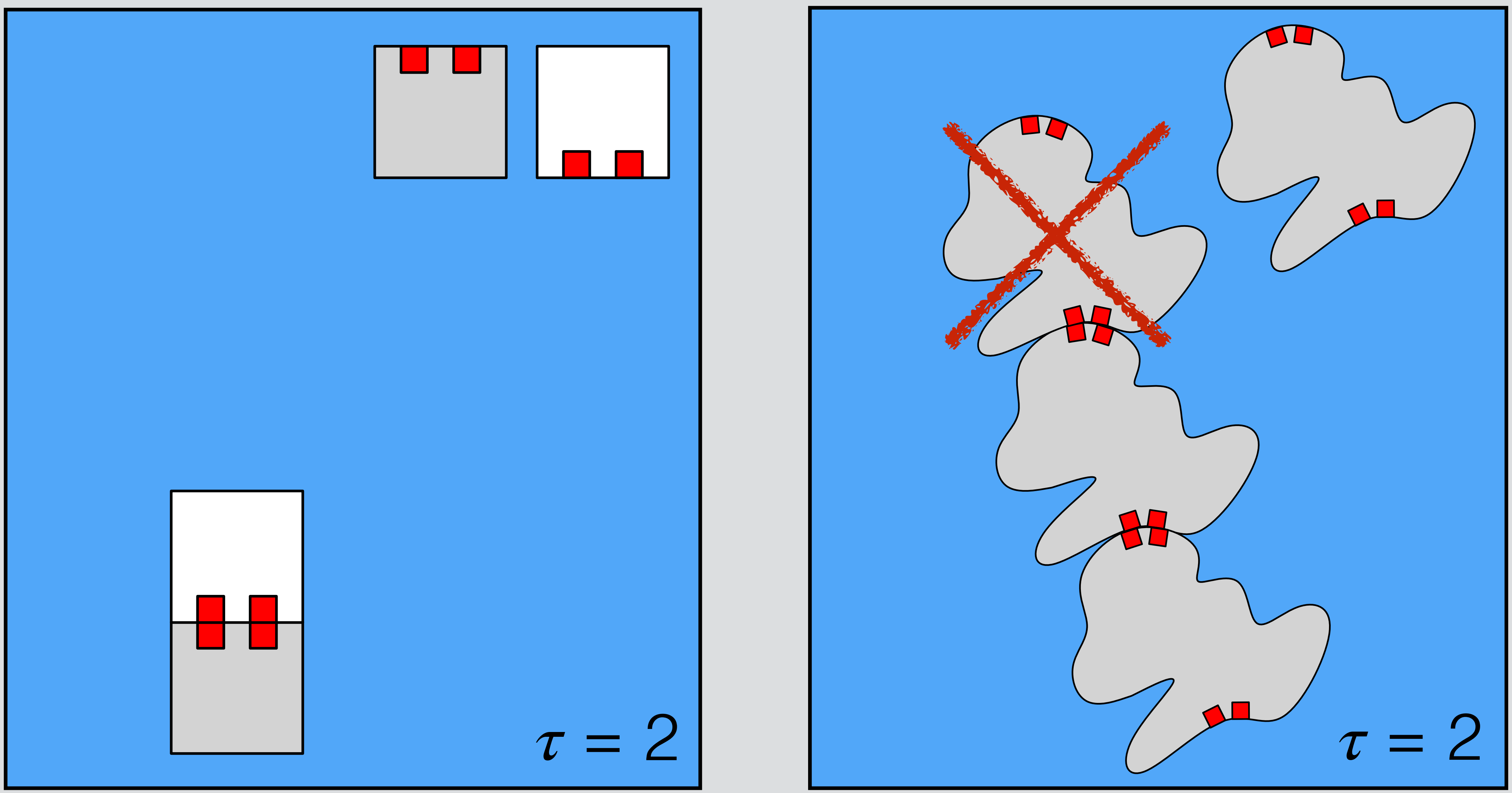
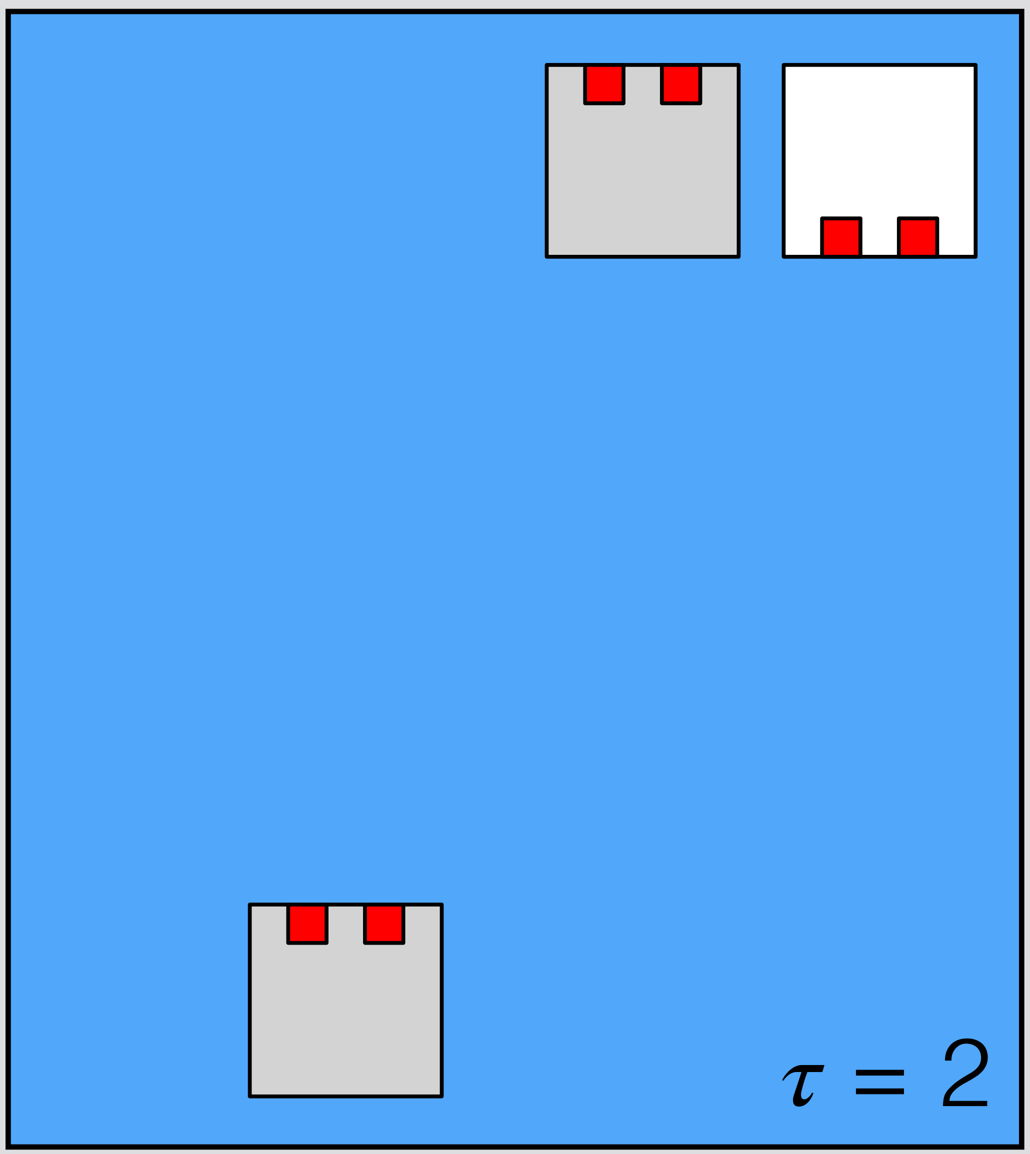
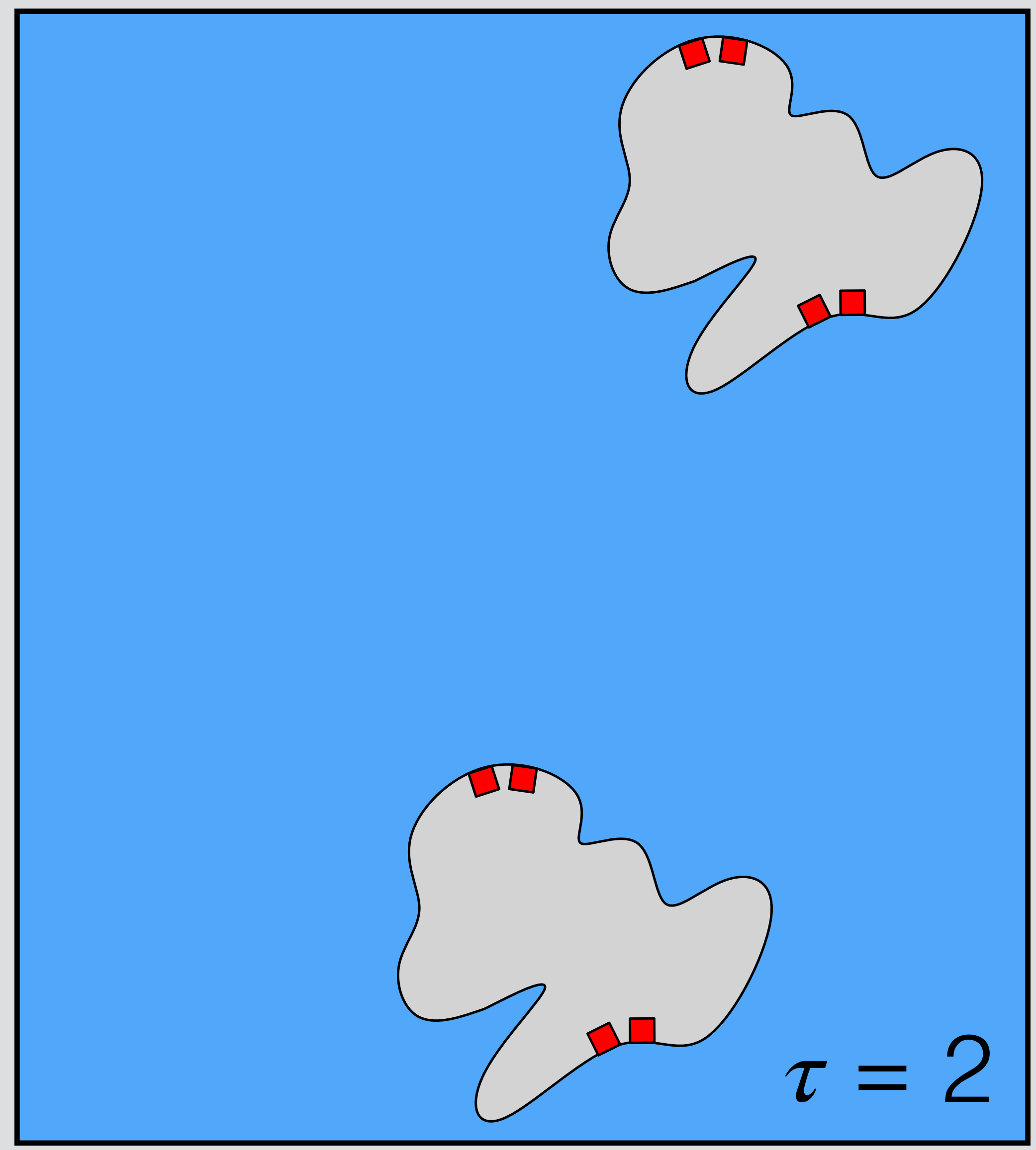


Proof of Universality

Challenge #2: unwanted rotations.





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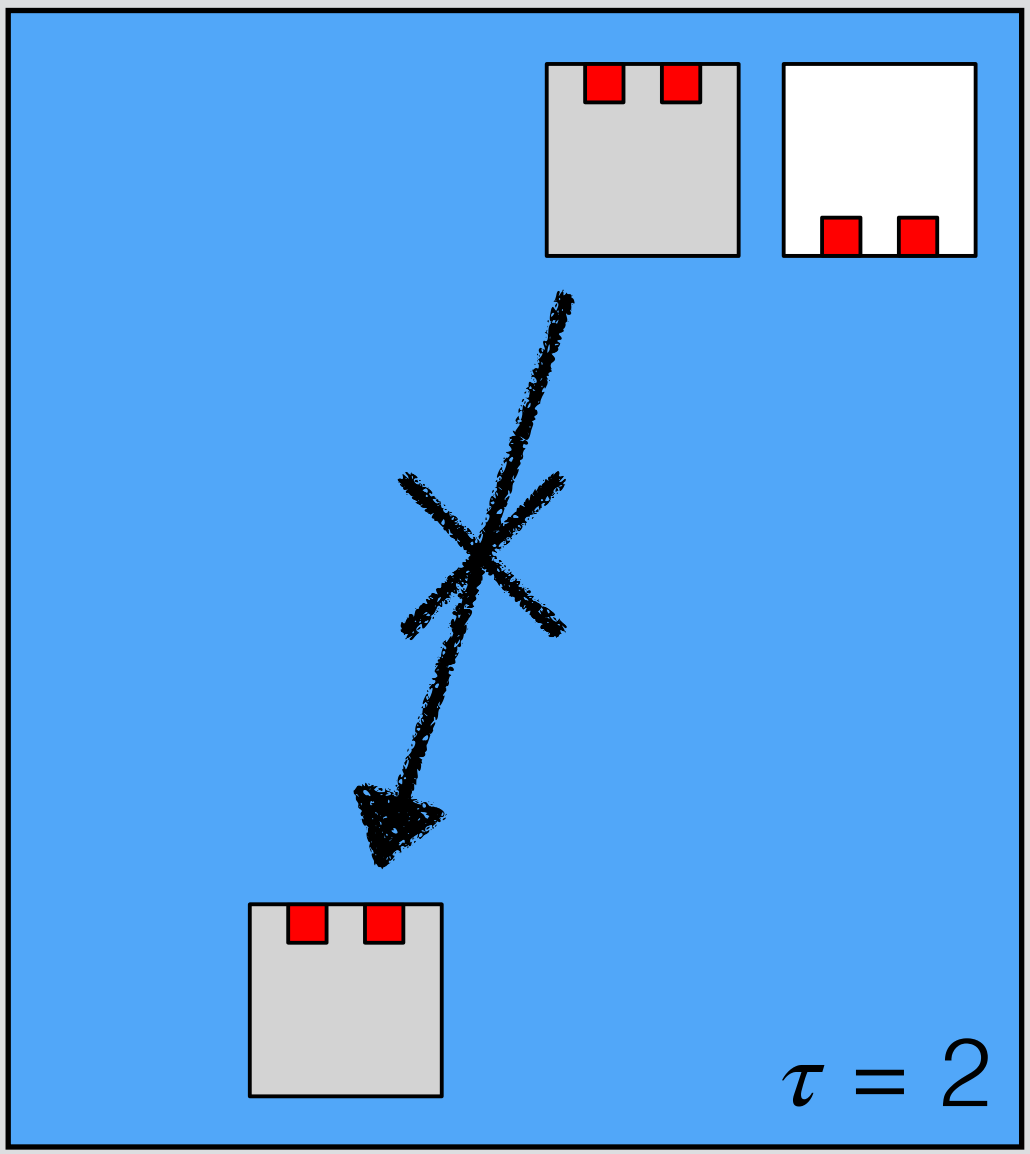

