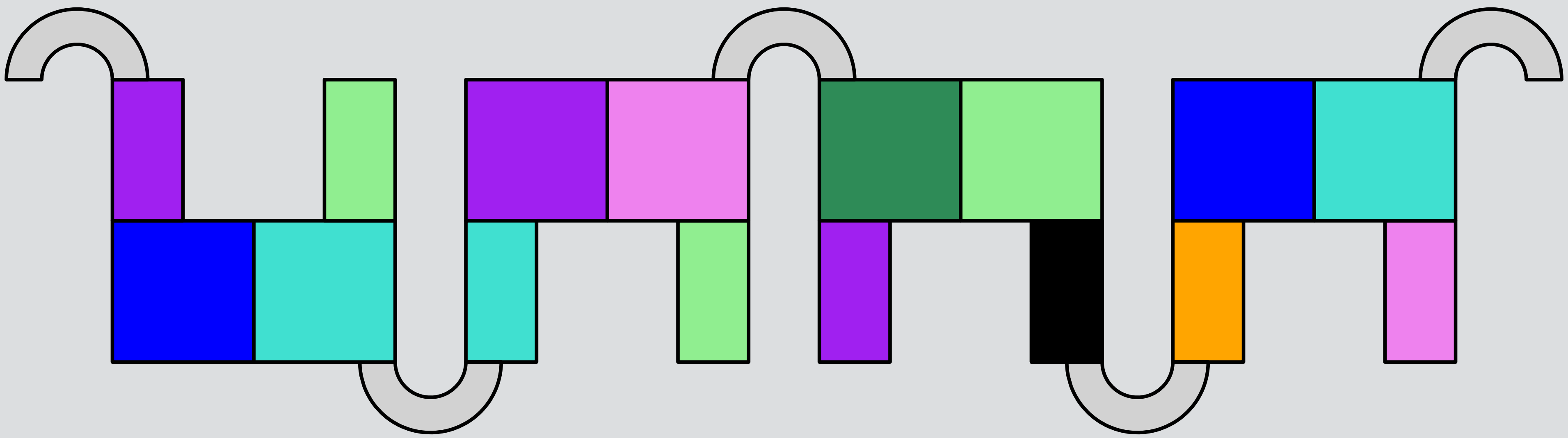
Tight bounds for active self-assembly using an insertion primitive



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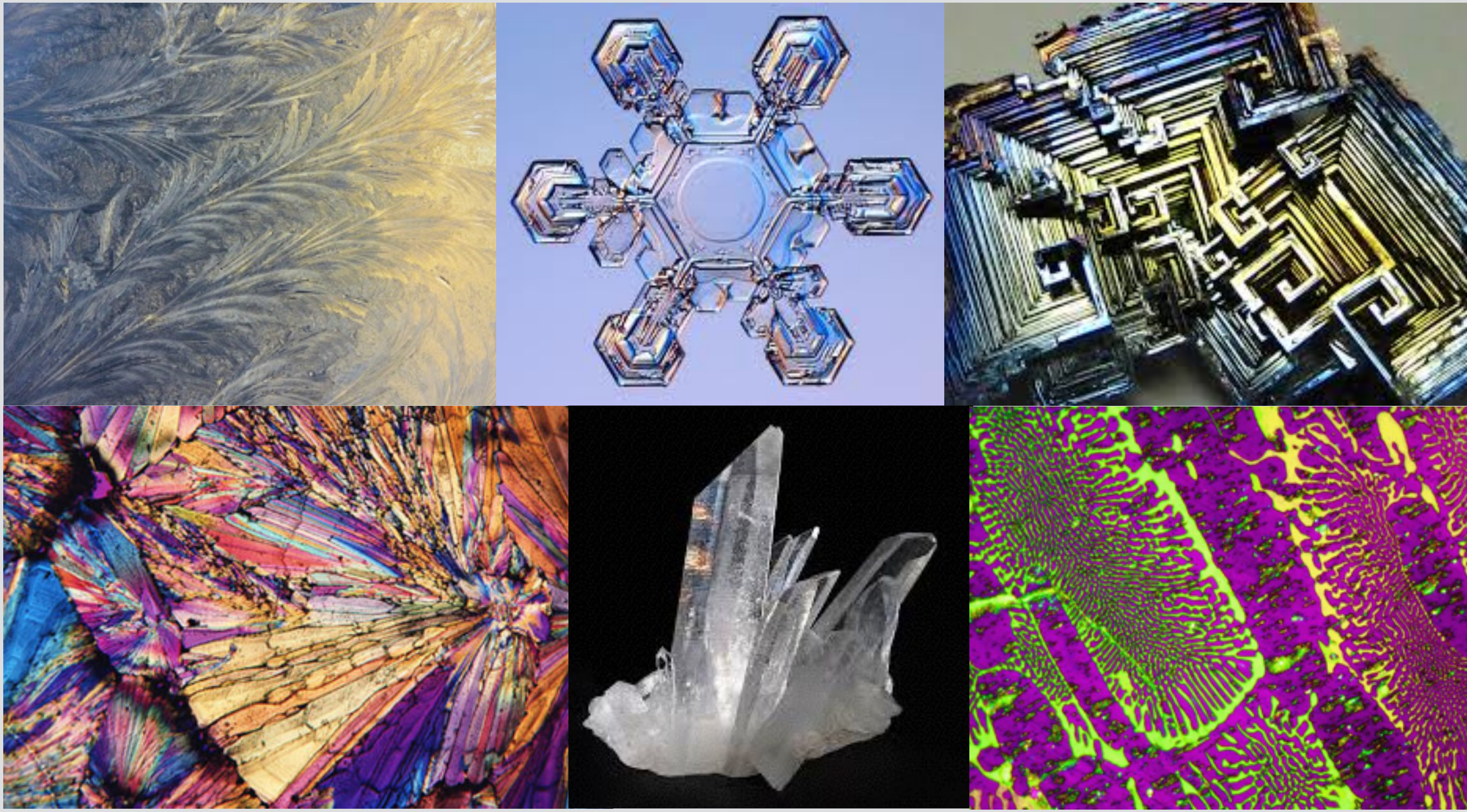
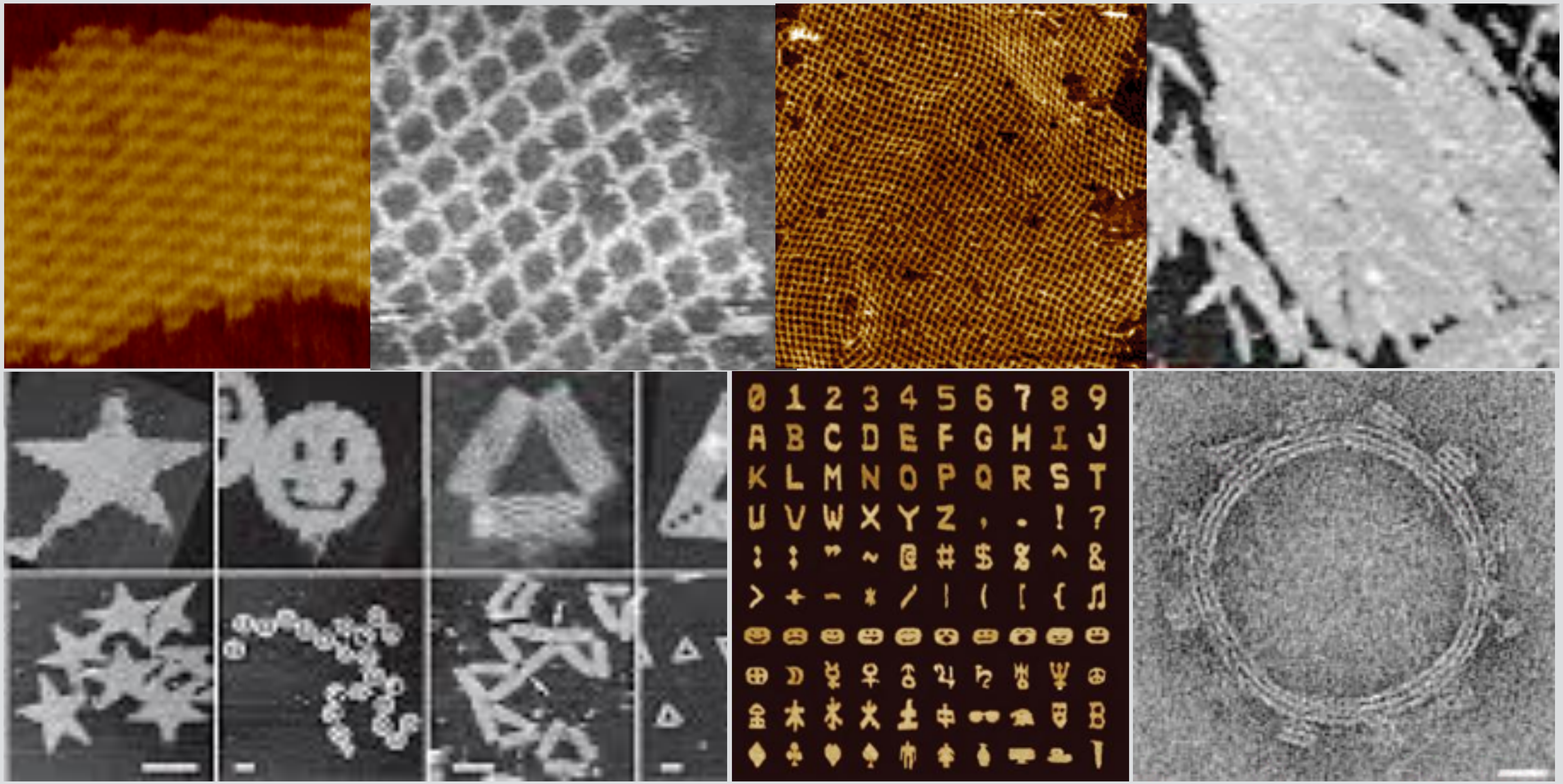
Natural Nanoscale Self-Assembly



