Some Results on Hidden Edge Guards

Sarah Cannon*, Diane L. Souvaine*, Andrew Winslow*
{scanno01,dls,awinslow}@cs.tufts.edu

1 Introduction

We consider guarding the interior of a polygon by selecting edges of the polygon to be *edge guards*. An edge is considered to guard a location in the polygon if there exists a line segment connecting the location to a point in the edge. In contrast to the original study of *closed edge guards* by Avis and Toussaint [1] in which the endpoints are included in the edge, we consider the more recent *open edge guards* suggested by Benbernou et al. [2] in which endpoints are excluded from the edge guard. Intuitively, these guards do not see 'around corners' when they are incident to reflex vertices, unlike closed edge guards.

Here we study guarding polygons using open edge guards under the constraint that no pair of guards may see each other. These *hidden guards* were first considered by Shermer [4] who gave several results, including examples of polygons which are not guardable using hidden vertex guards. The study of hidden edges has only been initiated recently by Kranakis et al. [3] who showed that computing the largest hidden open edge set (ignoring guarding) cannot be approximated within an arbitrarily small constant factor unless P = NP.

We show that there exist simple polygons which do not admit a set of hidden open edge guards sufficient to guard the entire polygon. We also show that the same result holds when restricting the problem to monotone polygons. Finally, we present tight combinatorial bounds on the size of the smallest hidden open edge guard set sufficient to guard a polygon in the case that such a set exists. It should also be mentioned that equivalent results to those presented hold for closed edge guards using signficantly simpler constructions and proofs.

2 Hidden Open Edge Guards

First we consider is whether every polygon is guardable by hidden edge guards. Shermer [4] showed that some polygons are not guardable by hidden vertex guards, but every polygon is guardable by point guards. Here we show that there exist polygons that cannot be guarded by open edge guards. See Figure 1.

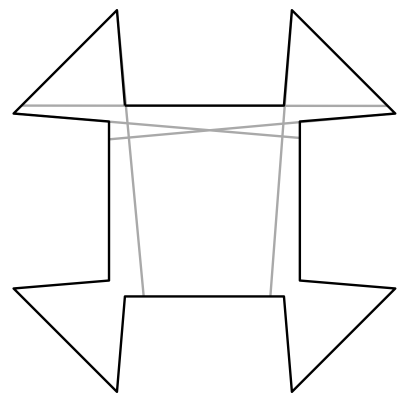

Figure 1: A polygon that cannot be guarded by open hidden edge guards. The grey lines indicate visibility.

Lemma 2.1 *The simple polygon in Figure 1 cannot be guarded using hidden open edge guards.*

Proof Any hidden edge guard set contains at most one edge from the set of four diagonal 'ear edges' in the corners of the polygon and the set of four long edges forming the center square region. We now enumerate the remaining possibilities for guard sets in Figure 2. As seen, no maximal set of hidden open edges is sufficient to guard the entire polygon.

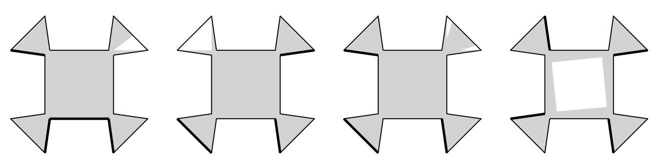

Figure 2: An enumeration of the possible maximal hidden open edge guard sets with symmetric cases omitted. The thick edges represent guard edges.

Now we show that even restricting the problem to monotone polygons is not enough to guarantee the polygon is guardable.

Lemma 2.2 *The monotone polygon in Figure 3 cannot be guarded using hidden open edge guards.*

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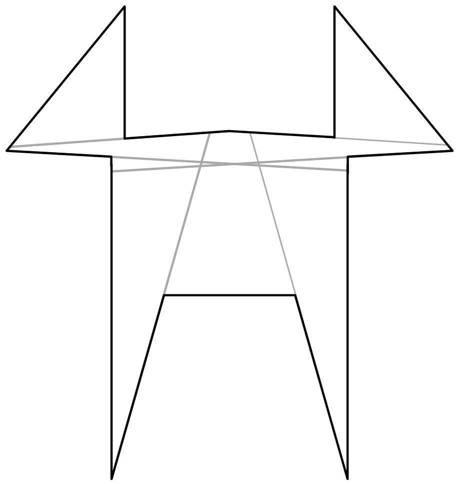

Figure 3: A monotone polygon that cannot be guarded by any set of hidden open edge guards. The grey lines indicate lines of visibility.

Proof First consider guarding the pair of ear regions without using any of the three edges that form each ear as seen in Figure 4. Any maximal combination of non-ear edges fails to guard either ear completely. Additionally, the region in each ear that cannot be seen is not visible from any edge in the other ear. Therefore, one edge per ear is needed to completely guard it.

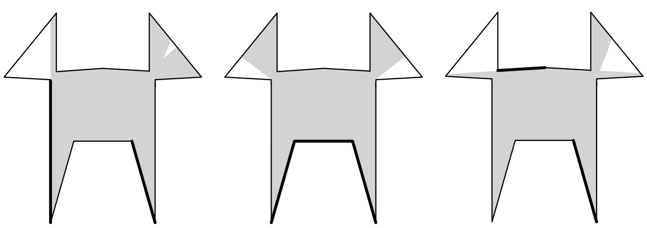

Figure 4: All maximal combinations of open edge guards that exclude the 6 ear edges.

Next, consider combinations of ear edge pairs, one for each ear. In Figure 5 it is shown by enumeration that for each ear edge pair, the pair cannot be augmented to form a hidden open edge guard set for the polygon. Thus the polygon cannot be guarded with hidden open edge guards.

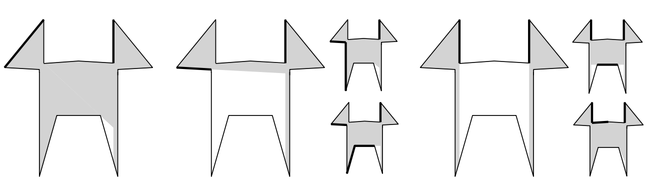

Figure 5: All combinations of ear edge pairs and the possible combinations of edges to finish guarding the polygon.

The lower bound example is seen in Figure 6.

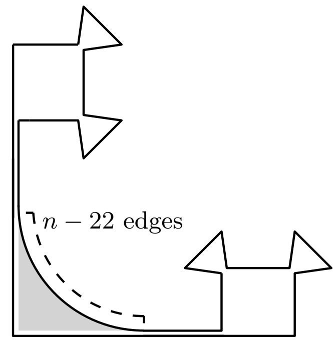

Figure 6: A polygon that admits a minimum hidden open edge guard set of size $n - 22$. The grey region is only guardable with the $n - 22$ edges along the arc.

Lemma 2.3 *There exist polygons for all $n \geq 22$ that admit open hidden edge guard sets and require $n - 22$ hidden open edge guards.*

Now excluding the polygons which cannot be guarded by hidden edge guards, we give bounds on the minimum number of edge guards of each type necessary to guard any polygon.

Lemma 2.4 *Any hidden open edge set has at most $n-2$ edges.*

Proof Two adjacent edges may both be in the same hidden open edge guard set only if their common vertex is reflex. Every polygon has at least three convex vertices, and thus at least two distinct edges must be excluded from the hidden guard set.

References

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