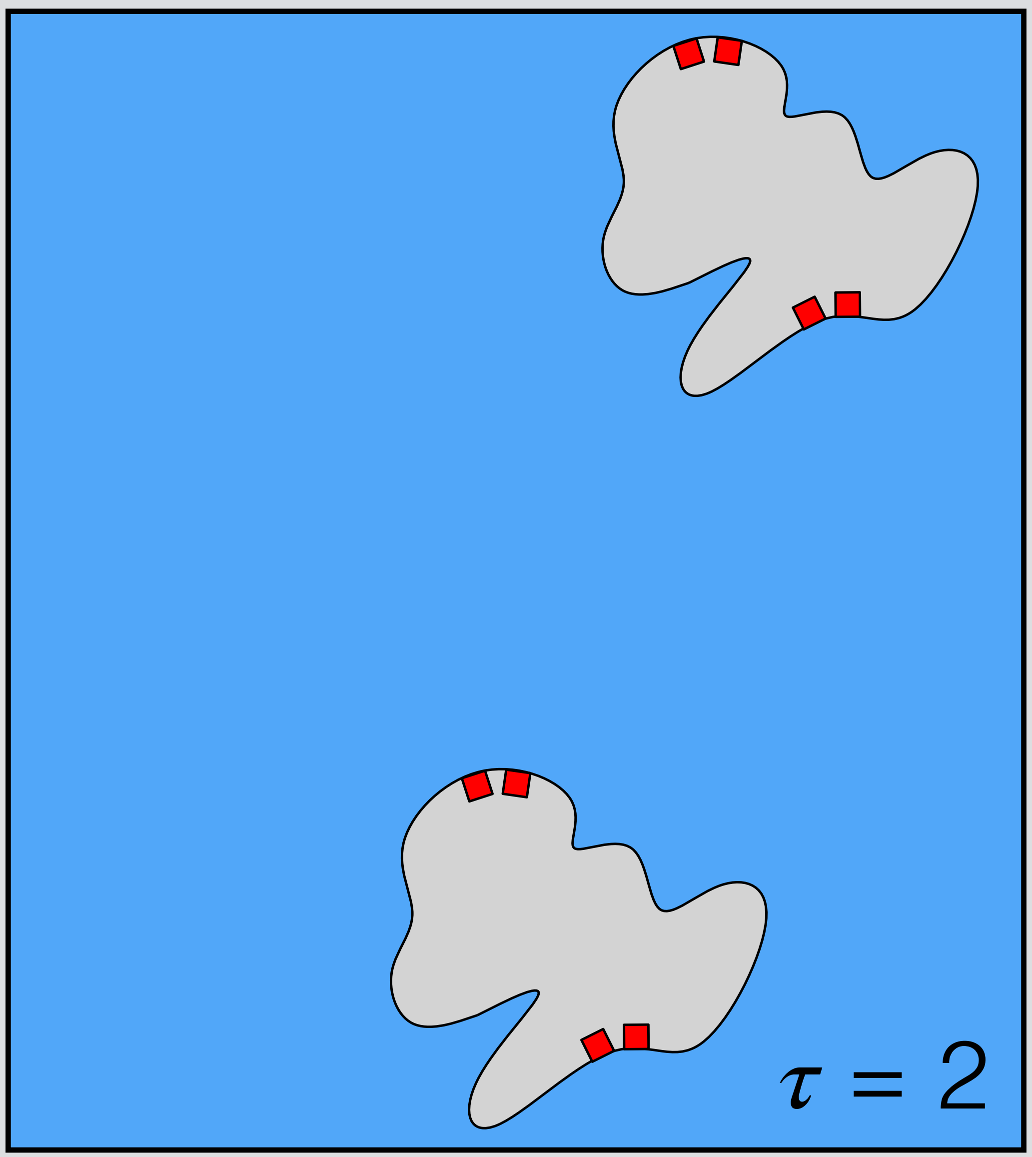

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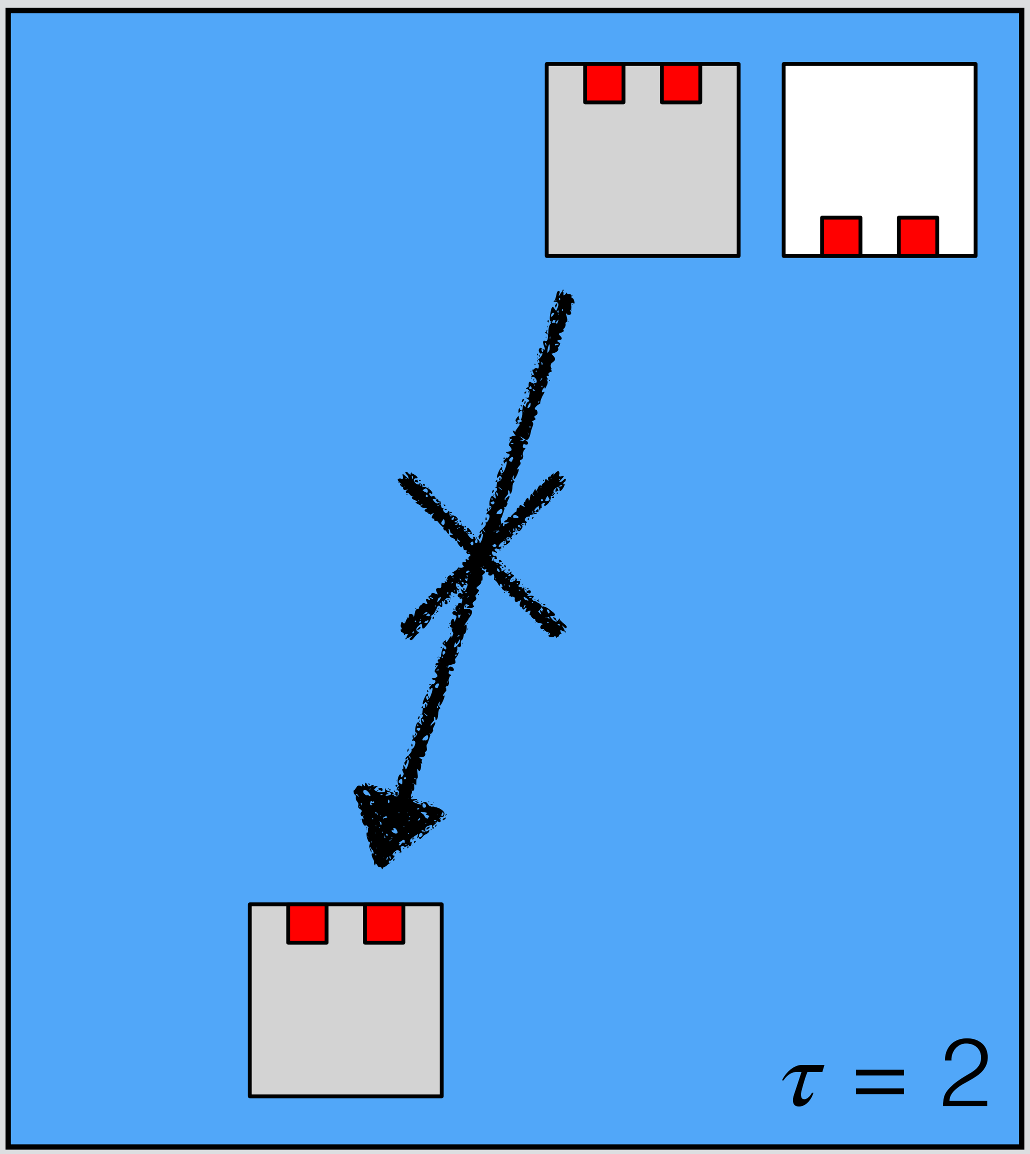

