A Tour of DNA Tile Self-Assembly

Andrew Winslow
Department of Computer Science, UTRGV
A Tour of DNA Tile Self-Assembly

Andrew Winslow
Department of Computer Science, UTRGV
Self-Assembly

Simple particles coalescing into complex superstructures.
Self-Assembly

Simple particles coalescing into complex superstructures.
Self-Assembly

Simple particles coalescing into complex superstructures.
Self-Assembly

Simple particles coalescing into complex superstructures.
Crystallization
Morphogenesis
Natural self-assembly
Synthetic DNA self-assembly
DNA

Base pairs

Single strand

GAAGTTTGCCGTTAGAACGTGTAATCCGCTTTGTTAAGACCCCCGTCTAAGCA
Sticky ends
Sticky ends
[Wei, Dai, Yin 2012]
DNA tiles

[Seeman 1982]
DNA tile self-assembly
Tile types

Seed tile
Tile types

Seed tile
Tile types

Seed tile
Tile types

Seed tile
Temperature 2
Temperature 1
Temperature 1
Temperature 1
abstract Tile Assembly Model (aTAM) [Winfree 1998]
Temperature 2
Sierpinski triangle
DNA Sierpinski triangles
[Papadakis, Rothemund, Winfree 2004]:
scale bars = 100 nm
Temperature 2
Temperature 2
Binary counter
DNA binary counters

[Evans, 2014]

80 nm
DNA tile self-assembly:

Tile self-assembly:
DNA tile self-assembly:

Tile self-assembly:
The aTAM of Winfree
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness

Intrinsic universality

1995 - 2000 - 2005 - 2010 - Now
The aTAM of Winfree
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness

Intrinsic universality

The aTAM problems

Benchmark problems

aTAM is “capable”

What is a “capable” model of self-assembly?
The benchmarks of a capable self-assembly model
The benchmarks of a capable self-assembly model

Algorithmic behavior demonstrated by Universal computation
The benchmarks of a capable self-assembly model

Efficient NxN square assembly demonstrated by Universal computation

Usable Algorithmic behavior
The benchmarks of a capable self-assembly model

Usable
Algorithmic behavior

demonstrated by
Universal computation

Efficient NxN square assembly

Efficient = few tile types
(program size, time)
The benchmarks of a capable self-assembly model

Algorithmic behavior demonstrated by Universal computation

Usable algorithmic behavior demonstrated by few tile types

Efficient NxN square assembly
The benchmarks of a capable self-assembly model

Algorithmic behavior demonstrated by Universal computation

Usable algorithmic behavior demonstrated by Few tile types Efficient N x N square assembly
Blocked cellular automata
Blocked cellular automata
Blocked cellular automata
Blocked cellular automata
Blocked cellular automata
Blocked cellular automata
Blocked cellular automata

Computationally universal by [Lindgren, Nordahl 1990]
Universal computation
(via blocked CA simulation)
Universal computation
(via blocked CA simulation)
[Winfree 1998]
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move left, change to s_1”
Universal computation
(solution by “zig-zag” TM simulation)

```
1 1 1 1 1 0 1
```

“write 1, move right, change to s₁”

```
1 1 0 1 1 0 1
```

“write 1, move left, change to s₁”

```
1 1 0 0 1 0 1
```
Universal computation
(solution by “zig-zag” TM simulation)

```
1 1 1 0 1 0 1
```
“write 0, move right, change to $s_0$”

```
1 1 1 1 1 1 0 1
```
“write 1, move right, change to $s_1$”

```
1 1 0 1 1 1 0 1
```
“write 1, move left, change to $s_1$”

```
1 1 0 0 1 0 1
```
“write 0, move right, change to $s_0$”

“write 0, move right, change to $s_0$”

“write 1, move right, change to $s_1$”

“write 1, move left, change to $s_1$”
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move left, change to $s_1$”
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move left, change to s₁”
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move left, change to s₁”
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move right, change to s₁”
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move right, change to $s_1$”
Universal computation
(solution by “zig-zag” TM simulation)