Synthetic Nanoscale Self-Assembly



All made of DNA!

DNA Self-Assembly

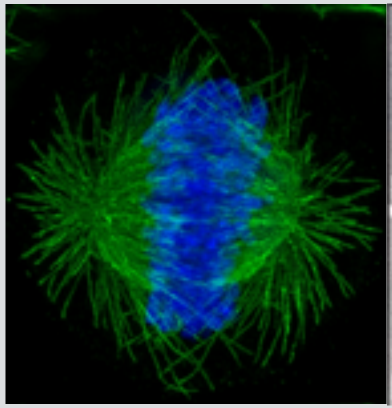
- Crystal-like growth: single particles attach to a growing lattice structure, starting as a *seed*.

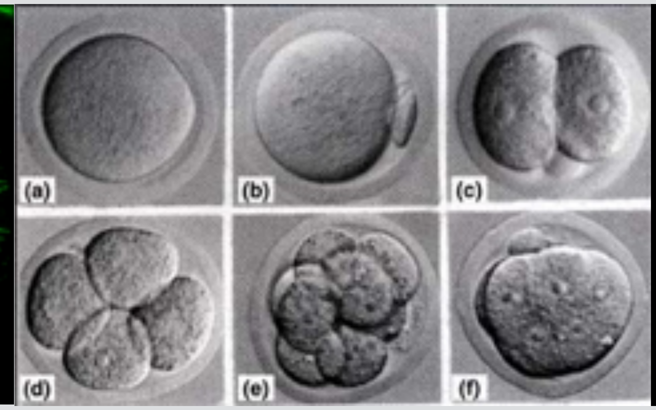
- Particles far from seed can only attach after many other particles.

- Limits growth rates to polynomial.

DNA Self-Assembly

- Crystal-like growth: single particles attach to a growing lattice structure, starting as a *seed*.
- Particles far from seed can only attach after many other particles.
- Limits growth rates to polynomial.
- But some natural systems grow exponentially!



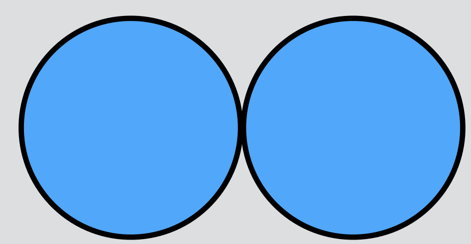


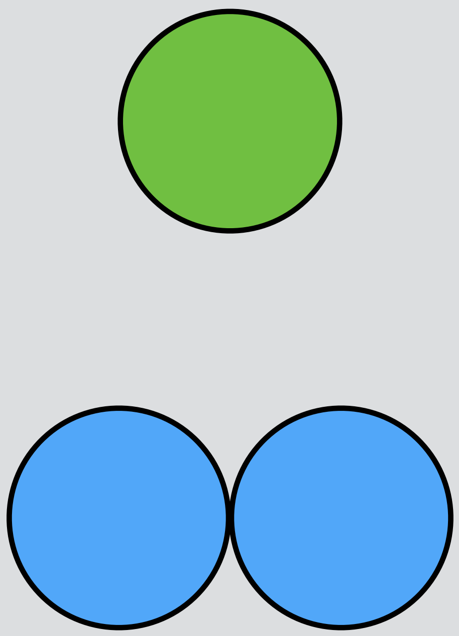
Passive vs. Active Self-Assembly

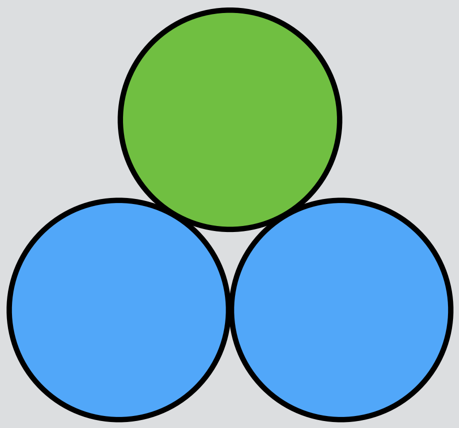
- Most current DNA self-assembly uses crystal-like **passive** growth: bonds and geometry do not change.