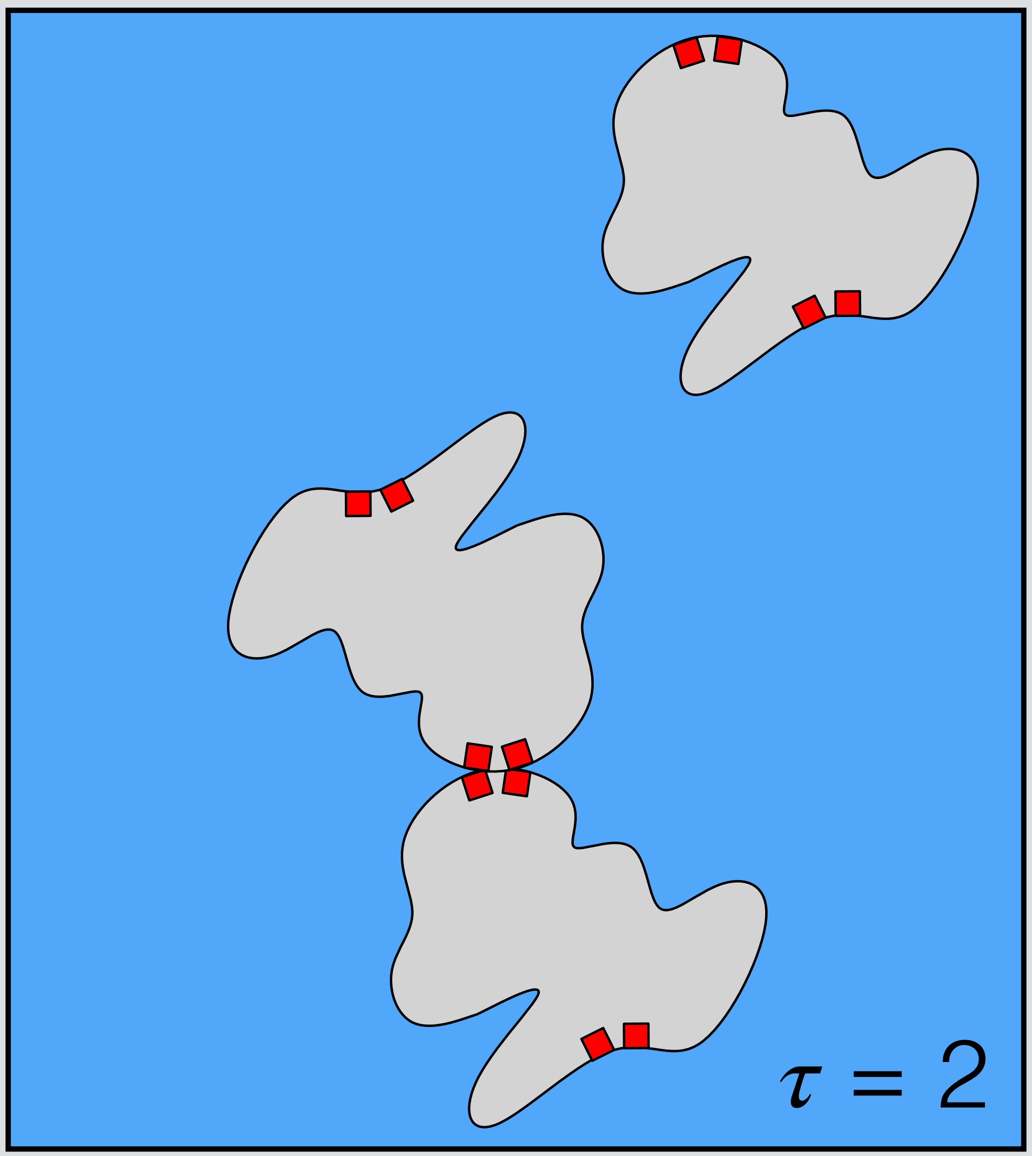

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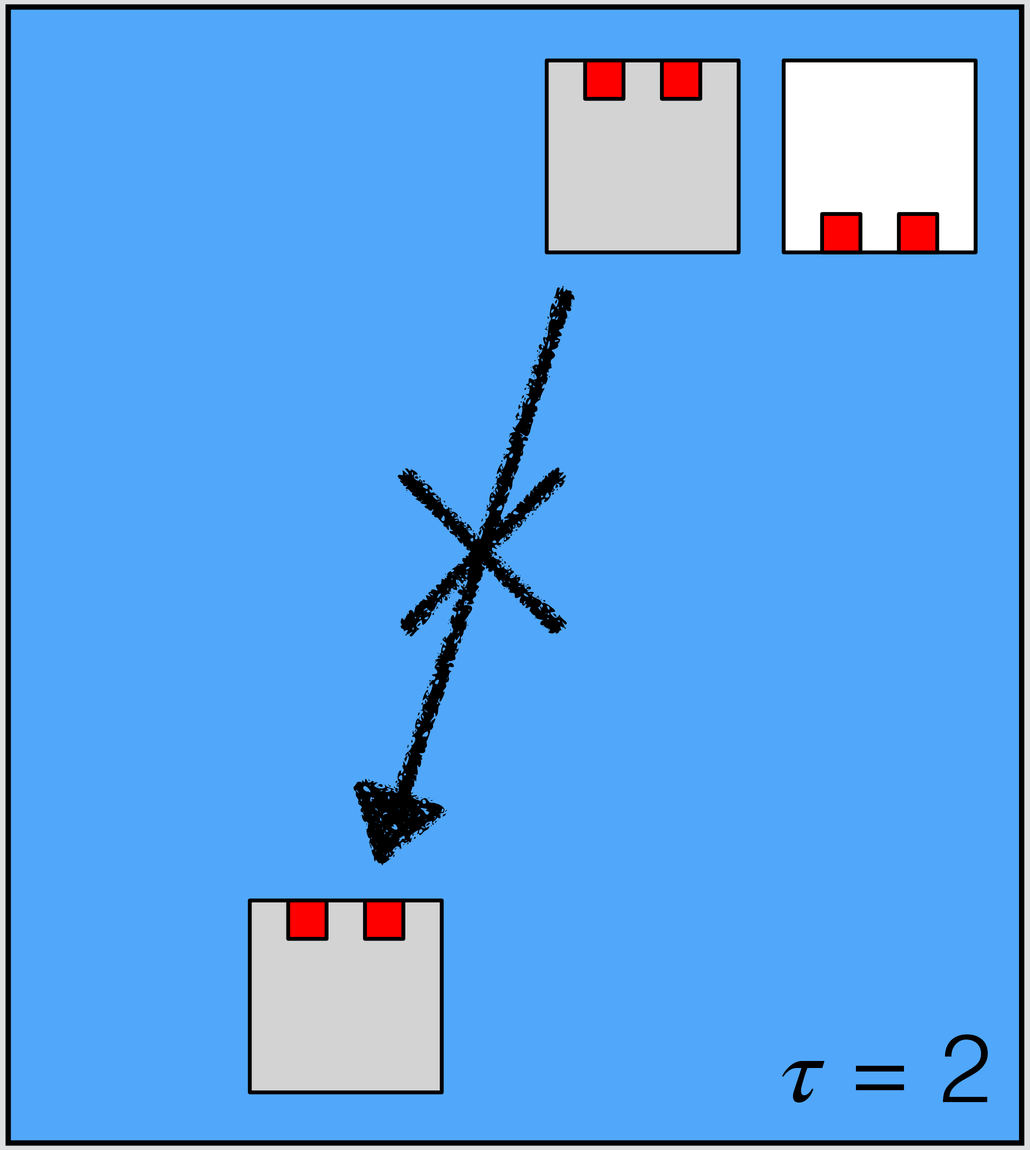
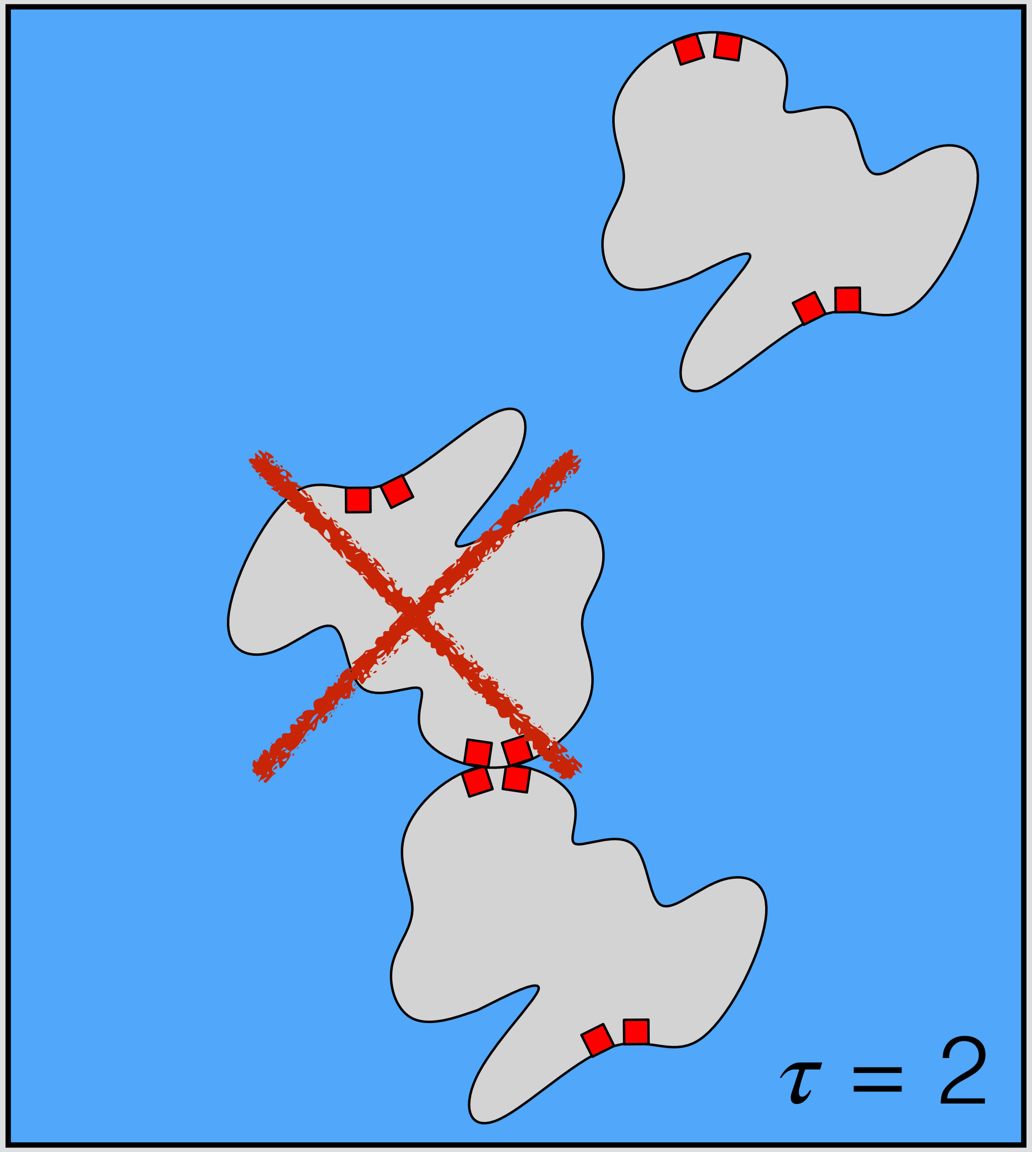


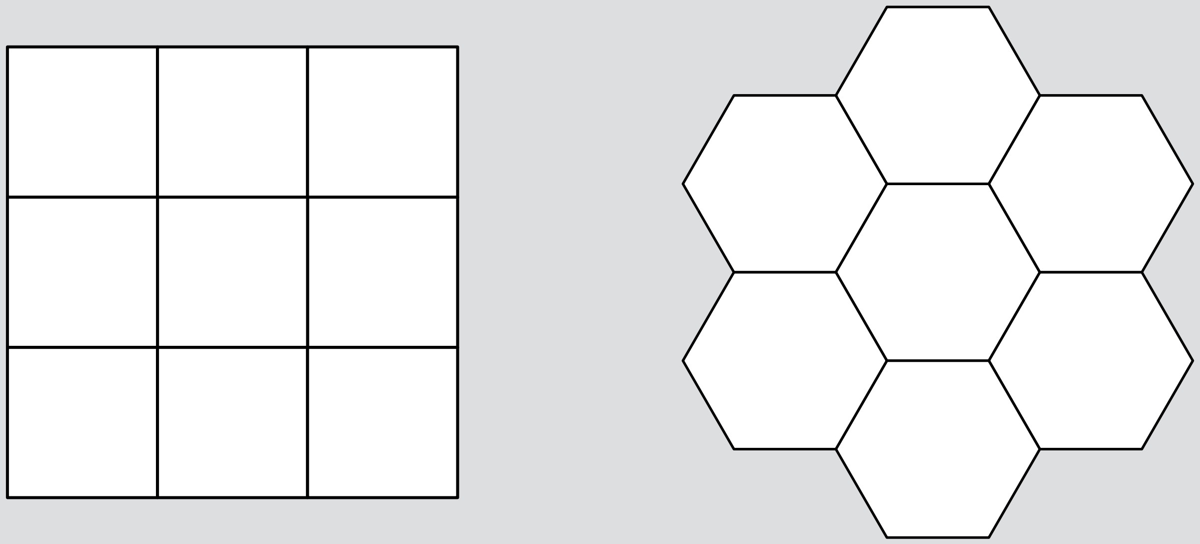


Proof of Universality

- Idea: use a chain of simulations from aTAM tile set at $\tau \geq 2$ to single-tile pfbTAM tile set at τ.
  - Simulation #1: eliminate strength-τ glues.
  - Simulation #2: eliminate unwanted rotations.
  - Simulation #3: encode tile set as a single tile.

Reduction #1: Eliminating strength-τ glues.

- Simulate with a system of hexagonal tiles:
  - With no rotation.
  - With <u>no strength-τ glues</u>.
  - With a multi-tile seed.
- Works because of square vs. hex lattice differences.



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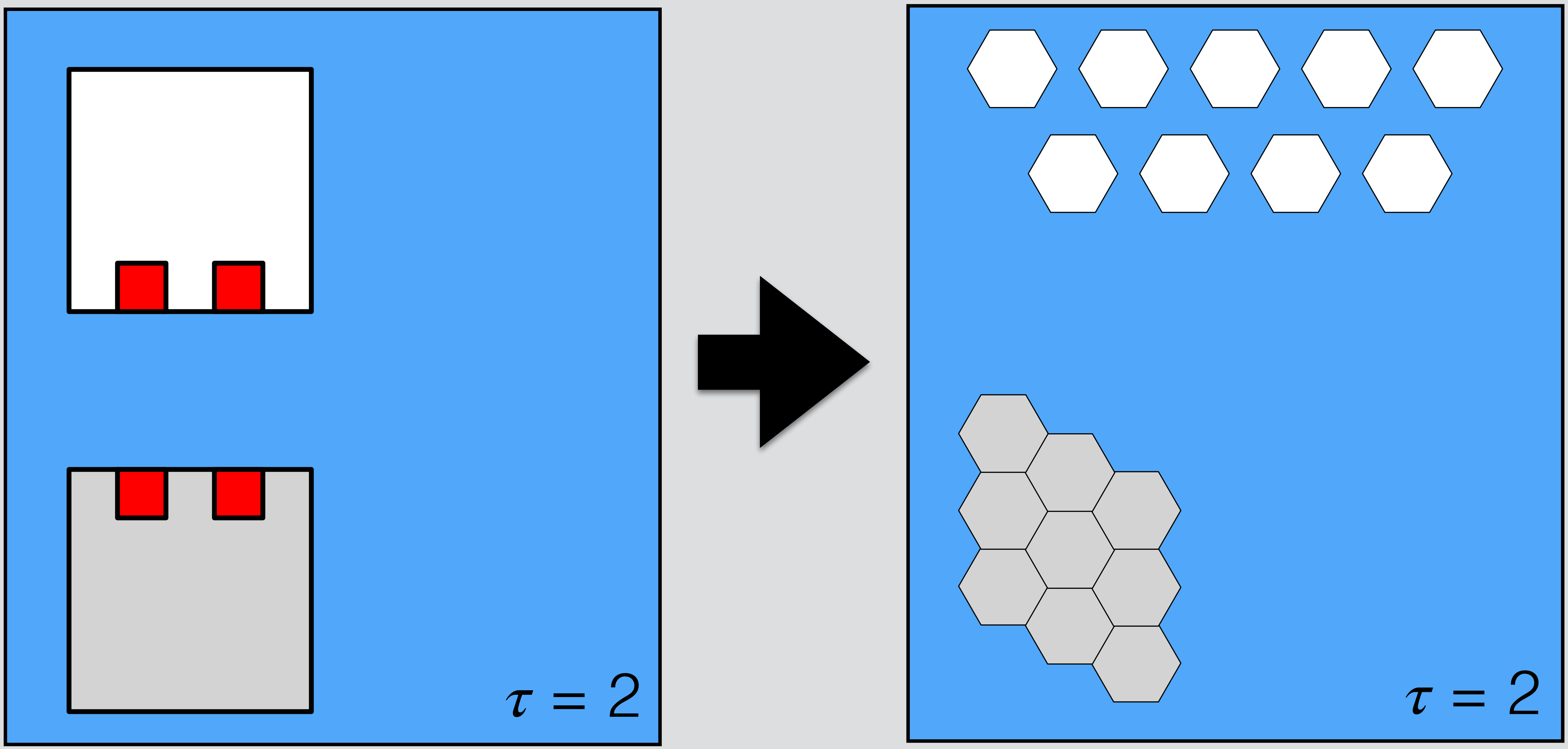

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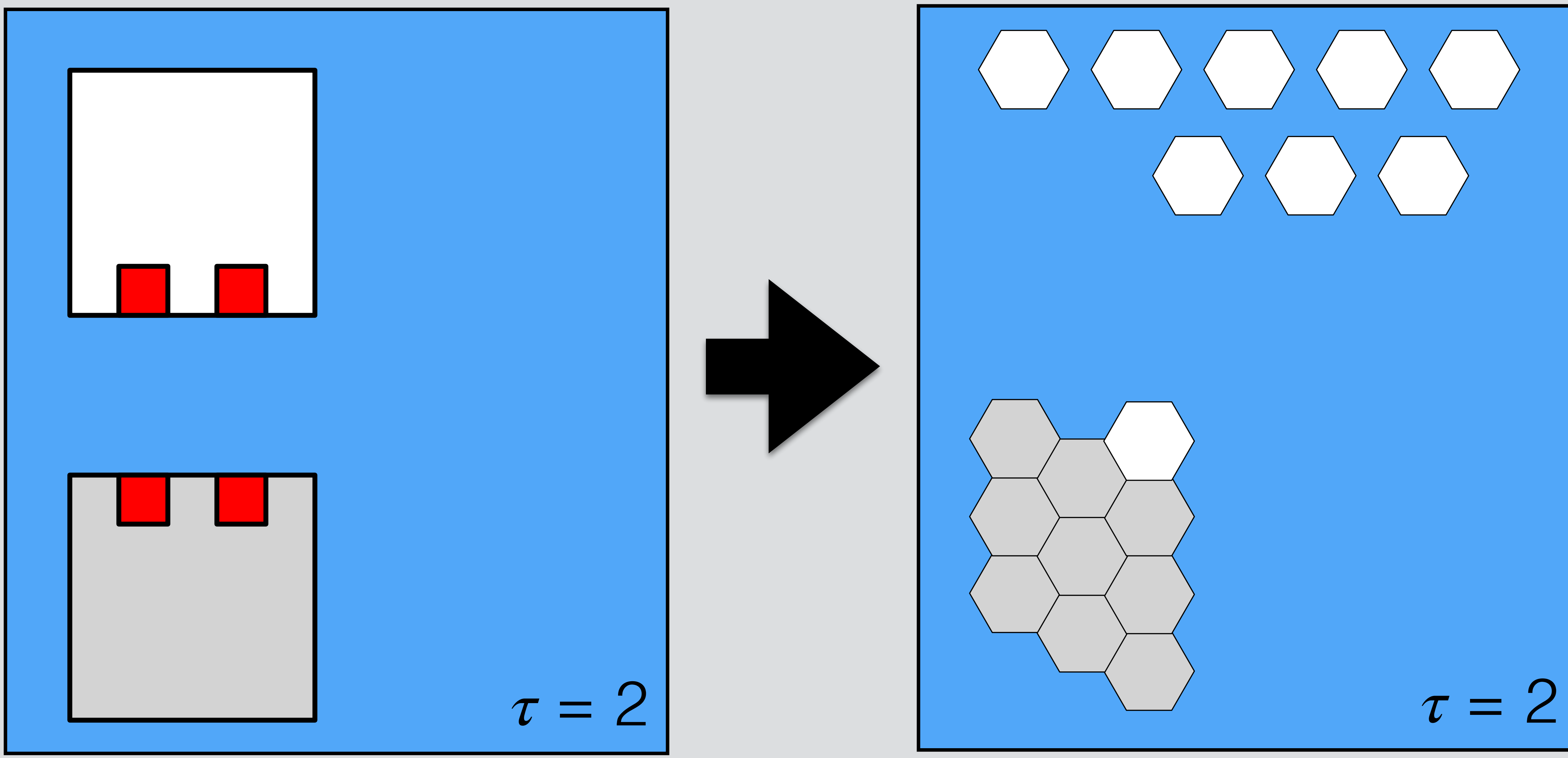