“write 1, move right, change to $s_1$”
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation
(solution by “zig-zag” TM simulation)

“write 0, move right, change to s_0”
Universal computation
(solution by “zig-zag” TM simulation)

“write 0, move right, change to s₀”
Universal computation
(solution by “zig-zag” TM simulation)

“write 0, move right, change to s_0”
Universal computation
(solution by “zig-zag” TM simulation)
Universal computation  
(solution by “zig-zag” TM simulation)

“write 0, move right, change to s₀”
Universal computation

“write 0, move right, change to s₀”
Universal computation

“write 0, move right, change to $s_0$”
Universal computation
Universal computation

[Rothemund, Winfree 2000]
The benchmarks of a capable self-assembly model

Algorithmic behavior demonstrated by Universal computation

Usable algorithmic behavior demonstrated by few tile types Efficient NxN square assembly
The benchmarks of a capable self-assembly model

Algorithmic behavior demonstrated by Universal computation

Usable algorithmic behavior demonstrated by few tile types Efficient NxN square assembly
Efficient assembly of N\times N square
Efficient assembly of NxN square
Efficient assembly of N×N square
Efficient assembly of \(N \times N\) square
Efficient assembly of $N \times N$ square

$2N - 1$ tile types
Efficient assembly of NxN square
Efficient assembly of NxN square

\[ \Theta(\log(N)) \quad \Theta(N) \]

\[ \Theta(\log(N)) \quad \Theta(N) \]
Efficient assembly of $N \times N$ square

$\Theta(\log(N))$  $\Theta(N)$

$\Theta(N)$  $\Theta(\log(N))$
Efficient assembly of $N \times N$ square
Efficient assembly of \( N \times N \) square

\[ \Theta(\log(N)) \quad \Theta(N) \]

Tile types:

\[ O(\log(N)) \]

\[ \Omega(\log(N)/\log\log(N)) \text{ for most } N \]

[Rothemund, Winfree 2000]
Efficient assembly of NxN square

\[ \Theta(log(N)) \quad \Theta(N) \]

Tile types:
\[ O(log(N)/\log\log(N)) \]
\[ \Omega(log(N)/\log\log(N)) \] for most N

[Rothemund, Winfree 2000]
[Adleman et al. 2001]
The aTAM of Winfree

Benchmark problems
The aTAM of Winfree

Benchmark problems

The temperature-1 problem
The benchmarks of a capable self-assembly model

Universal computation

NxN square assembly w/O(\log(N)) tile types

Temperature 2
The benchmarks of a capable self-assembly model

Universal computation

NxN square assembly w/O(log(N)) tile types

Temperature 2
Temperature 2 can do \textit{cooperative bonding}.
Computation uses cooperative bonding:

Is this required?
Efficient square assembly uses cooperative bonding:

Is this required?
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation
The temperature-1 problem:

Is the aTAM at temperature 1 computationally universal?
The temperature-1 problem:

Is the aTAM at temperature 1 computationally universal?
Open since 2000, conjecture: no.
The temperature-1 problem:

Is the aTAM at temperature 1 computationally universal?
Open since 2000, conjecture: no.

“weak cooperation”
Can an augmented temperature-1 aTAM be computationally universal?
The temperature-1 problem:

Is the aTAM at temperature 1 computationally universal? Open since 2000, conjecture: no.

“weak cooperation”
Can an augmented temperature-1 aTAM be computationally universal?

Yes, several ways.
Reading a bit w/o cooperation

\[ \ldots \quad \ldots \]

\[ \ldots \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad \ldots \]

\[ \ldots \quad \ldots \]

\[ \ldots \quad \ldots \]
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation
Reading a bit w/o cooperation

\[ \ldots \quad \ldots \quad \ldots \]

\[ \ldots \quad \begin{array}{ccc}
\text{1} \\
\text{1} \\
\text{1} \\
\end{array} \quad \ldots \]

\[ \ldots \quad \ldots \quad \ldots \]
Reading a bit w/o cooperation
Reading a bit w/o cooperation
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
A planarity problem
Planarity is the barrier to reading without cooperative bonding.

