CSCI 3310 Homework: Trees

1 Degree

Problem 1. Prove or disprove that $\forall n \in \{5, 6, \ldots \} \exists$ a tree with $n$ vertices and degree 4.

Problem 2. Prove or disprove that there exists an graph $G = (V, E)$ such that:

- $\forall v \in V$, $1 \leq \deg(v) \leq 2$.
- Exactly three vertices have degree 1.

*Hint: use the Handshaking Lemma.*

Problem 3. Draw a graph $G = (V, E)$ such that:

- $G$ is connected.
- $\Delta(G) \leq 2$.
- $G$ cannot be 2-colored.

Is every such graph $G$ Hamiltonian?

2 (Minimum) Spanning Trees

Problem 4. Find a spanning tree of the graph in Figure 1.

Problem 5. Compute a minimum spanning tree of the graph in Figure 1.

![Figure 1: A weighted undirected graph.](image)

Problem 6. Find a spanning tree of the graph in Figure 1.

Problem 7. Find a minimum spanning tree of the graph in Figure 2.

Problem 8. How many spanning trees does the graph in Figure 3 have?

Problem 9. How many minimum spanning trees does the graph in Figure 3 have?
3 Rooted Trees

Problem 10. Give parent, children, ancestors, descendants, siblings, height, and depth of vertices $d$, $m$ and $h$ in the rooted tree in Figure 4. What is the arity of the tree?

Problem 11. Draw a complete 6-ary tree of height 1.

Problem 12. Draw a full 3-ary tree that is not complete.

Problem 13. Draw a balanced binary tree that is not full.
Problem 14. Draw a full 3-ary tree that is not balanced.

4 Proofs Involving Trees

Problem 15. Write a recurrence relation for the maximum number of leaves in a 4-ary rooted tree of height $h$. Find a closed form for your recurrence relation.

Problem 16. Prove by induction that your closed form for Problem 15 is correct.

Problem 17. Use your solution to Problem 16 to give a closed form for the maximum number of vertices in a 4-ary rooted tree of height $h$. 