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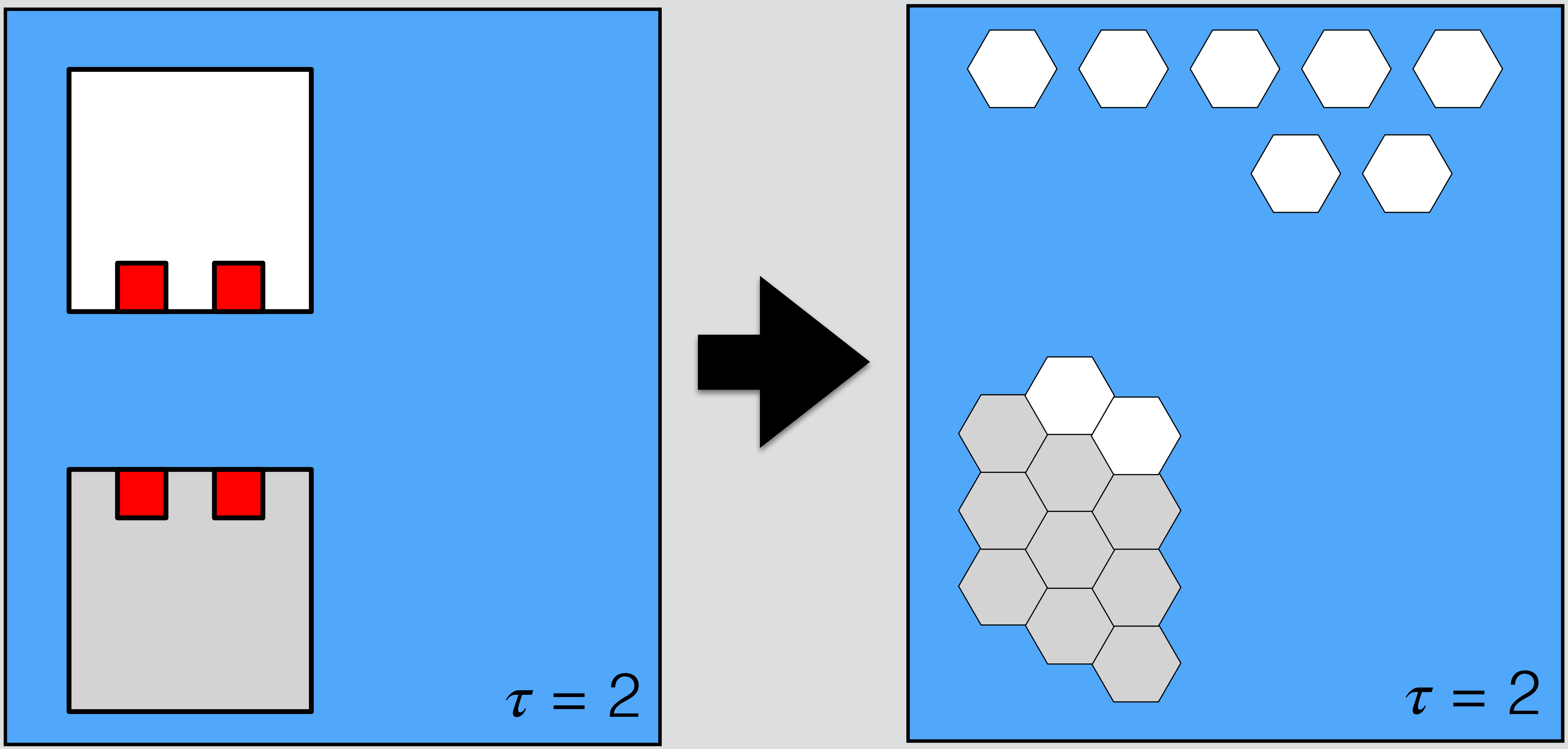

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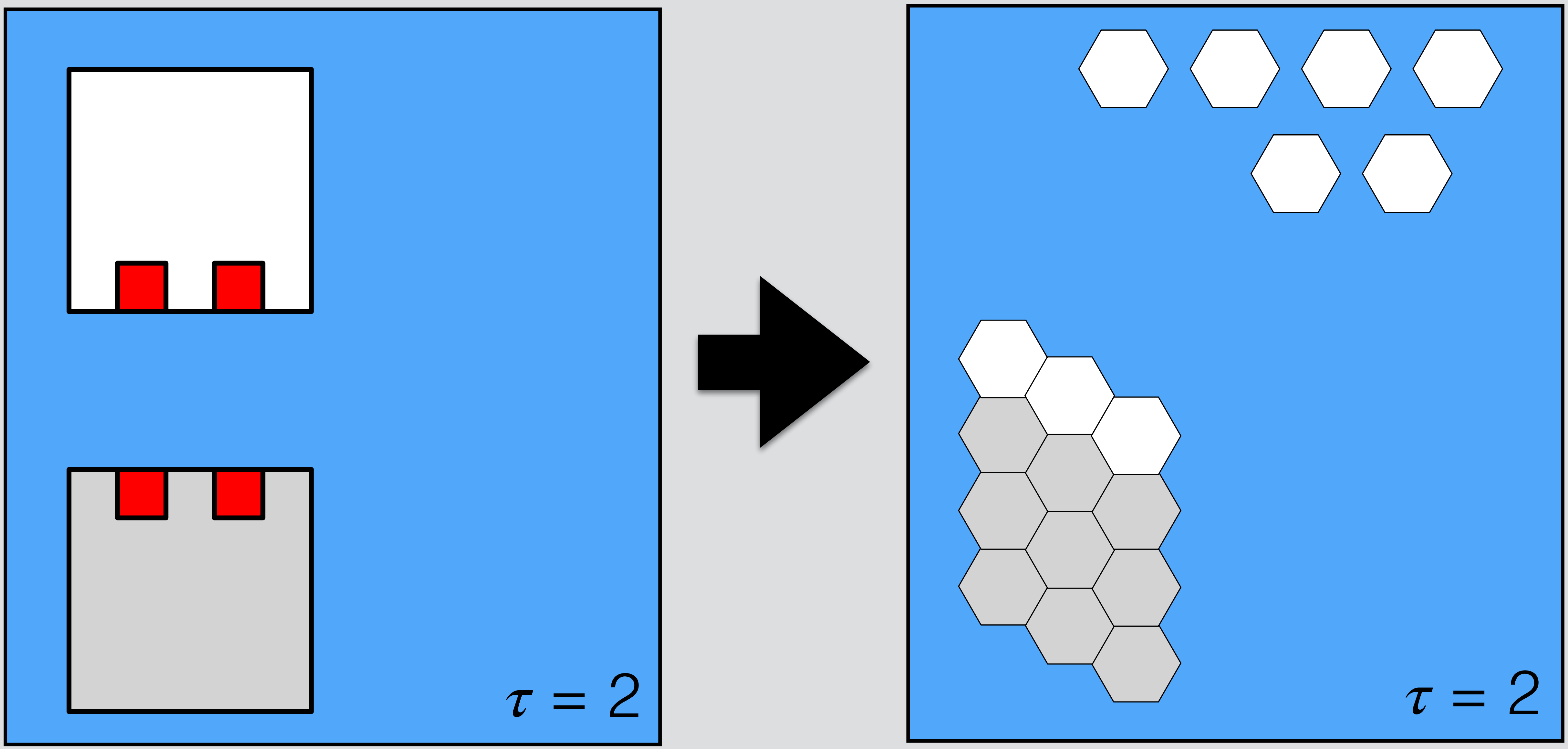

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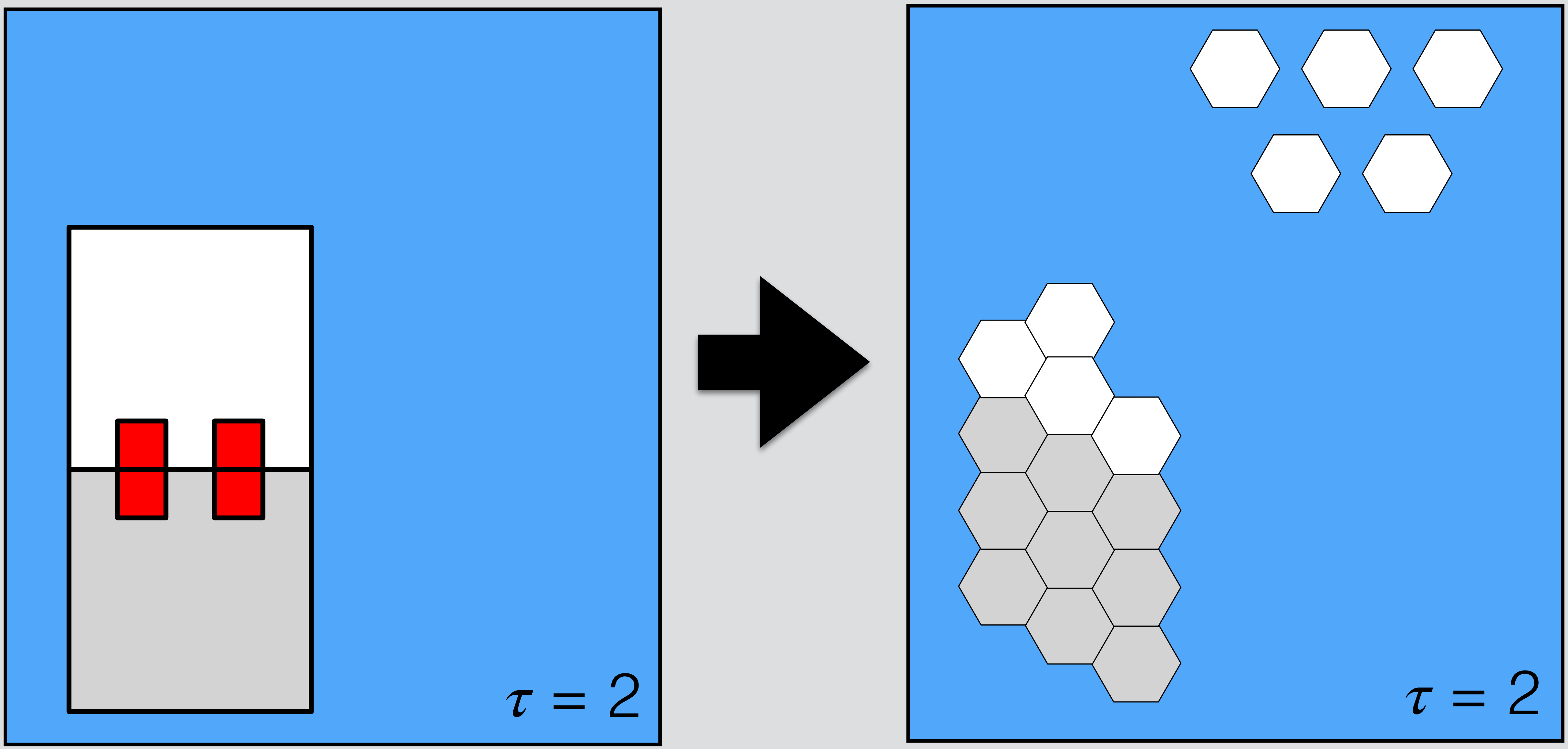

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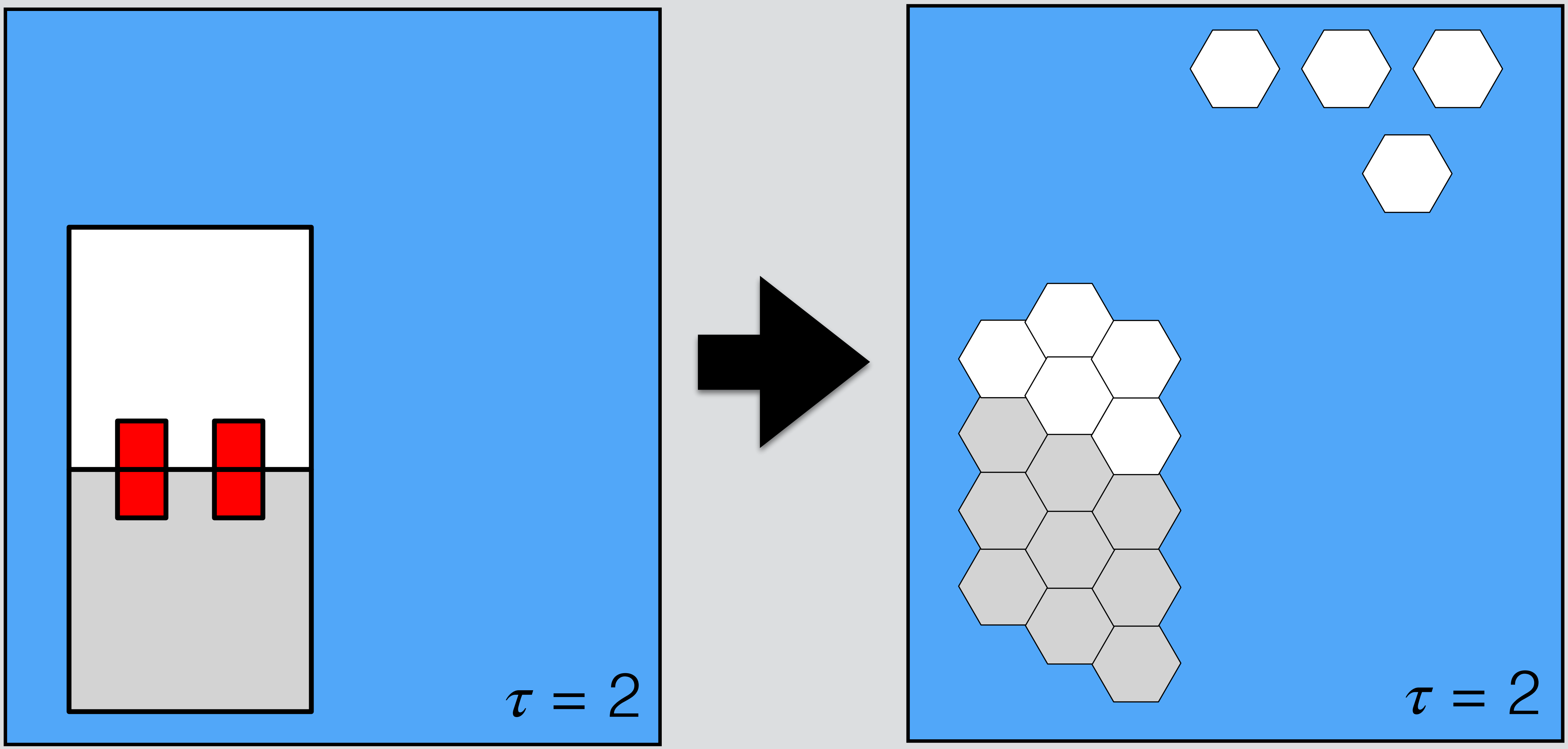