Can read, but might get trapped.
Planarity is the barrier to reading without cooperative bonding.

Can read, but might get trapped.
Approach 1: 3D

[Cook, Fu, Schweller 2012]
Approach 1: 3D

[Cook, Fu, Schweller 2012]
Approach 1: 3D

[Cook, Fu, Schweller 2012]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]

Temperature 1
Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]
Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]
Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]
Approach 4: Polyomino Tiles
[Hendricks et al. 2014], [Fekete et al. 2015]
Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]
Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]
Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]
The aTAM of Winfree

The temperature-1 problem

Benchmark problems

Universality via weak cooperation

The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness

One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)
One-handed assembly (aTAM)

Producible assemblies:

Temperature 1
One-handed assembly (aTAM)

Terminal assemblies:

Temperature 1
Two-handed assembly (2HAM)
Two-handed assembly (2HAM)
Two-handed assembly (2HAM)

Producible assemblies:
Two-handed assembly (2HAM)

Terminal assemblies:
Two-handed assembly (2HAM)
Two-handed assembly (2HAM)

Producible assemblies:

Temperature 2
Two-handed assembly (2HAM)

Terminal assemblies:
Temperature 2

aTAM

Producible w/seed:

Producible w/seed:

2HAM

Producible:

Producible:
Is a seed necessary for some results?

no spurious nucleation

universal computation?

efficient shape construction?
Is a seed necessary for some results? No!

Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be \textit{simulated} by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be \textit{simulated} by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be \textit{simulated} by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be *simulated* by a 2HAM system.
Every aTAM system can be \textit{simulated} by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be **simulated** by a 2HAM system.
Every aTAM system can be simulated by a 2HAM system.
Every aTAM system can be *simulated* by a 2HAM system.
Every aTAM system can be \textit{simulated} by a 2HAM system.  
[Cannon et al. 2013]
Bricks
Temperature 2
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness

The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness
The aTAM of Winfree

The temperature-1 problem

Benchmark problems

Universality via weak cooperation

Handedness

Intrinsic universality

Intrinsic universality

In cellular automata:
Intrinsic universality

In cellular automata:
State of aTAM computation:
(circa 2008)

Temperature-1 universal computation is open.
(and very hard)
State of aTAM computation:
(circa 2008)

Temperature-1 universal computation is open.
(and very hard)

Easier goal: disprove stronger positive temp-1 claim.
State of aTAM computation:
(circa 2008)

Temperature-1 universal computation is open.
(and very hard)

Easier goal: disprove stronger positive temp-1 claim.

Claim: some temp-1 system can simulate every temp-2 system.
State of aTAM computation:
(circa 2008)

Temperature-1 universal computation is open.
(and very hard)

Easier goal: disprove stronger positive temp-1 claim.

Claim: some temp-1 system can simulate every temp-2 system.

Claim: temp-1 is *intrinsically universal* for temp-2.
Simulation: same behavior (with scale and fuzz)
Simulation: same behavior (with scale and fuzz)

Temperature 1

Temperature 2
Simulation: same behavior (with scale and fuzz)

Temperature 1  ↔  Temperature 2
Simulation: same behavior (with scale and fuzz)

Temperature 1

Temperature 2
Is intrinsic universality too much to ask?
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps.

[Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.

Temperature 2
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.

U

Temperature 2

T

Temperature 6
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.

Temperature 2

Temperature 6
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.

Temperature 2

Temperature 4
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps.

[Doty et al. 2012]

There exists a temp-2 system that simulates every system.

Temperature 2

Temperature 4
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps.  
[Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.

Temperature 2

Temperature 4
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps. [Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Is intrinsic universality too much to ask?

No: temp-2 is intrinsically universal for all temps.