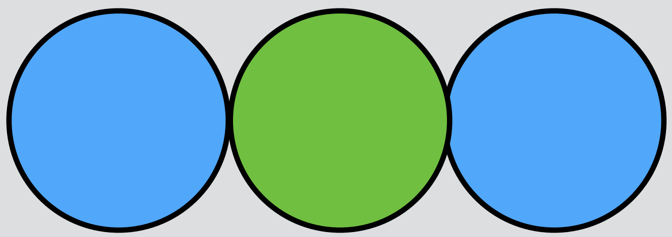
- Some natural systems use **active** growth: bonds and geometry change.

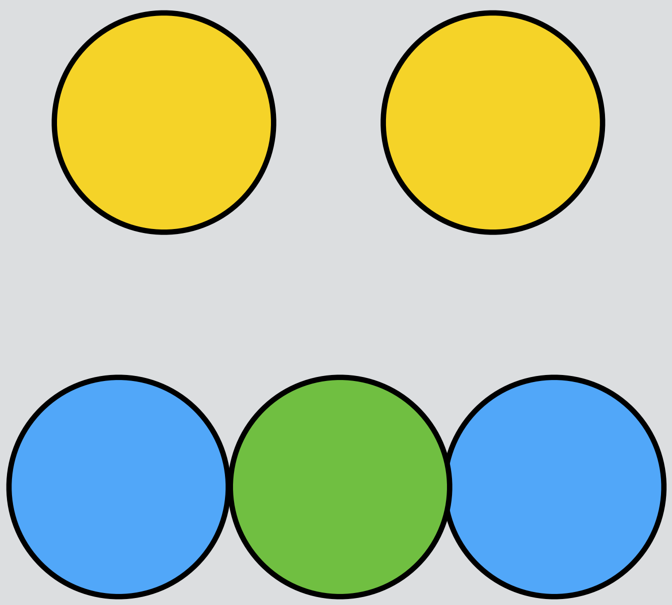
- Active growth enables exponential growth rates.