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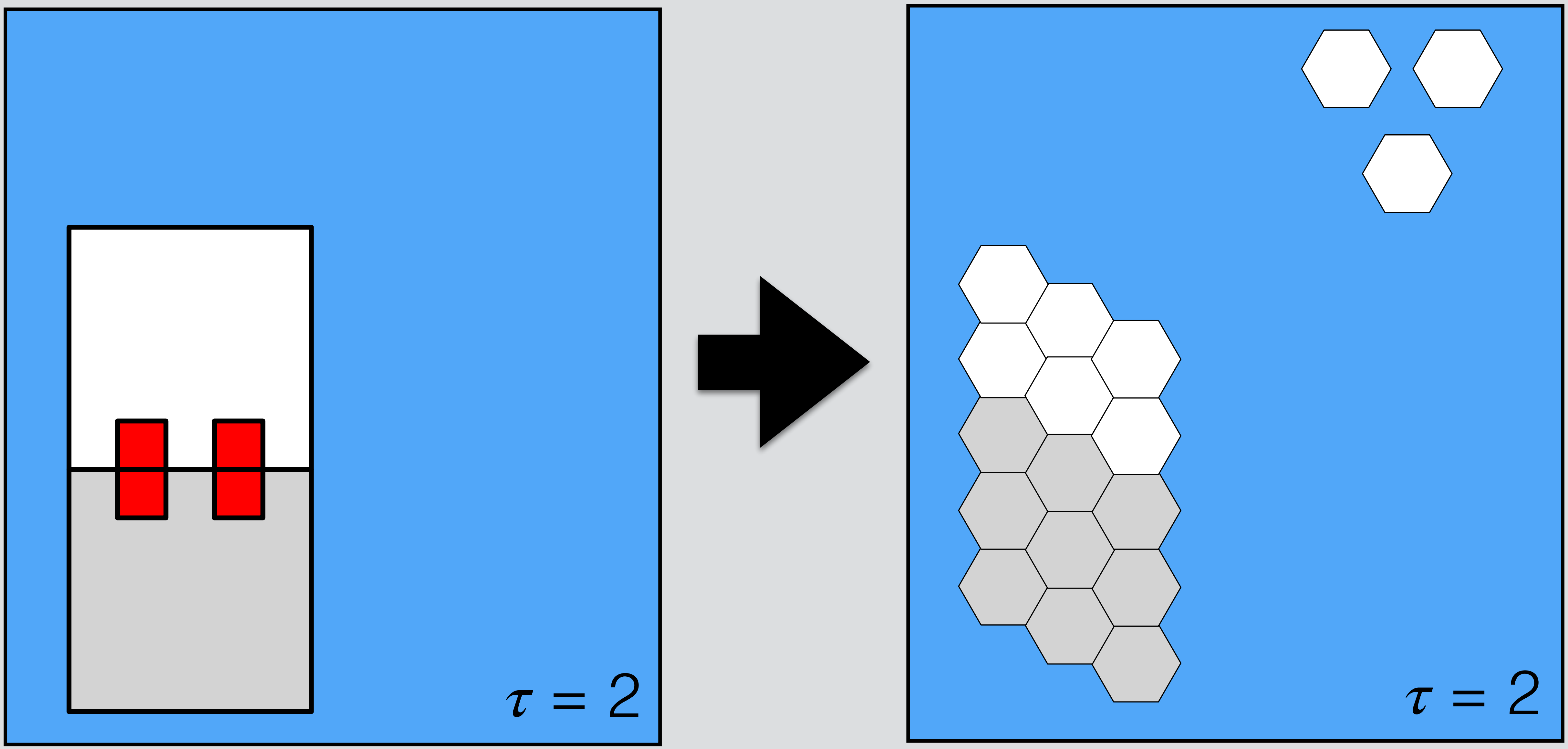

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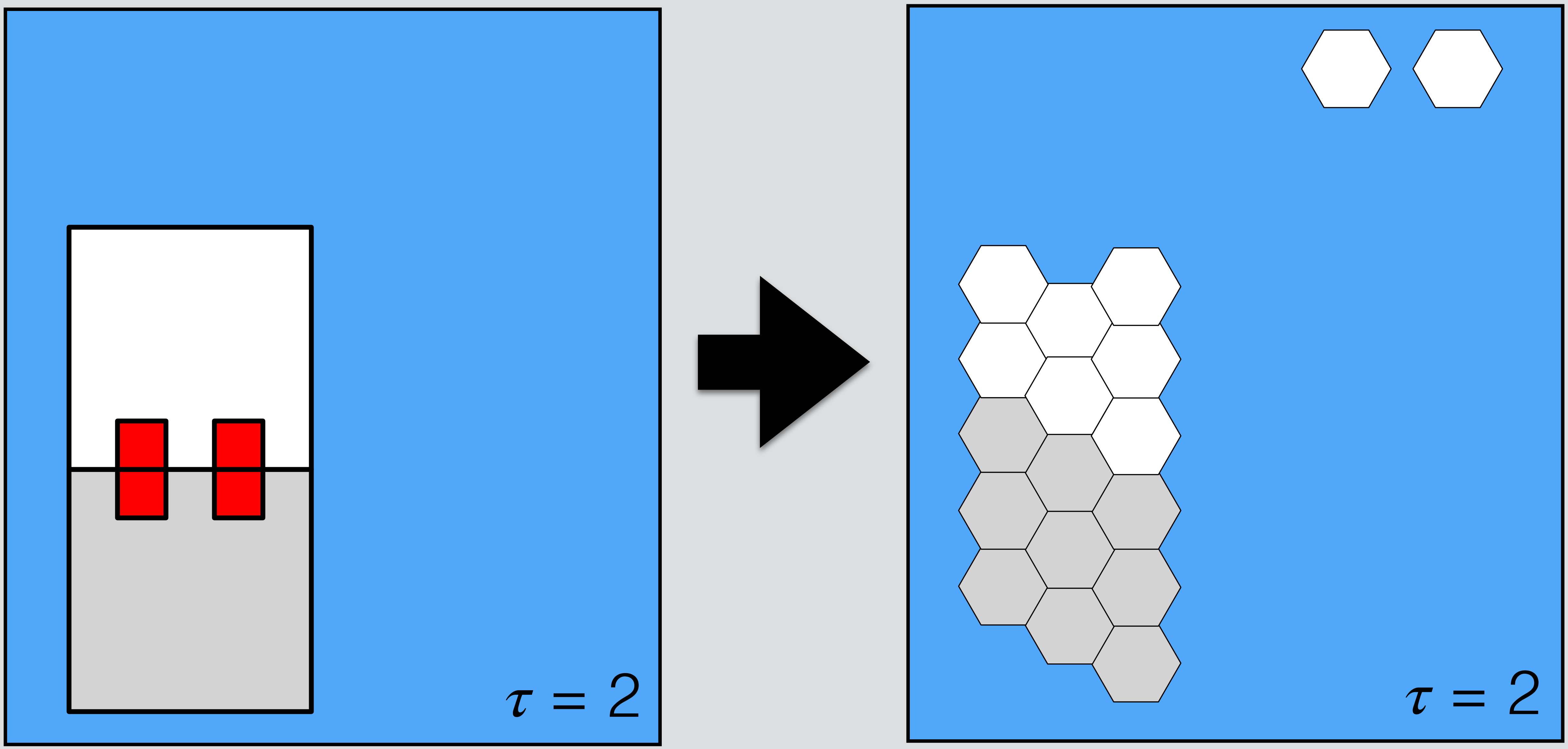

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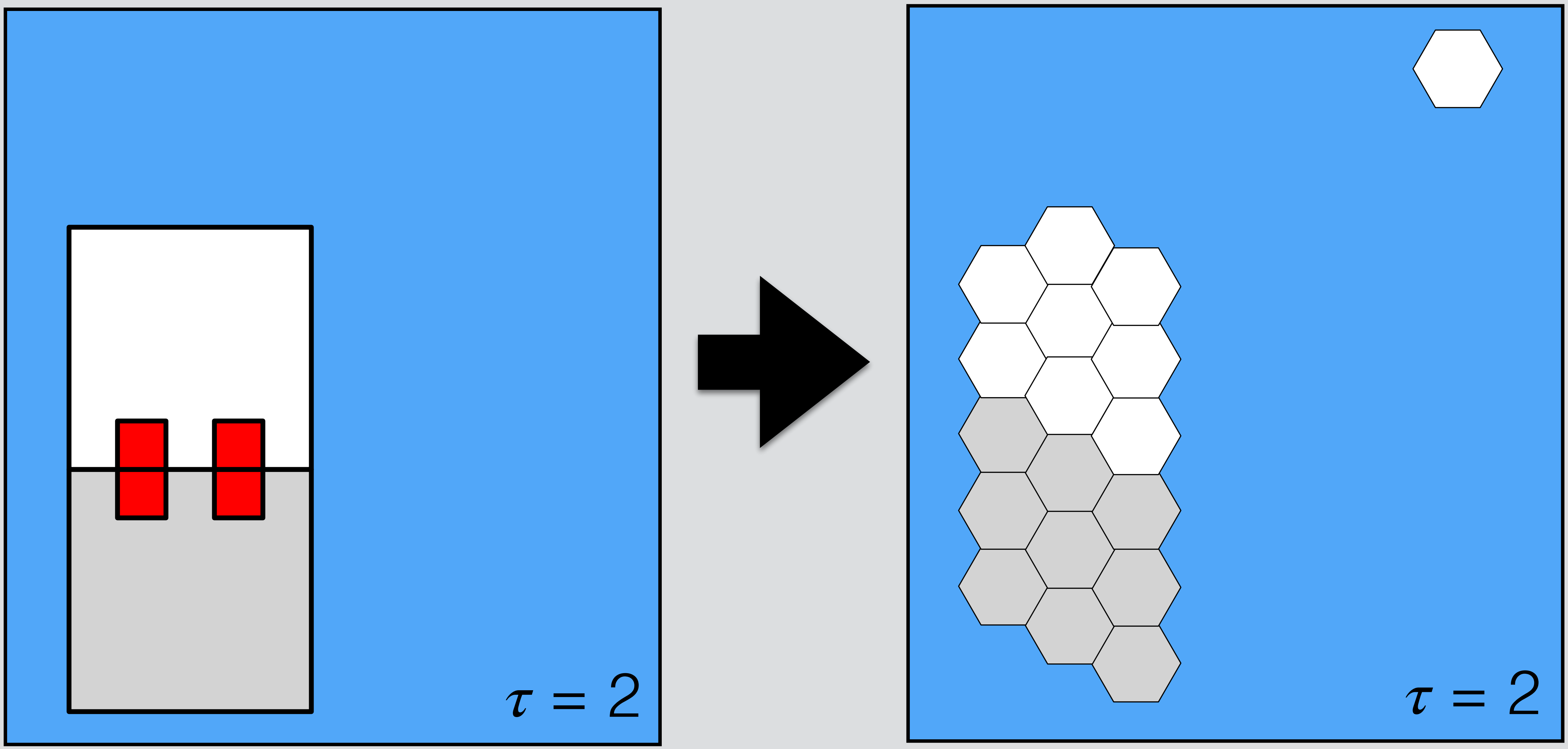

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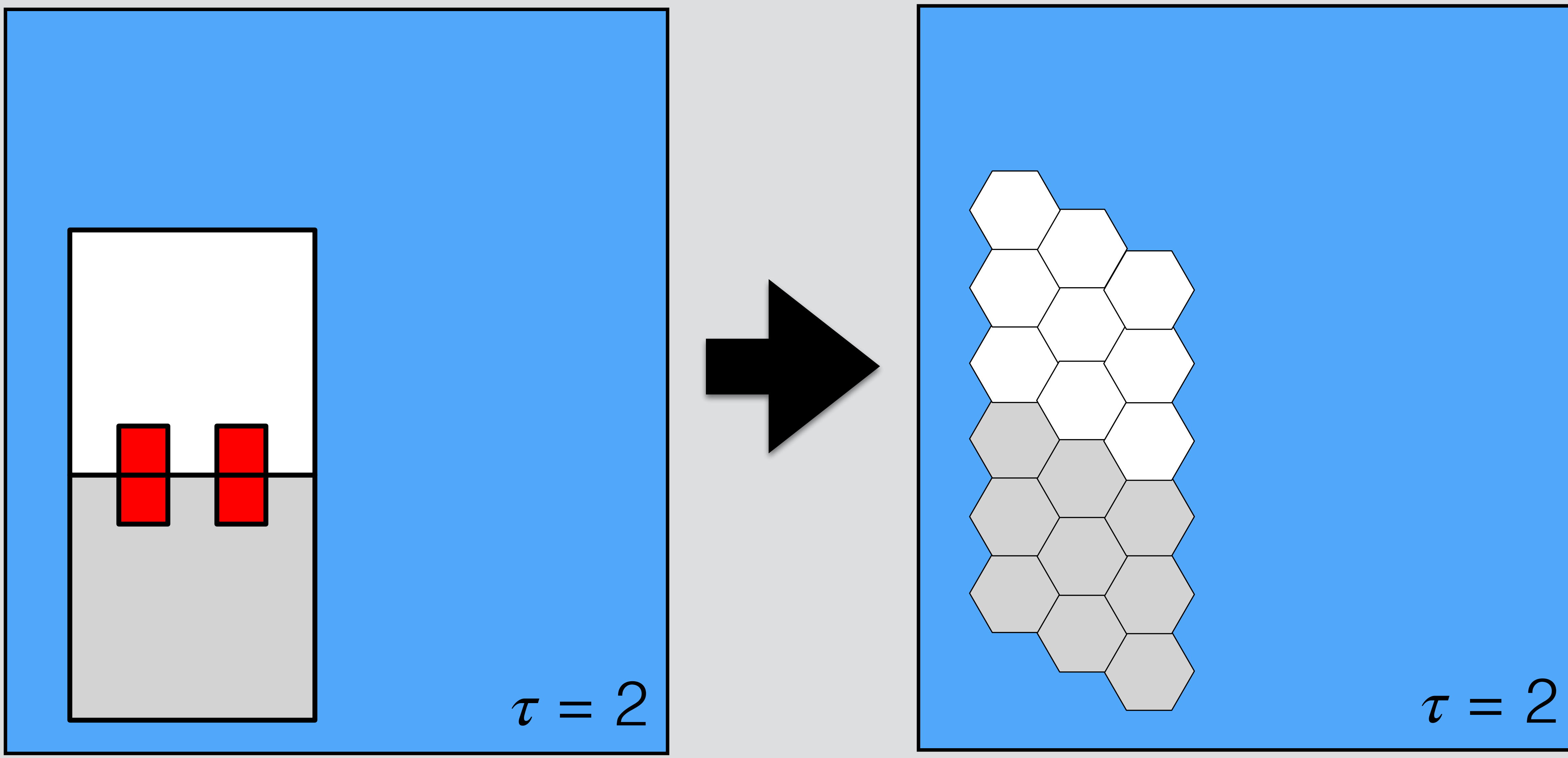

Reduction #2: Eliminate unwanted rotations.