[Doty et al. 2012]

There exists a temp-2 system that simulates every system.
Claim: temp-1 is *intrinsically universal* for temp-2.
Claim: temp-1 is *intrinsically universal* for temp-2.

There is a temp-2 system not simulated by any temp-1 system.

Specifically: [Meunier et al. 2014]
at temperature 2 assembles

for all $a, b, c \geq 0$
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.
Claim: simulation will fail.

Assembled by temperature-1 “simulation”:

Shapes assembled by at temperature 2:

for all $a, b, c \geq 0$
Claim: simulation will fail.

Assembled by temperature-1 “simulation”:

Shapes assembled by at temperature 2:

for all $a, b, c \geq 0$
Fruits of intrinsic universality

The tile assembly model is intrinsically universal
David Doty*  Jack H. Lutz†  Matthew J. Patitz‡  Robert T. Schweller§  Scott M. Summers¶  Damien Woods‖

Intrinsic universality in tile self-assembly requires cooperation
Pierre-Etienne Meunier*  Matthew J. Patitz†  Scott M. Summers‡  Guillaume Theyssier§  Andrew Winslow¶  Damien Woods‖

The two-handed tile assembly model is not intrinsically universal
Erik D. Demaine*  Matthew J. Patitz†  Trent A. Rogers‡  Robert T. Schweller§  Scott M. Summers¶  Damien Woods‖

Signal Transmission across Tile Assemblies: 3D Static Tiles Simulate Active Self-assembly by 2D Signal-Passing Tiles
Jacob Hendricks1,*, Jennifer E. Padilla2,**  Matthew J. Patitz1,*  and Trent A. Rogers3,*

One Tile to Rule Them All: Simulating Any Tile Assembly System with a Single Universal Tile*,**
Erik D. Demaine1, Martin L. Demaine1, Sándor P. Fekete2, Matthew J. Patitz3, Robert T. Schweller4, Andrew Winslow5, and Damien Woods6

It’s a Tough Nanoworld: in Tile Assembly, Cooperation is not (strictly) more Powerful than Competition
Florent Becker*  Pierre-Étienne Meunier†

The Simulation Powers and Limitations of Hierarchical Self-Assembly Systems
Jacob Hendricks(§§), Matthew J. Patitz, and Trent A. Rogers
Department of Computer Science and Computer Engineering, University of Arkansas, Fayetteville, AR, USA
{jhendrick,patitz,tar003}@uark.edu

Two Hands Are Better Than One (up to constant factors)
Sarah Cannon*  Erik D. Demaine†  Martin L. Demaine†  Sarah Eisenstat†  Matthew J. Patitz†  Robert Schweller†  Scott M. Summers§  Andrew Winslow*
Fruits of intrinsic universality

Intrinsic universality and the computational power of self-assembly

Damien Woods*
The aTAM of Winfree
The aTAM of Winfree

Benchmark problems
The aTAM of Winfree

Benchmark problems

The temperature-1 problem
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation
- The aTAM of Winfree
- The temperature-1 problem
- Benchmark problems
- Universality via weak cooperation
- Handedness

Timeline:
- 1995
- 2000
- 2005
- 2010
- Now
The aTAM of Winfree

Benchmark problems

The temperature-1 problem

Universality via weak cooperation

Handedness

Intrinsic universality

A Brief Tour of Theoretical Tile Self-Assembly

Andrew Winslow

Université Libre de Bruxelles, Brussels, Belgium
awinslow@ulb.ac.be

Abstract. The author gives a brief historical tour of theoretical tile self-assembly via chronological sequence of reports on selected topics in the field. The result is to provide context and motivation for the these results and the field more broadly.
A Brief Tour of Theoretical Tile Self-Assembly

Andrew Winslow

Université Libre de Bruxelles, Brussels, Belgium

awinslow@ulb.ac.be

Abstract. The author gives a brief historical tour of theoretical tile self-assembly via chronological sequence of reports on selected topics in the field. The result is to provide context and motivation for the these results and the field more broadly.