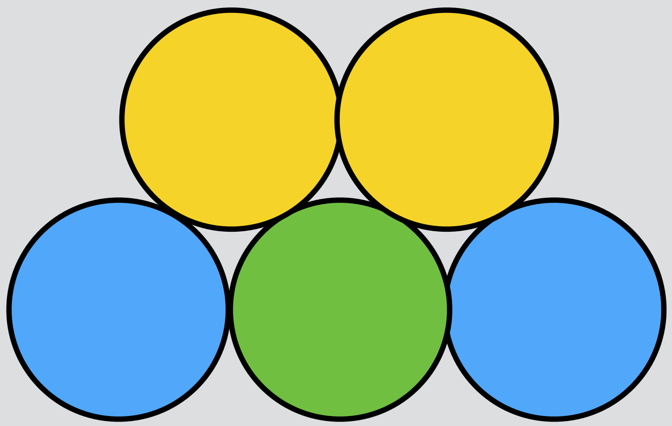


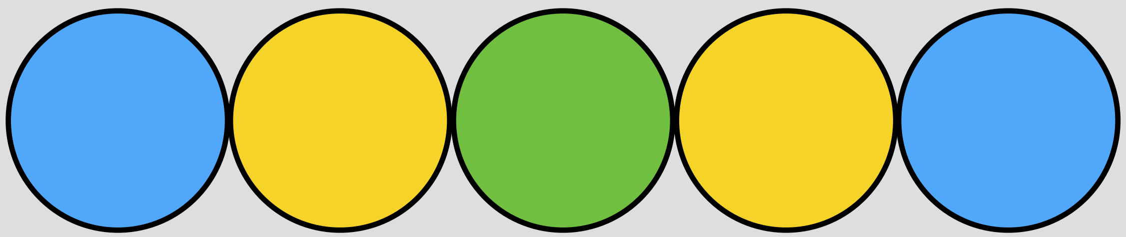


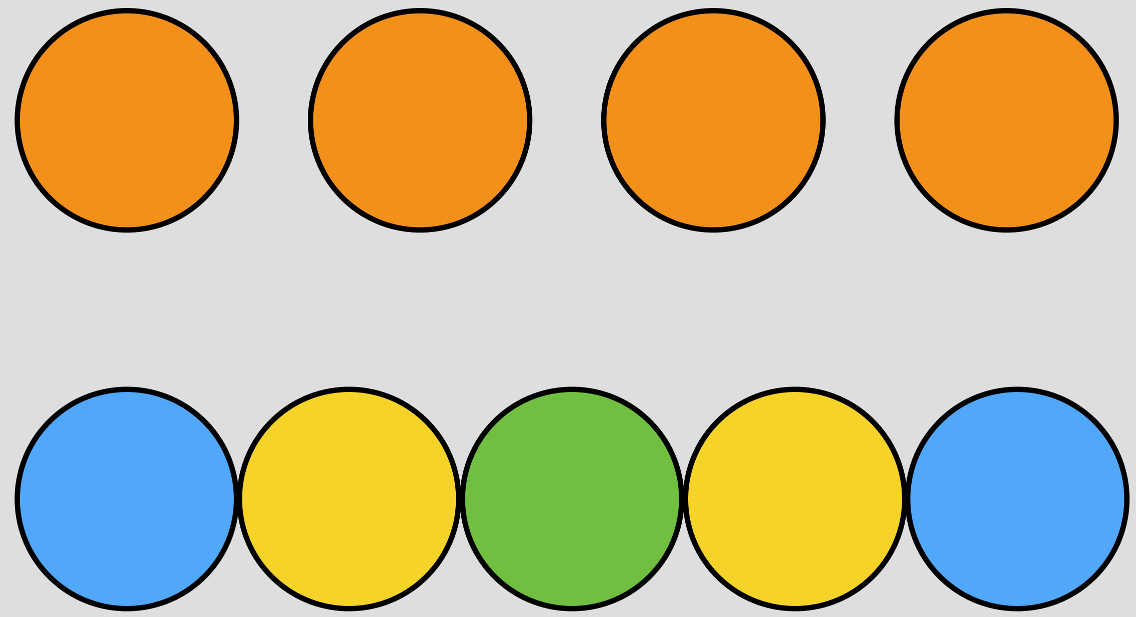


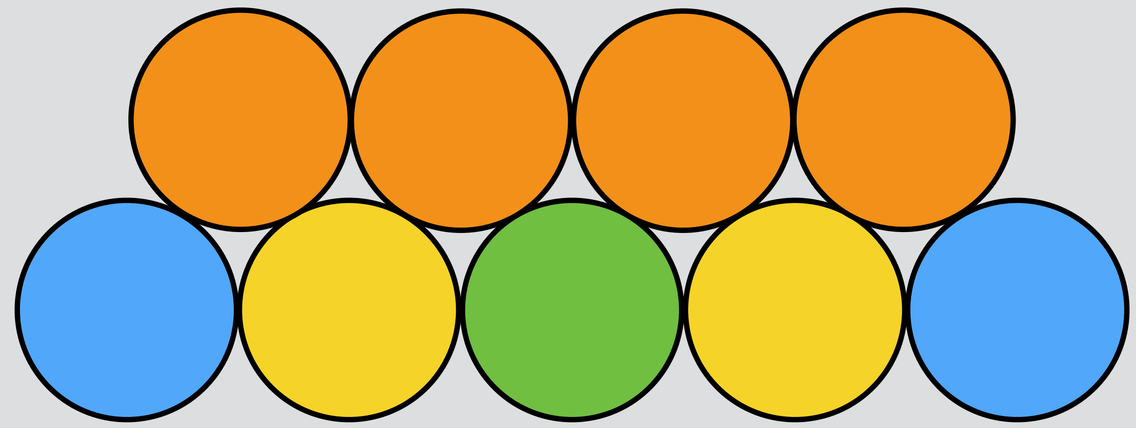


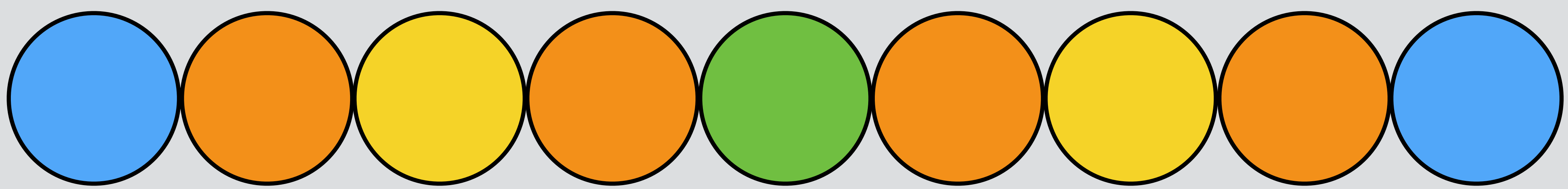


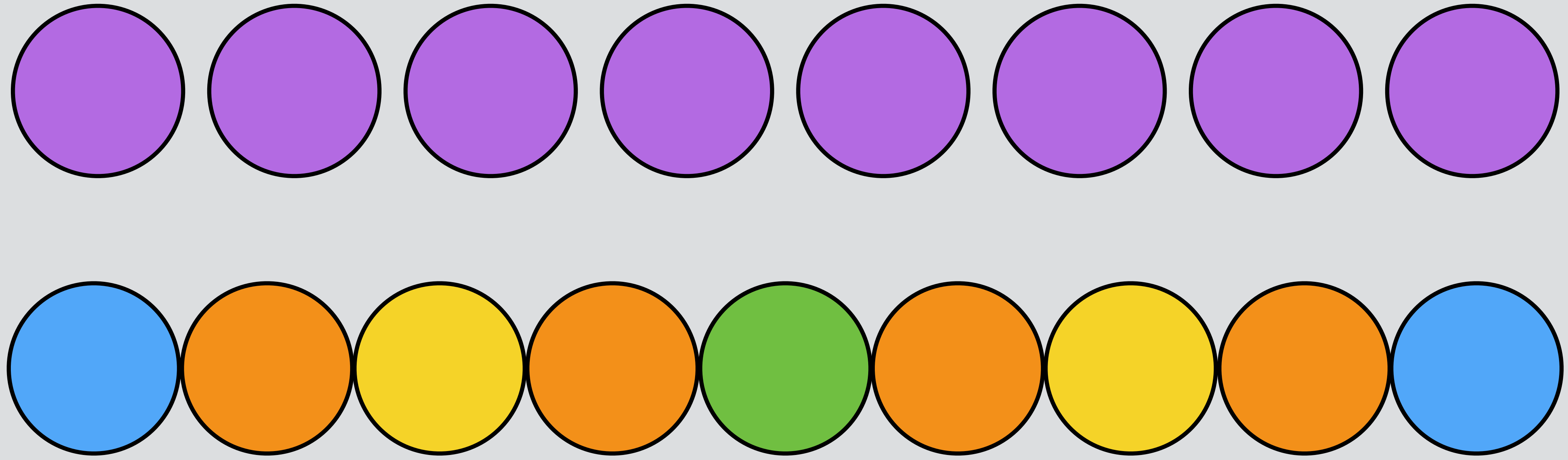


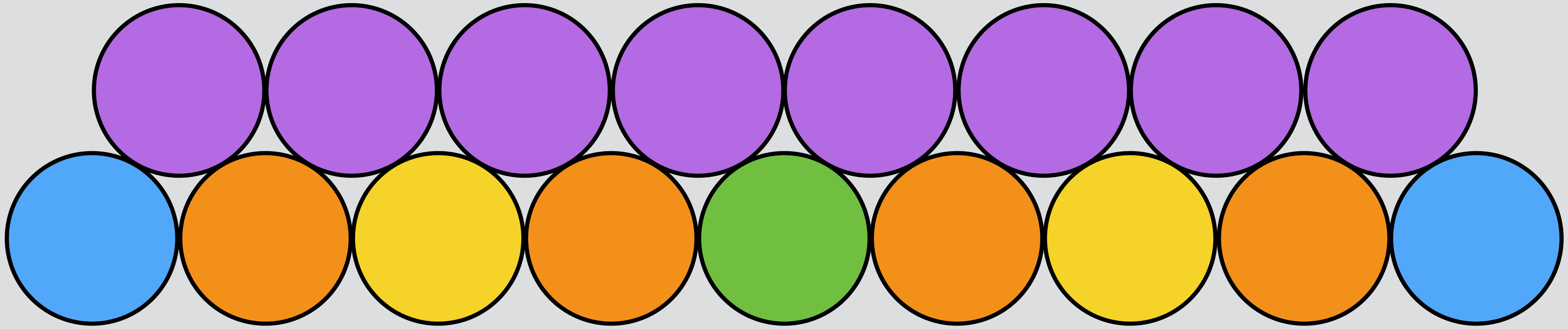


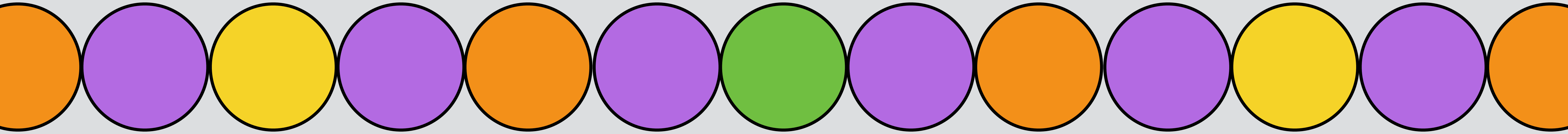




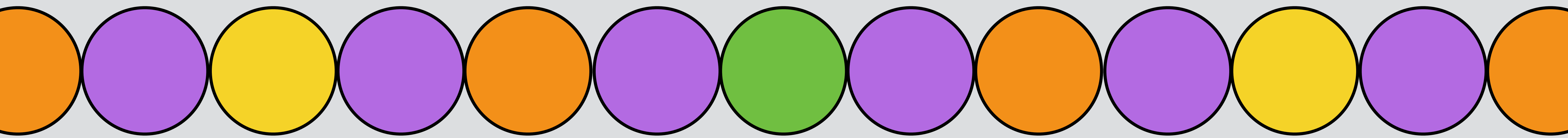






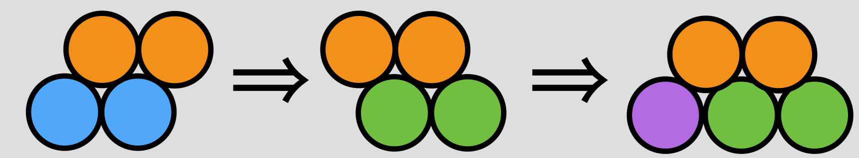


Exponential growth!

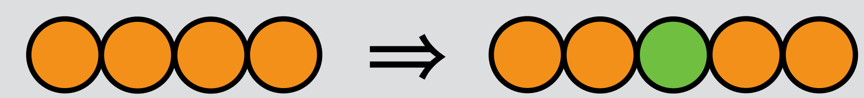


Active self-assembly models

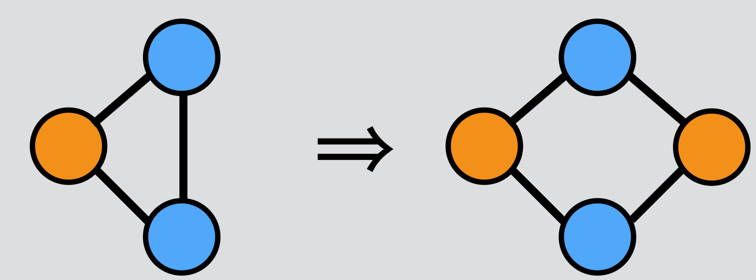
Nubots [Woods et al. ITCS 2012]:
2D, flexible and rigid bonds, stateful particles.