- Simulate with a system of hexagonal tiles:
  - With rotation.
  - With a multi-tile seed.
- Use minimal glue sets of [Cannon et al. 2013] to maintain exposed glue invariants.

Reduction #3: Encode tile set as a single tile.

- Interleave hexagonal tile sides into a single equilateral+equiangular polygonal tile.

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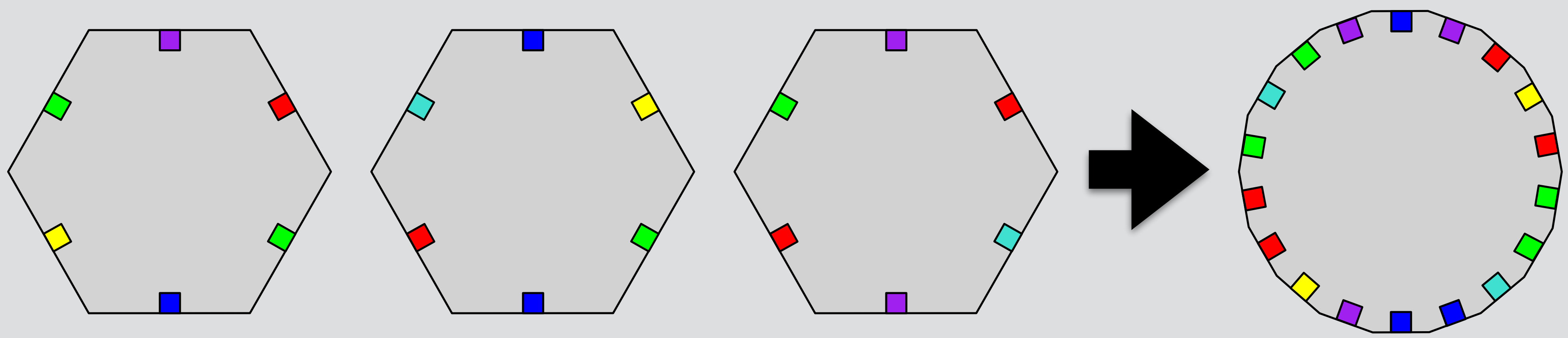

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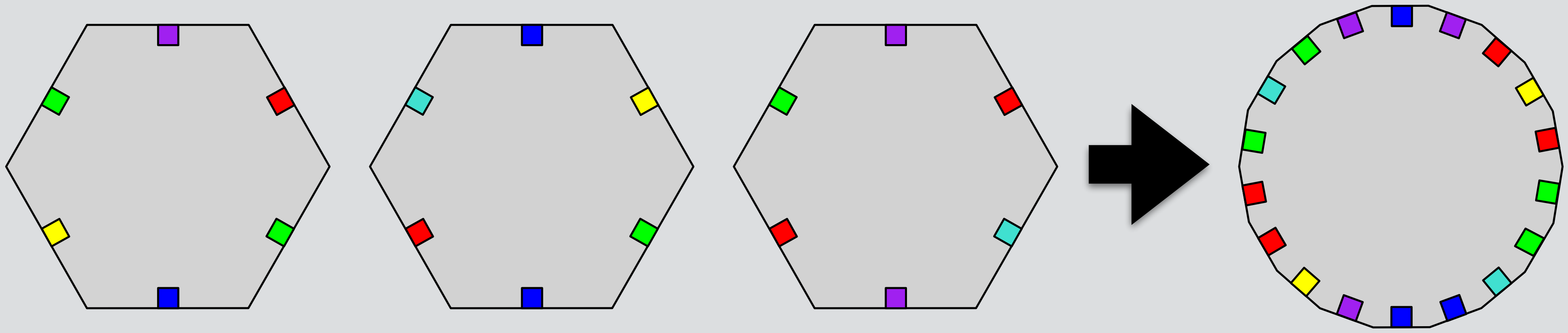

- Tile's rotation determines hexagonal tile simulated.

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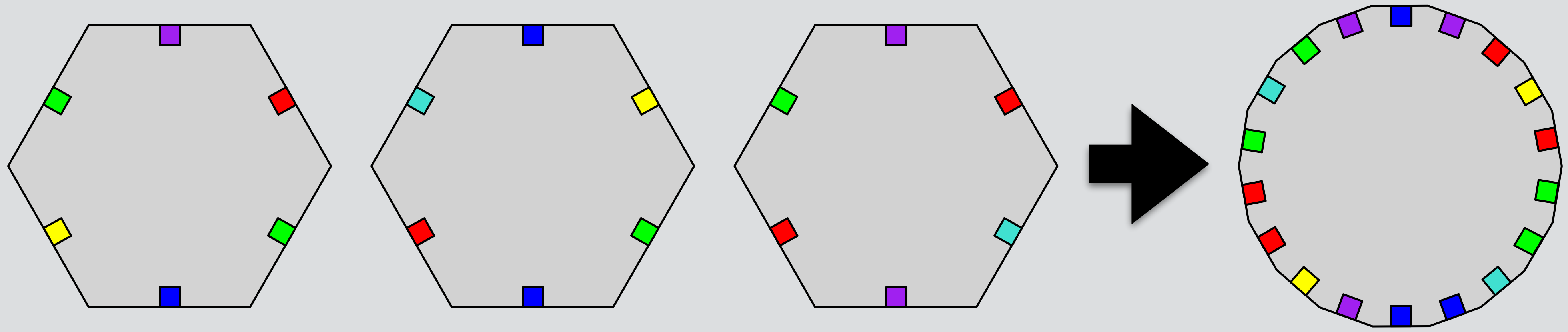
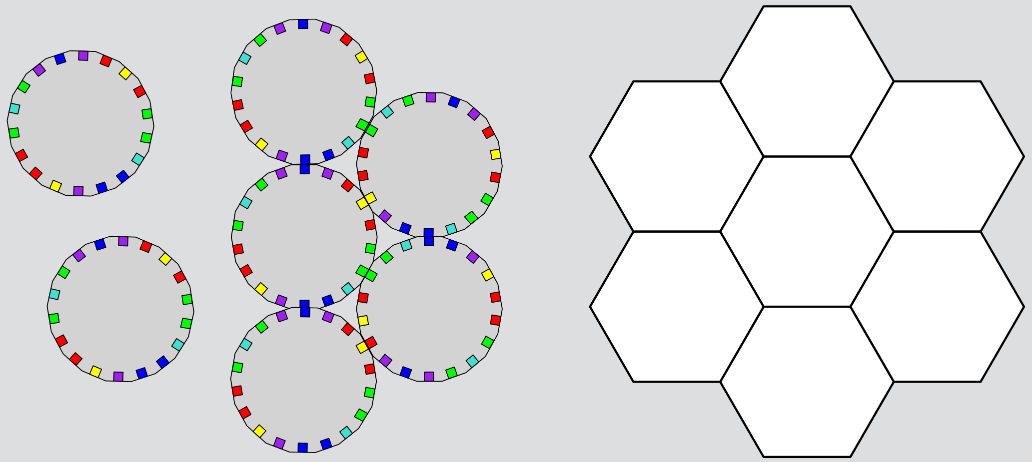


- Tile's rotation determines hexagonal tile simulated.

- No strength-τ glues implies only hex lattice formed.



Reduction #3: Encode tile set as a single tile.

- Interleave hexagonal tile sides into a single equilateral+equiangular polygonal tile.

- Use a single strength-τ glue to replace multi-tile seed with single-tile seed.

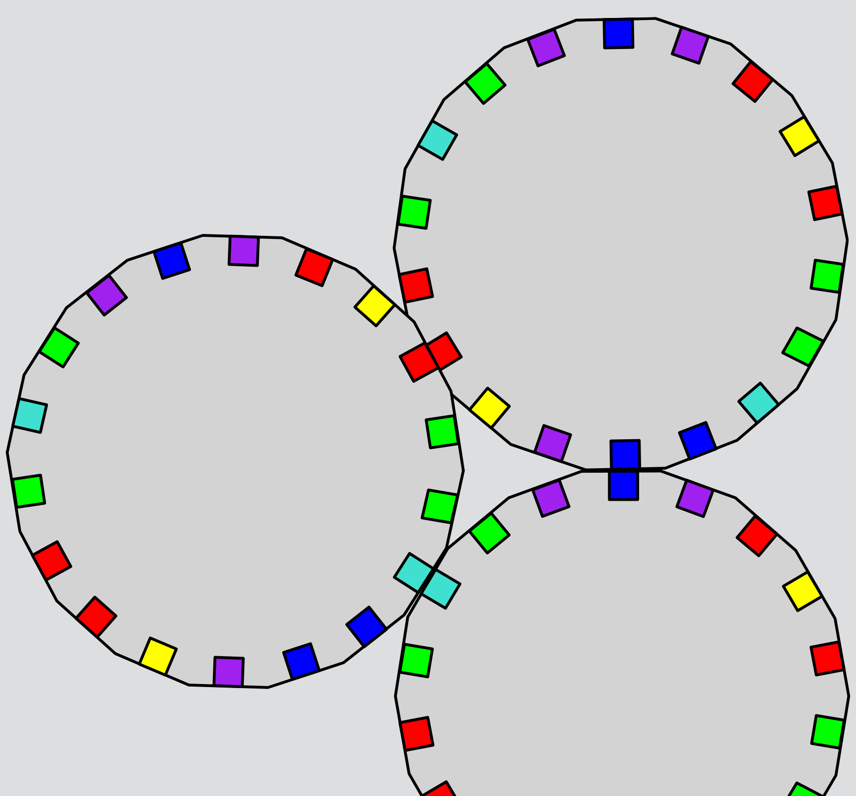
    - *Self-seeding.*

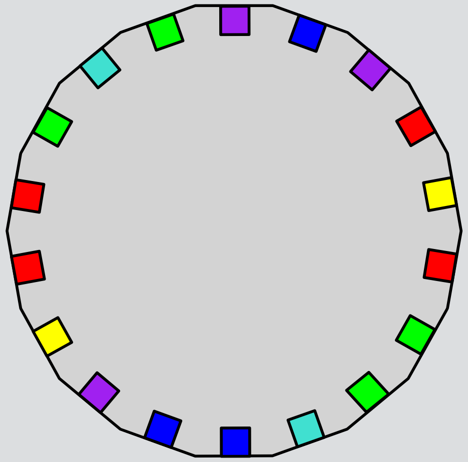
Reduction #3: Encode tile set as a single tile.

- Interleave hexagonal tile sides into a single equilateral+equiangular polygonal tile.

- Use a single strength-τ glue to replace multi-tile seed with single-tile seed.

    - Only usable when seed assembly is one tile.





