CSCI 3310 Homework: Integers

1 Base Conversion

Problem 1. Convert $101001_2$ to ternary, decimal, and hexadecimal.

Problem 2. Convert $120120_3$ to binary, decimal, and hexadecimal.

Problem 3. Convert $35121_6$ to binary, ternary, and decimal.

2 Modular Arithmetic

Problem 4. Give three numbers equal to 5 mod 7.

Problem 5. Compute the following values:
   - $100 \mod 9$
   - $-100 \mod 9$
   - $1000 \mod 77$
   - $-1000 \mod 77$

Problem 6. Write $\{x \in \mathbb{Z} : x \mod 10 < x\}$ in roster notation.

3 Prime Factorizations

Problem 7. Give the canonical prime factorization of 504.

Problem 8. Give the canonical prime factorization of 540.

Problem 9. Give the canonical prime factorization of 208740.

Problem 10. Give the canonical prime factorization of 514395.

4 GCD, LCM, Relative Primality

For the following problems, use the prime factorizations from the previous section - no calculator is needed. Write the solutions as prime factorizations.

Problem 11. Compute gcd(504, 540). Are 504 and 540 relatively prime?

Problem 12. Compute lcm(504, 540).

Problem 13. Compute gcd(208740, 514395). Are 208740 and 514395 relatively prime?

5 Summations

Write the summations without a Σ, i.e., in a closed form.

Problem 15. Write \( \sum_{i=1}^{99} i^2 \).

Problem 16. Write \( \sum_{i=0}^{99} 2^i \).

Problem 17. Write \( \sum_{i=1}^{99} 10^i \).

6 One-Term Recurrence Relations

Problem 18. Give a closed form for \( a_n = a_{n-1} + 10 \) with \( a_1 = 0 \).

Problem 19. Give a closed form for \( a_n = 7a_{n-1} \) with \( a_1 = 4 \).

Problem 20. Give a closed form for \( a_n = -3a_{n-1} \) with \( a_1 = 15 \).

7 Multi-Term Recurrence Relations

Problem 21. Give a closed form for \( a_n = 5a_{n-1} - 6a_{n-2} \) with \( a_0 = 2, a_1 = 5 \).

Problem 22. Give a closed form for \( a_n = 5a_{n-1} - 4a_{n-2} \) with \( a_0 = 3, a_1 = 9 \).

Problem 23. Give a closed form for \( a_n = 5a_{n-1} - 4a_{n-2} \) with \( a_0 = 11, a_1 = 32 \).

Problem 24. Give a closed form for \( a_n = 4a_{n-2} \) with \( a_0 = 2, a_1 = 16 \).