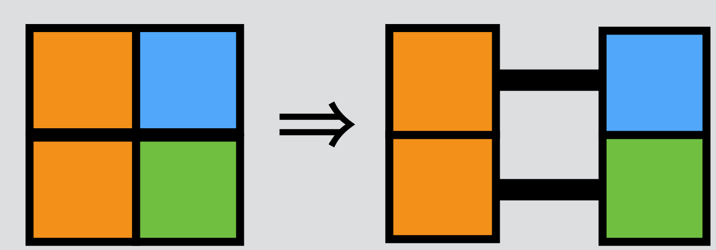
Insertion systems [Dabby, Chen SODA 2013]:
1D, fixed shape, stateless particles.



Graph grammars [Klavins et al. ICRA 2004]:
Geometry-less, stateless particles.



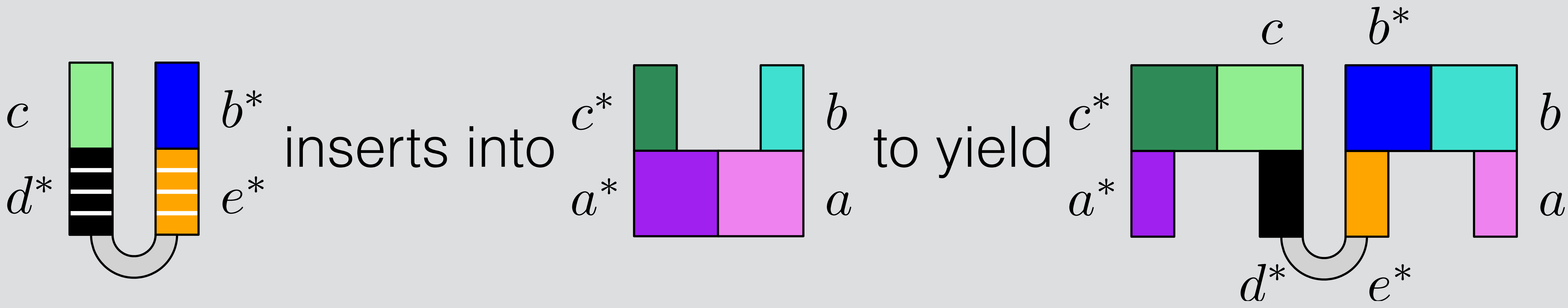
Crystalline robots [Rus, Vona ICRA 1999]:
3D, stateful particles, global communication.



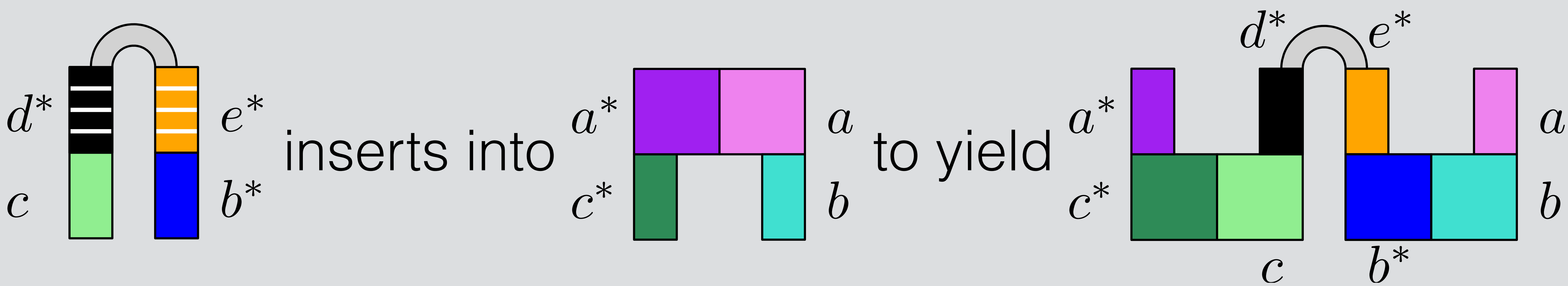
Insertion systems

- Introduced by [Dabby, Chen SODA 2013].
- A model of active self-assembly.
    - Implementable in DNA.
    - Capable of exponential growth.
    - Grows a linear structure by insertion of particles.
- Our work: bound capabilities of insertion systems.