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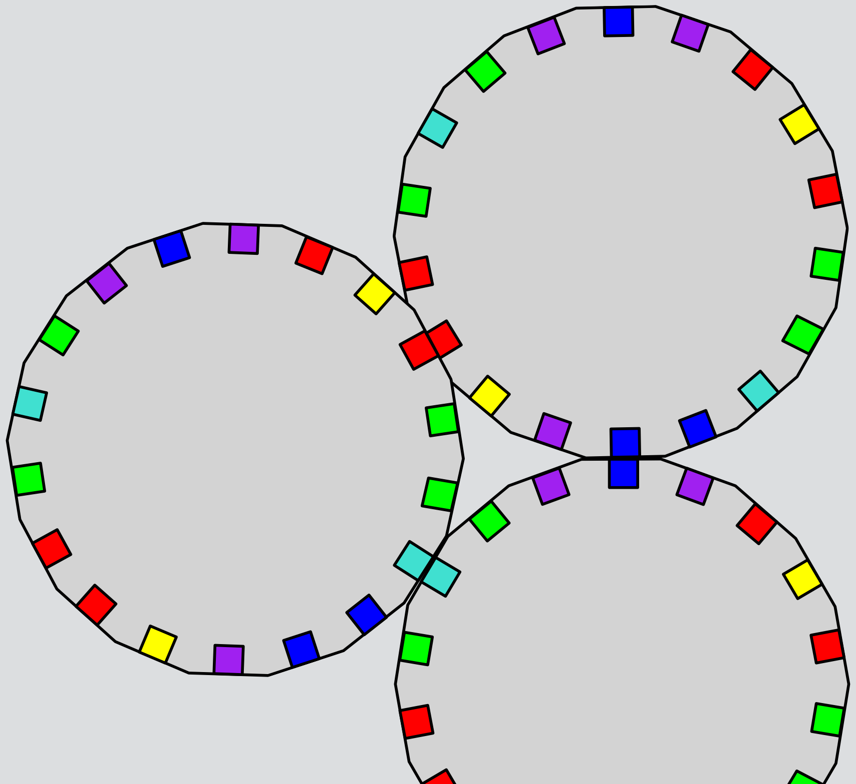
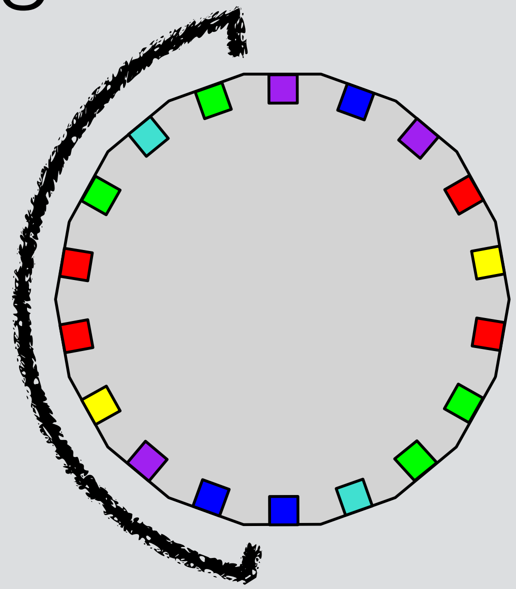


Safe region



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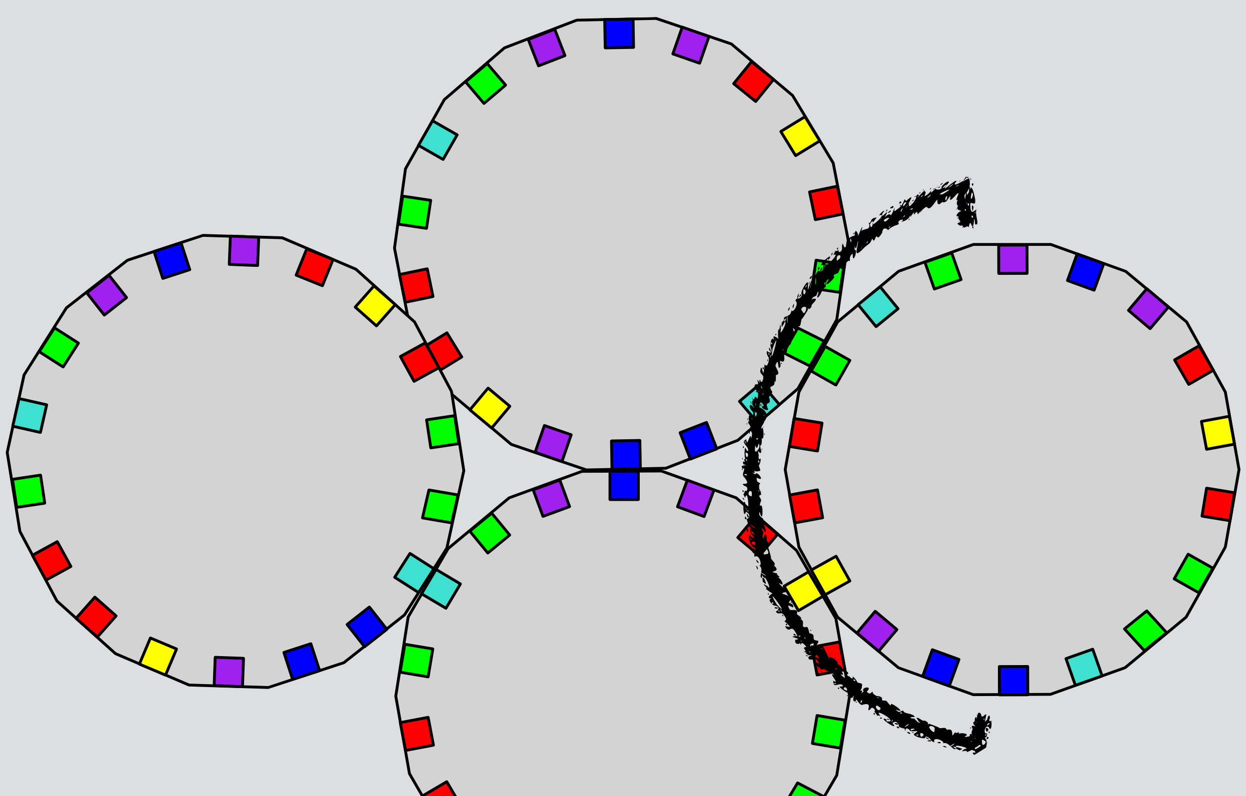


Safe region
no longer usable

Proof of Universality

- Idea: use a chain of simulations from aTAM tile set at $\tau \geq 2$ to single-tile pfbTAM tile set at τ.
- For input aTAM tile set of t tiles, resulting pfbTAM tile has O(t) sides.
- Also works for aTAM tile sets at $\tau = 1$. (Easy)

What about a single tile without rotation?

Single Non-rotatable Tile

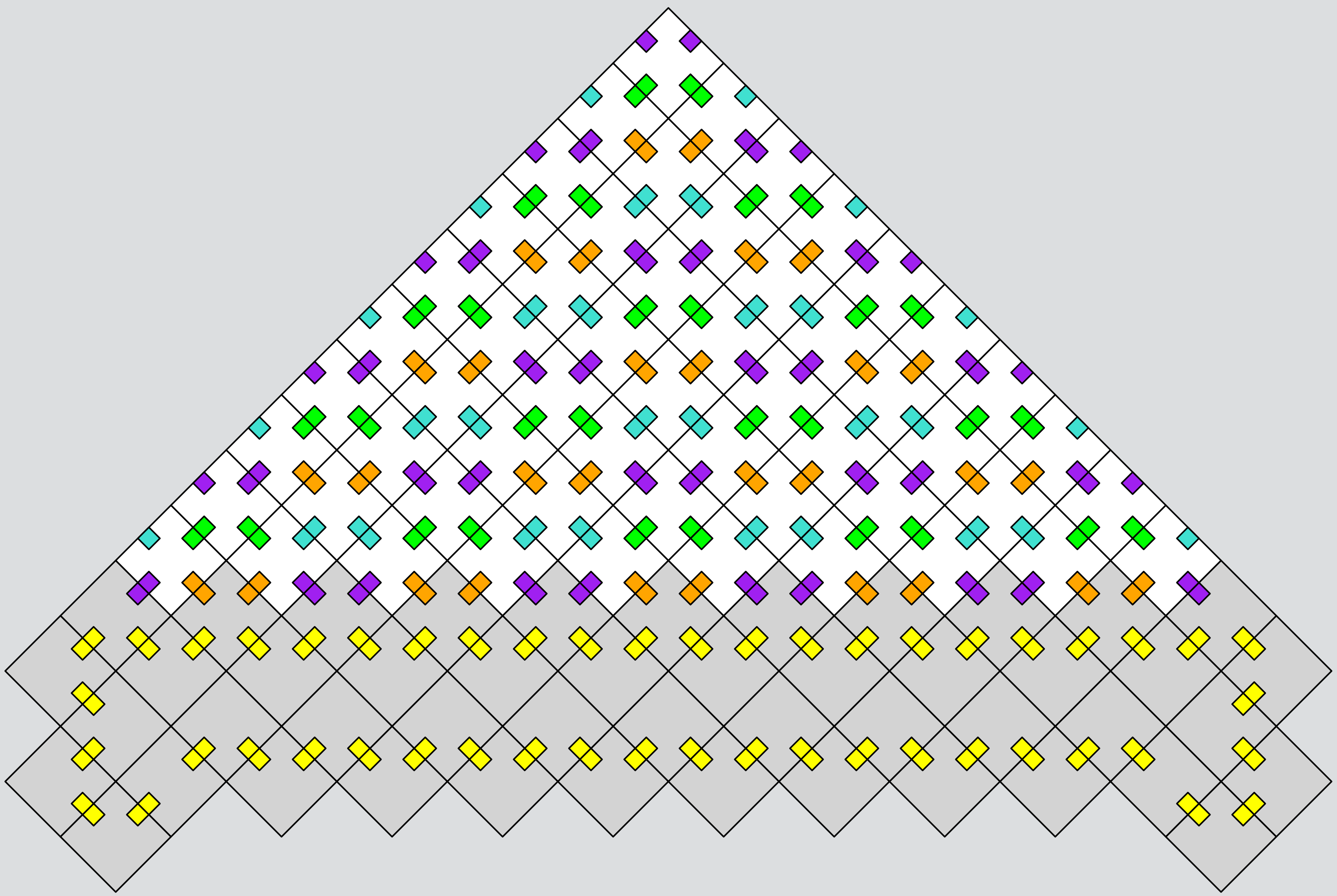
- Theorem: can compute t steps of any blocked cellular automaton machine at $\tau = 3$, starting with a seed assembly of size $\Theta(t)$.

Single Non-rotatable Tile

- Theorem: can compute t steps of any blocked cellular automaton machine at τ = 3, starting with a seed assembly of size Θ(t).
  - Simulation #1: Blocked cellular automata with wedge-shaped τ = 2 aTAM tile set.
  - Simulation #2: wedge-shaped τ = 2 aTAM tile set with single polygonal tile at τ = 3.

Single Non-rotatable Tile

Simulation #1: BCA with wedge-shaped $\tau = 2$ aTAM tile set.



Single Non-rotatable Tile

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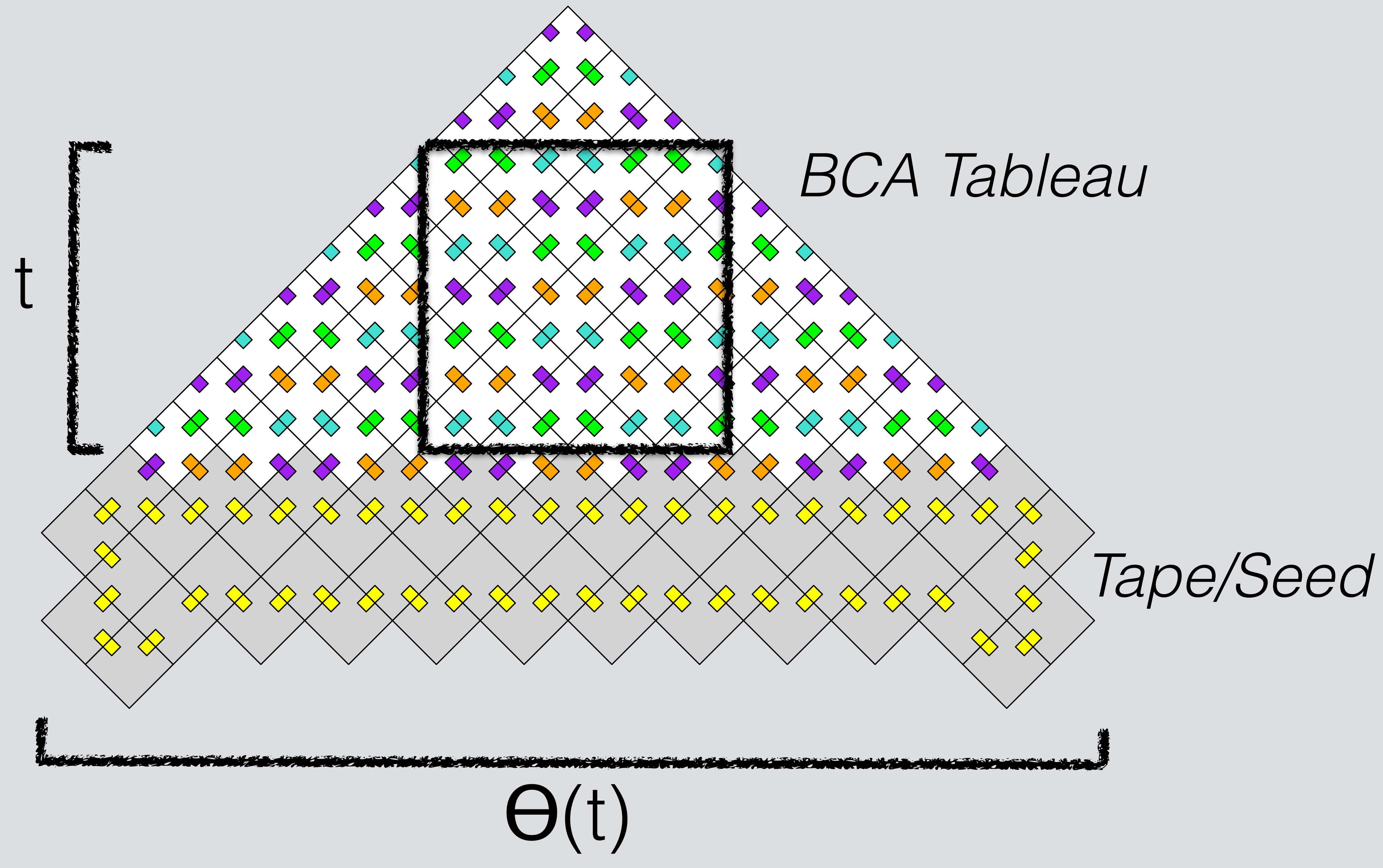
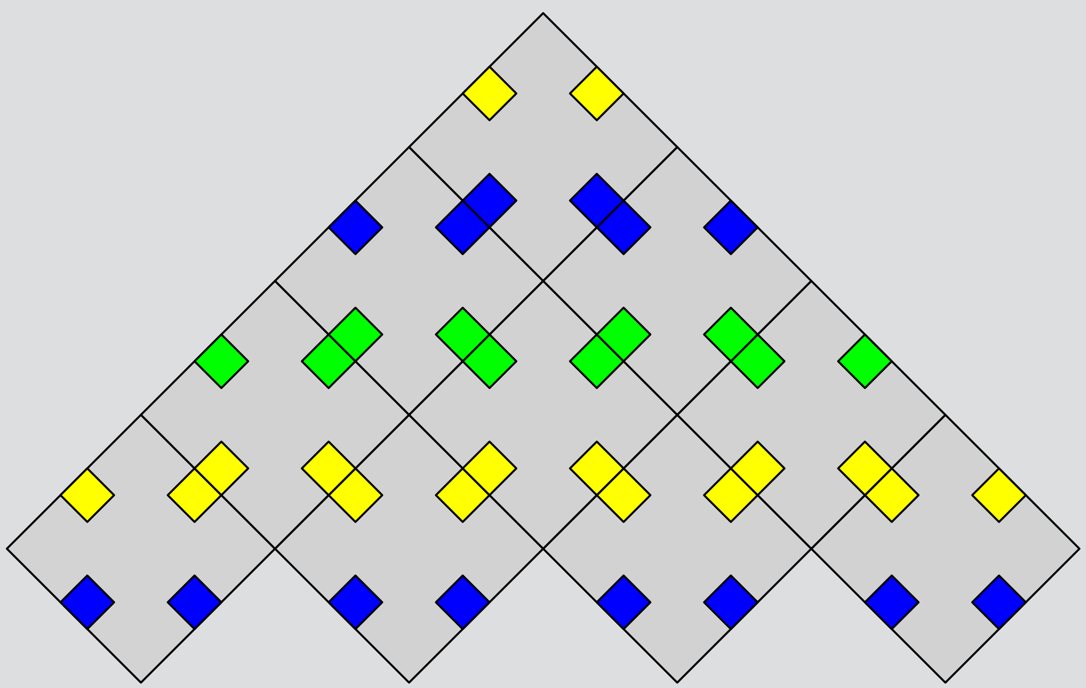


Single Non-rotatable Tile

Simulation #2: wedge-shaped $\tau = 2$ aTAM tile set with single polygonal tile at $\tau = 3$.