Definitions and examples



$(c, d^*, e^*, b^*)^+$ inserts into $(a^*, c^*)(b, a)$ to yield $(a^*, c^*)(c, d^*, e^*, b^*)(b, a)$



$(d^*, c, b^*, e^*)^-$ inserts into $(c^*, a^*)(a, b)$ to yield $(c^*, a^*)(d^*, c, b^*, e^*)(a, b)$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,3^*)^-$ $(3,x,4,b)^+$

Initiator: $(a,1^*)(b^*,a^*)$

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^{*},a,3^{*})^{-}$ $(3,x,4,b)^{+}$

Initiator: $(a,1^{*})(b^{*},a^{*})$

$(a,1^{*})(b^{*},a^{*})$

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^*,a,3^*)^{-}$ $(3,x,4,b)^{+}$

Initiator: (a,1*)(b*,a*)

$$(a,1^*)(b^*,a^*)$$
$$\Downarrow$$
$$(a,1^*)(1,x,2,b)(b^*,a^*)$$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,3^*)^-$ $(3,x,4,b)^+$

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$$\Downarrow$$
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Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,3^*)^-$ $(3,x,4,b)^+$

Initiator: $(a,1^*)(b^*,a^*)$

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$$(a,1^*)(1,x,2,b)(x,2^*,a,3^*)(b^*,a^*)$$
$$\Downarrow$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,3^*)(3,x,4,b)(b^*,a^*)$$

Terminal polymer of length 5

Insertion time

- Each monomer type has a concentration in [0,1].
- Concentrations of all types in a system must sum to ≤ 1.
- An insertion occurs after time t with:
    - t an exponential random variable with rate c.
    - c is the total concentration of insertable monomers.

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^{*},a,3^{*})^{-}$ $(3,x,4,b)^{+}$

Initiator: $(a,1^{*})(b^{*},a^{*})$

Insertion system:

Monomer types:	$(1,x,2,b)^+$	$(x,2^*,a,3^*)^-$	$(3,x,4,b)^+$
Concentrations:	0.25	0.25	0.5

Initiator: $(a,1^*)(b^*,a^*)$

Insertion system:

Monomer types:	$(1,x,2,b)^+$	$(x,2^*,a,3^*)^-$	$(3,x,4,b)^+$
Concentrations:	0.25	0.25	0.5

Initiator: $(a,1^*)(b^*,a^*)$

$(a,1^*)(b^*,a^*)$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,3^*)^-$ $(3,x,4,b)^+$

Concentrations: 0.25 0.25 0.5

Initiator: $(a,1^*)(b^*,a^*)$

$$(a,1^*)(b^*,a^*)$$

$$\Downarrow t_1$$

$$(a,1^*)(1,x,2,b)(b^*,a^*)$$

$$\Downarrow t_2$$

$$(a,1^*)(1,x,2,b)(x,2^*,a,3^*)(b^*,a^*)$$

$$\Downarrow t_3$$

$$(a,1^*)(1,x,2,b)(x,2^*,a,3^*)(3,x,4,b)(b^*,a^*)$$

Terminal polymer of length 5

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^{*},a,3^{*})^{-}$ $(3,x,4,b)^{+}$

Concentrations: 0.25 0.25 0.5

Initiator: $(a,1^{*})(b^{*},a^{*})$

$$(a,1^{*})(b^{*},a^{*})$$
$$\Downarrow t_1$$
$$(a,1^{*})(1,x,2,b)(b^{*},a^{*})$$
$$\Downarrow t_2$$
$$(a,1^{*})(1,x,2,b)(x,2^{*},a,3^{*})(b^{*},a^{*})$$
$$\Downarrow t_3$$
$$(a,1^{*})(1,x,2,b)(x,2^{*},a,3^{*})(3,x,4,b)(b^{*},a^{*})$$

Terminal polymer of length 5

Expected time: $t_1 + t_2 + t_3$, *with*

$E[t_1] = E[t_2] = 4$, $E[t_3] = 2$.

$4 + 4 + 2 = 12$

Insertion system:

Monomer types:	$(1^*,2,2,1^*)^+$	$(x,0^*,2^*,x)^-$	$(x,2^*,0,x)^-$
Concentrations:	0.5	0.1	0.4

Initiator: $(0,1)(1,0^*)$

$(0,1)(1,0^*)$

Insertion system:

Monomer types:	$(1^*,2,2,1^*)^+$	$(x,0^*,2^*,x)^-$	$(x,2^*,0,x)^-$
Concentrations:	0.5	0.1	0.4

Initiator: $(0,1)(1,0^*)$

$$(0,1)(1,0^*)$$
$$\Downarrow t_1$$
$$(0,1)(1^*,2,2,1^*)(1,0^*)$$

Insertion system:

Monomer types: $(1^*,2,2,1^*)^+$ $(x,0^*,2^*,x)^-$ $(x,2^*,0,x)^-$

Concentrations: 0.5 0.1 0.4

Initiator: $(0,1)(1,0^*)$

$$(0,1)(1,0^*)$$
$$\Downarrow t_1$$
$$(0,1)(1^*,2,2,1^*)(1,0^*)$$
$$\Downarrow t_2$$
$$(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(1,0^*)$$

Insertion system:

Monomer types: $(1^*,2,2,1^*)^+$ $(x,0^*,2^*,x)^-$ $(x,2^*,0,x)^-$

Concentrations: 0.5 0.1 0.4

Initiator: $(0,1)(1,0^*)$

$$
\begin{aligned}
&(0,1)(1,0^*)\\
&\Downarrow t_1\\
&(0,1)(1^*,2,2,1^*)(1,0^*)\\
&\Downarrow t_2\\
&(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(1,0^*)\\
&\Downarrow t_3\\
&(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(x,2^*,0,x)(1,0^*)
\end{aligned}
$$

Insertion system:

Monomer types: $(1^*,2,2,1^*)^+$ $(x,0^*,2^*,x)^-$ $(x,2^*,0,x)^-$

Concentrations: 0.5 0.1 0.4

Initiator: $(0,1)(1,0^*)$

$$(0,1)(1,0^*)$$
$$\Downarrow t_1$$
$$(0,1)(1^*,2,2,1^*)(1,0^*)$$
$$\Downarrow t_2$$
$$(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(1,0^*)$$
$$\Downarrow t_3$$
$$(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(x,2^*,0,x)(1,0^*)$$

Terminal polymer of length 5

Insertion system:

Monomer types: $(1^*,2,2,1^*)^+$ $(x,0^*,2^*,x)^-$ $(x,2^*,0,x)^-$

Concentrations: 0.5 0.1 0.4

Initiator: $(0,1)(1,0^*)$

$$(0,1)(1,0^*)$$
$$\Downarrow t_1$$
$$(0,1)(1^*,2,2,1^*)(1,0^*)$$
$$\Downarrow t_2$$
$$(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(1,0^*)$$
$$\Downarrow t_3$$
$$(0,1)(x,0^*,2^*,x)(1^*,2,2,1^*)(x,2^*,0,x)(1,0^*)$$

Terminal polymer of length 5

Expected time: $t_1 + \max(t_2, t_3)$, *with*

$E[t_1] = 2$, $E[t_2] = 10$, $E[t_3] = 2.5$.

$2 + 10.5 = 12.5$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

Concentrations: 0.5 0.4 0.1

Initiator: $(a,1^*)(b^*,a^*)$

$(a,1^*)(b^*,a^*)$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

Concentrations: 0.5 0.4 0.1

Initiator: $(a,1^*)(b^*,a^*)$

$$(a,1^*)(b^*,a^*)$$

$$\Downarrow t_1$$

$$(a,1^*)(1,x,2,b)(b^*,a^*)$$

Insertion system:

Monomer types:	$(1,x,2,b)^{+}$	$(x,2^*,a,1^*)^{-}$	$(x,2^*,a,x)^{-}$
Concentrations:	0.5	0.4	0.1

Initiator: $(a,1^*)(b^*,a^*)$

$$(a,1^*)(b^*,a^*)$$
$$\Downarrow t_1$$
$$(a,1^*)(1,x,2,b)(b^*,a^*)$$
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$$(a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

Concentrations: 0.5 0.4 0.1

Initiator: (a,1*)(b*,a*)

$$(a,1^*)(b^*,a^*)$$
$$\Downarrow t_1$$
$$(a,1^*)(1,x,2,b)(b^*,a^*)$$
$$\Downarrow t_2$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$$

Expected time: $t_1 + t_2$, *with*

$E[t_1] = E[t_2] = 2.$

$2 + 2 = 4$

Insertion system:

Monomer types:	$(1,x,2,b)^+$	$(x,2^*,a,1^*)^-$	$(x,2^*,a,x)^-$
Concentrations:	0.5	0.4	0.1

Initiator: $(a,1^*)(b^*,a^*)$

$$(a,1^*)(b^*,a^*)$$
$$\Downarrow t_1$$
$$(a,1^*)(1,x,2,b)(b^*,a^*)$$
$$\Downarrow t_2$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

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Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

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Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

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$$\Downarrow t_3$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(b^*,a^*)$$
$$\Downarrow t_4$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$$

Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,1^*)^-$ $(x,2^*,a,x)^-$

Concentrations: 0.5 0.4 0.1

Initiator: $(a,1^*)(b^*,a^*)$

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$$\Downarrow t_4$$
$$(a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$$

Expected time: $t_1 + t_2 + t_3 + t_4$, *with*

$E[t_1] = E[t_2] = E[t_3] = E[t_4] = 2.$

$2 + 2 + 2 + 2 = 8$

Expressive power

Expressive Power

- Insertion systems: initiator + set of monomers = set of polymers, with terminal polymer subset.
- Context-free grammars: start symbol + set of rules = set of partial derivations, with string subset.
- Do insertion systems and context-free grammars have equal "expressive-ness"?

Expressive Power

- Insertion systems: initiator + set of monomers = set of polymers, with terminal polymer subset.

- Context-free grammars: start symbol + set of rules = set of partial derivations, with string subset.

- Do insertion systems and context-free grammars have equal “expressive-ness”? *Yes.*

Expressive Power

Theorem: every insertion system can be expressed as a context-free grammar. [Dabby, Chen 2013]

Theorem: every context-free grammar can be expressed as an insertion system. [This work]

Expressive Power

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Insertion system:

Monomer types: $(1,x,2,b)^+$ $(x,2^*,a,3)^-$ $(3,x,4,b)^+$

Concentrations: 0.25 0.25 0.25

Initiator: $(a,1)(b^*,a^*)$

Context-free grammar:

Production rules:

$S_{(a,1)(b^*,a^*)} \to S_{(a,1)(1,x)}S_{(2,b)(b^*,a^*)}$

$S_{(2,b)(b^*,a^*)} \to S_{(2,b)(x,2^*)}S_{(a,3)(b^*,a^*)}$

$S_{(a,3)(b^*,a^*)} \to S_{(a,3)(3,x)}S_{(4,b)(b^*,a^*)}$

$S_{(a,1)(1,x)} \to (a, 1)(1, x)$

$S_{(2,b)(x,2^*)} \to (2,b)(x,2^*)$

$S_{(a,3)(3,x)} \to (a,3)(3,x)$

$S_{(4,b)(x,4^*)} \to (4,b)(x,4^*)$

Start symbol: $S_{(a,1)(b^*,a^*)}$

Expressive Power

Theorem: every insertion system can be expressed as a context-free grammar. [Dabby, Chen 2013]

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Theorem: every insertion system can be expressed as a context-free grammar. [Dabby, Chen 2013]

Theorem: every context-free grammar can be expressed as an insertion system. [This work]

Rule: $A \rightarrow BC$

Derivation step: $eADe \Rightarrow eBCDe$

Monomer type: $(1,x,2,b)^+$

Insertion: $(a,1)(b^*,a^*) \Rightarrow (a,1)(1,x,2,b)(b^*,a^*)$

Rules completely replace non-terminals.

Insertions do not completely replace insertion sites.

Polymer length

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^*,a,1^*)^{-}$ $(x,2^*,a,x)^{-}$

Concentrations: 0.5 0.4 0.1

Initiator: (a,1*)(b*,a*)

(a,1*)(1,x,2,b)(x,2*,a,x)(b*,a*)

(a,1*)(1,x,2,b)(x,2*,a,1*)(1,x,2,b)(x,2*,a,x)(b*,a*)

(a,1*)(1,x,2,b)(x,2*,a,1*)(1,x,2,b)(x,2*,a,1*)(1,x,2,b)(x,2*,a,x)(b*,a*)

• • •

Context-free grammar:

Production rules: $S \rightarrow A$ $A \rightarrow aaA$ $A \rightarrow aa$

Start symbol: S

---

aa

aaaa

aaaaaa

• • •

Insertion system:

Monomer types: $(1,x,2,b)^{+}$ $(x,2^*,a,1^*)^{-}$ $(x,2^*,a,x)^{-}$

Concentrations: 0.5 0.4 0.1

Initiator: $(a,1^*)(b^*,a^*)$

$(a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$

$(a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$

$(a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(x,2^*,a,1^*)(1,x,2,b)(x,2^*,a,x)(b^*,a^*)$

•••

Constructing arbitrarily long polymers is easy if infinite polymers allowed.