Single Non-rotatable Tile

Simulation #2: wedge-shaped τ = 2 aTAM tile set with single polygonal tile at τ = 3.



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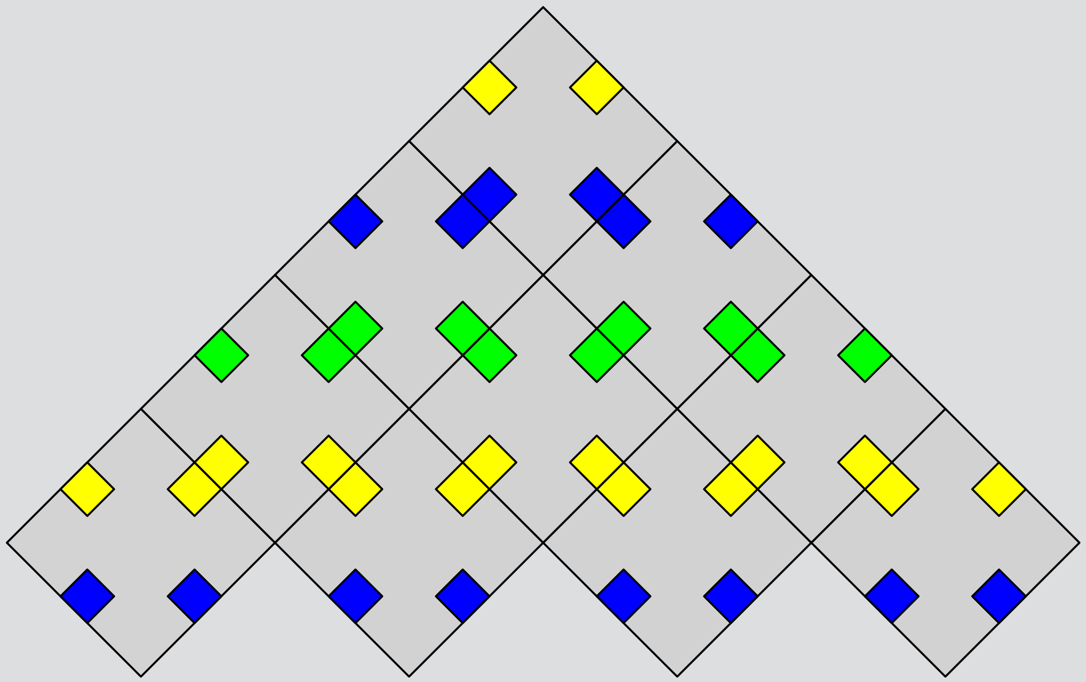
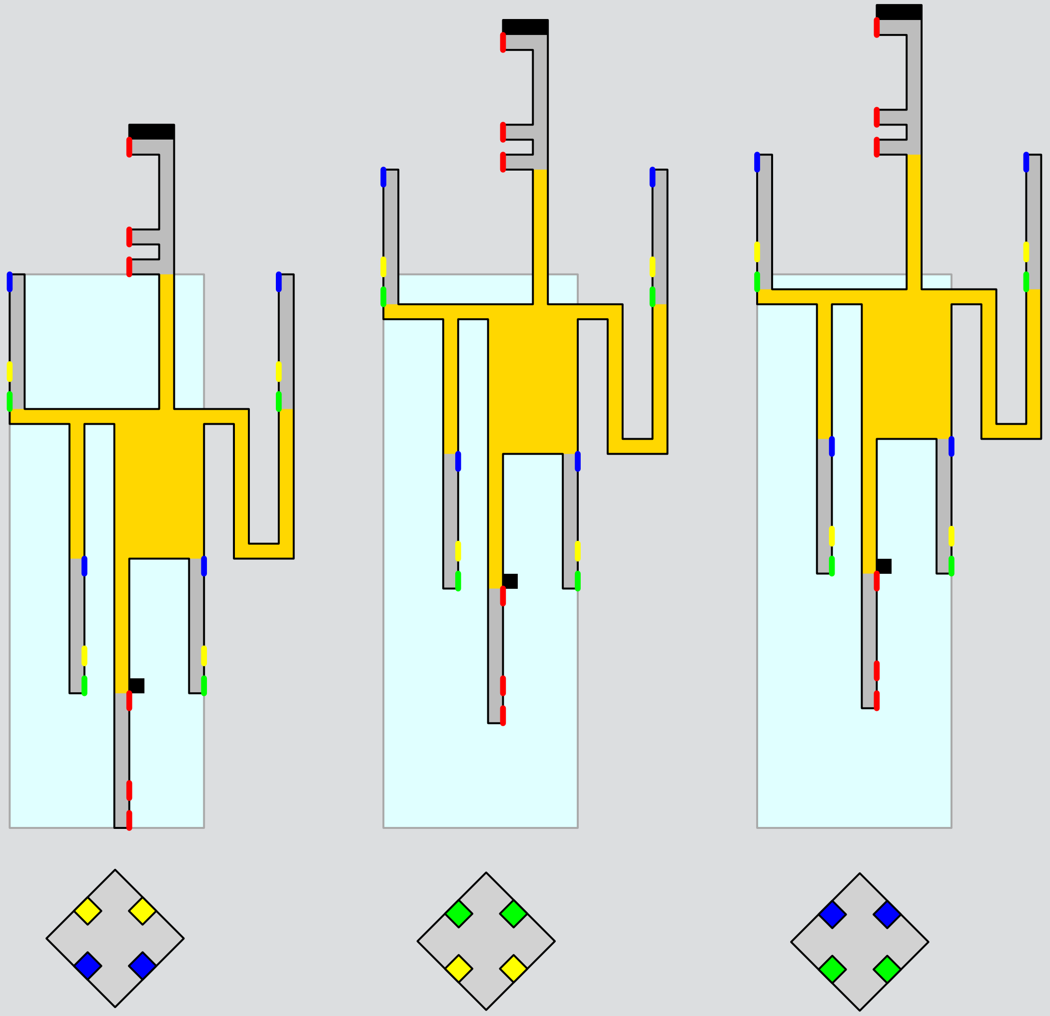
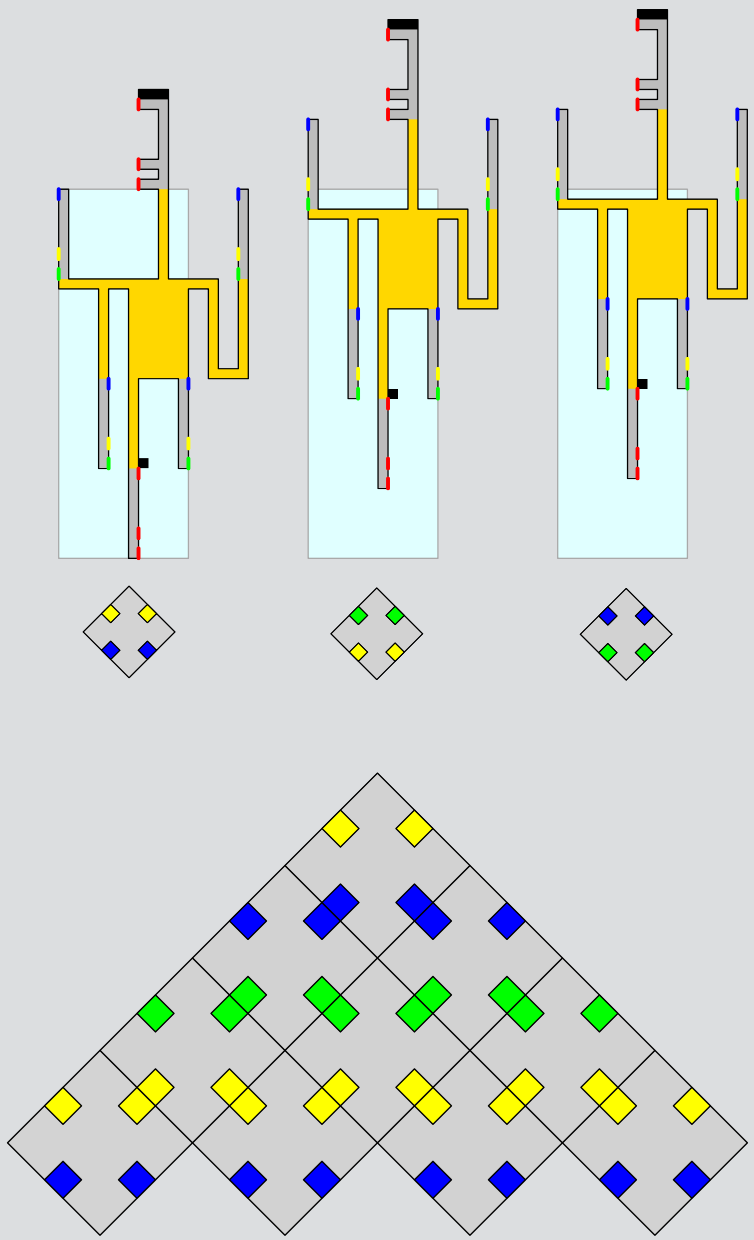
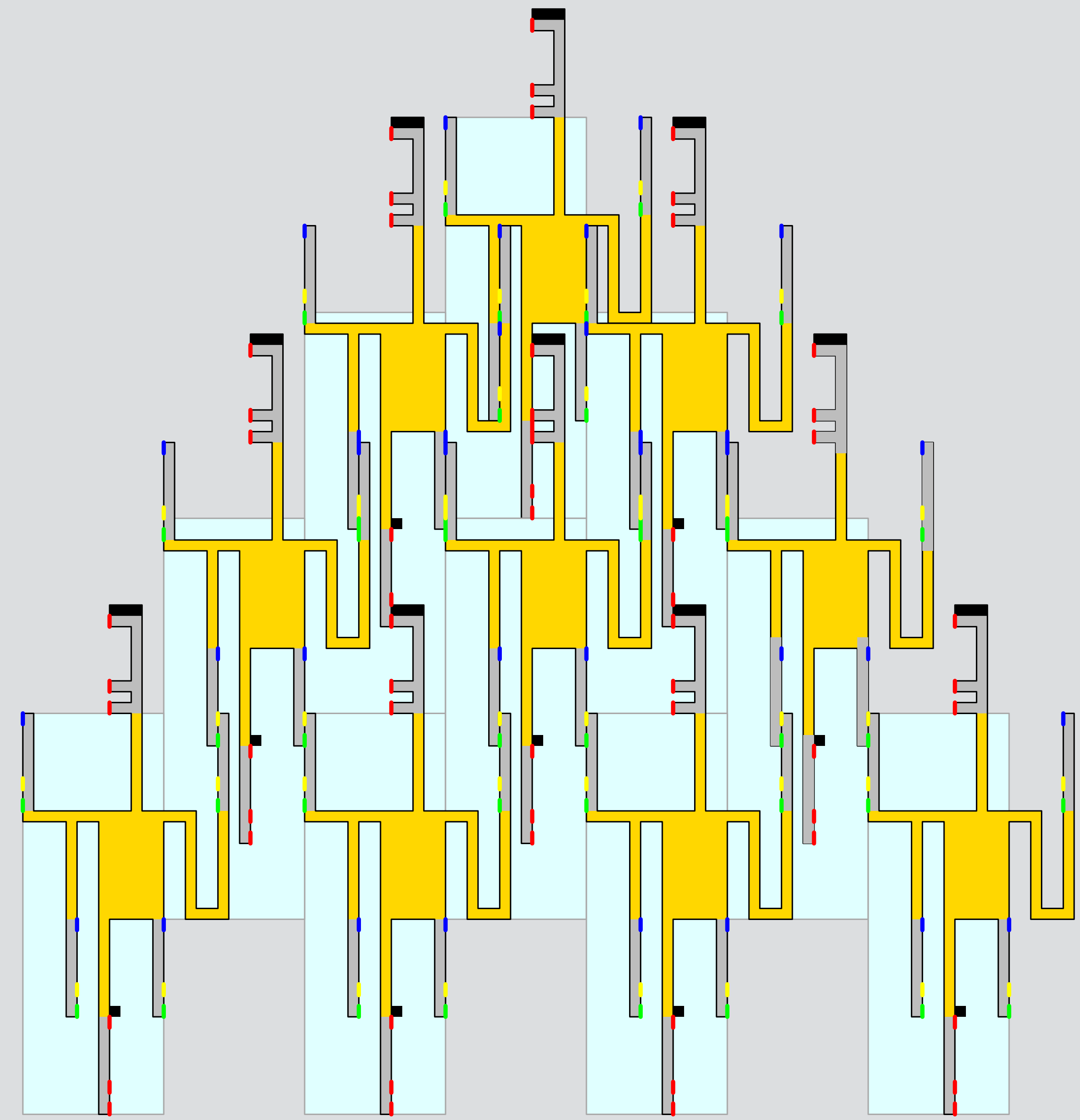




Single Non-rotatable Tile

Simulation #2: wedge-shaped $\tau = 2$ aTAM tile set
with single polygonal tile at $\tau = 3$.





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- Conjecture: finite assembly of n-tile shape requires seed with $\Omega(n^{1/2})$ tiles.

Conclusions

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- A single rotatable tile at $\tau = 2$ can simulate every tile assembly system, given an appropriate seed.
  - One tile to simulate them all. 
- A single non-rotatable tile at $\tau = 3$ can do non-trivial computation (linear in seed size).
  - How much more?

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