Context-free grammar:

Production rules: S→A A→aaA A→aa

Start symbol: S

---

aa

aaaa

aaaaaa

•••

Constructing long polymers

Theorem: a system with k monomer types constructing a finite number of polymers can construct:

- polymers of length $2^{\Theta(k^{1/2})}$ [Dabby, Chen 2013]
- polymers of length $2^{\Theta(k^{3/2})}$ [This work]
- only polymers of length $2^{O(k^{3/2})}$ [This work]

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Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

$\Downarrow$

…u,0)(0*,1,x,0*)(0,u*…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

⇓

…u,0)(0*,1,x,0*)(0,u*…

⇓

…u,0)(x,u*,1*,1*)(0*,1,x,0*)(0,u*…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

⇓

…u,0)(0*,1,x,0*)(0,u*…

⇓

…u,0)(x,u*,1*,1*)(0*,1,x,0*)(0,u*…

⇓

…u,0)(x,u*,1*,1*)(1, x, 2, 0)(0*,1,x,0*)(0,u*…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

⇓

…u,0)(0*,1,x,0*)(0,u*…

⇓

…u,0)(x,u*,1*,1*)(0*,1,x,0*)(0,u*…

⇓

…u,0)(x,u*,1*,1*)(1, x, 2, 0)(0*,1,x,0*)(0,u*…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

⇓

…u,0)(0*,1…

⇓

…1*,1*)(0*,1…

⇓

…2, 0)(0*,1…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

…u,0)(0,u*…

$\Downarrow$

…u,0)(0*,1…

$\Downarrow$

…1*,1*)(0*,1…

$\Downarrow$

…2, 0)(0*,1…

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.

- Ingredient 1: long insertion sequence with no repeated insertion sites.

$$\ldots 0,0)(0,0^{*}\ldots$$
$$\Downarrow O(1)$$
$$\ldots 0,0)(1,0^{*}\ldots$$
$$\Downarrow O(1)$$
$$\vdots$$
$$\Downarrow O(1)$$
$$\ldots r, r)(r,r^{*}\ldots$$

$\Theta(k^{3/2})$

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.
- Ingredient 1: long insertion sequence with no repeated insertion sites.
- Ingredient 2: duplication of each site in sequence.

$$\ldots 0,0)(0,0^{*}\ldots$$

$$\Downarrow O(1)$$

$$\ldots 0,0)(1,0^{*}\ldots 0,0)(1,0^{*}\ldots$$

$$\Downarrow O(1)$$

$$\ldots 0,\ 0)(2,0^{*}\ldots 0,0)(2,0^{*}\ldots 0,0)(2,0^{*}\ldots 0,0)(2,0^{*}\ldots$$

Constructing long polymers

- Consider *insertion sequences*: repeated insertions into the site resulting from previous insertion.
- Ingredient 1: long insertion sequence with no repeated insertion sites.
- Ingredient 2: duplication of each site in sequence.
- Combine these for long polymers.

Constructing long polymers

…0,0)(0,0*…

$\Downarrow$ O(1)

…0,0)(1,0*…0,0)(1,0*…

$\Downarrow$ O(1)

…0, 0)(2,0*…0,0)(2,0*…0,0)(2,0*…0,0)(2,0*…

$\Downarrow$ O(1)

⋮

$\Downarrow$ O(1)

…r, r)(r,r*…r, r)(r,r*…r, r)(r,r*…r,r)(r,r*…r,r)(r,r*…r,r)(r,r*…

$\Theta(k^{3/2})$

$2^{\Theta(k^{3/2})}$

Polymer growth speed

Insertion system:

Monomer types: $(b^*,a^*,a,b)^+$ $(b^*,x,x,b)^+$

Concentrations: 0.5 0.5

Initiator: $(a,b)(b^*,a^*)$

$$(a,b)(b^*,a^*)$$

$$\Downarrow 1$$

$$(a,b)(b^*,a^*,a,b)(b^*,a^*)$$

$$\Downarrow 2$$

$$(a,b)(b^*,a^*,a,b)(b^*,a^*,a,b)(b^*,a^*,a,b)(b^*,a^*)$$

$$\Downarrow 4$$

$$(a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*)$$

Each round of insertions takes O(1) expected time.

Construction of length $n = 2^i-1$ takes O(i) expected time.

O(log(n))

Insertion system:

Monomer types: $(b^*,a^*,a,b)^+$ $(b^*,x,x,b)^+$

Concentrations: 0.5 0.5

Initiator: $(a,b)(b^*,a^*)$

Constructing polymers in O(log(n)) expected time is easy if infinite polymers allowed.

$(a,b)(b^*,a^*)$

$\Downarrow 1$

$(a,b)(b^*,a^*,a,b)(b^*,a^*)$

$\Downarrow 2$

$(a,b)(b^*,a^*,a,b)(b^*,a^*,a,b)(b^*,a^*,a,b)(b^*,a^*)$

$\Downarrow$

$(a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*,a,b)(b^*,x,x,b)(b^*,a^*)$

Each round of insertions takes O(1) expected time.

Construction of length $n = 2^i-1$ takes O(i) expected time.

O(log(n))

Constructing polymers quickly

Theorem: a system constructing a finite number of polymers can **deterministically** construct a polymer of length n in:

- $O(\log^3(n))$ expected time [Dabby, Chen 2013]
- $O(\log^{5/3}(n))$ expected time [This work]
- only $\Omega(\log^{5/3}(n))$ expected time [This work]

Summary

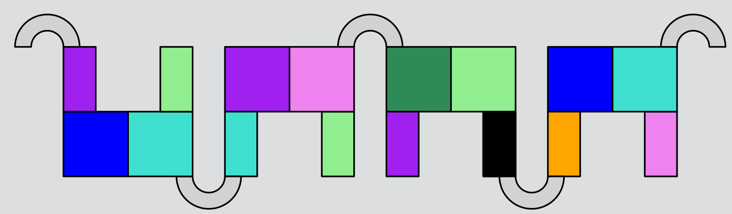
Theorem: insertion systems and context-free grammars are expressively equivalent.

Theorem: a system with k monomer types constructing a finite number of polymers can construct polymers of length $2^{\Theta(k^{3/2})}$ and this is the best possible.

Theorem: a system constructing a finite number of polymers can **deterministically** construct a polymer of length n in $\Theta(\log^{5/3}(n))$ expected time and this is the best possible.

Thank you.

Tight bounds for active self-assembly
using an insertion primitive



Caleb Malchik and Andrew Winslow